

Keeping Price Indices Representative Despite Constant Market Changes Using Auxiliary Information in Index Estimation Formulae

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Abstract

Compiling price indexes includes many challenges. The CPI (Consumer Price Index) calculation, for example, is performed in different stages, from an elementary aggregate level up to the general index level. This paper is focused on the study of the aggregation structure of the CPI. In particular, it shows how auxiliary information (such as market shares) can be added to the aggregation structure of the CPI in order to enhance the quality of the final estimates, comparing new proposed methodologies of index compilation to the classic ones. Preliminary results confirmed that there are noticeable differences considering indexes by type of store (specialized and non-specialized). Thus this seems to suggest that a new aggregation structure can improve the robustness and the representativeness of the estimates of prices movements. The study is developed through a longer period of time in order to evaluate in the long-term the impact of the use of auxiliary information. The chaining perspective will be also assessed. Moreover the new proposed methodology and the traditional methods are compared to the indexes (defined sub-indexes) computed by type of store.

Key Words: Price index; type of store; aggregation; chained index; CPI

1. Introduction

A price index is a measure of changes in a set of prices over time. In particular, the study presented in this paper was done in the context of a Consumer Price Index (CPI), which measures the rate at which the prices of consumption goods and services change from period to period (International Labour Office et al., 2004). To compile a CPI, a fixed basket of representative commodities is observed over time. The CPI basket is based on expenditures of a given target population (made of both families and individuals) living in private households during a certain reference period. To summarize the evolution of prices over time, a wide group of options is available: the choice of the best price index formula to use has been the object of intensive study and debate in recent years (see, for example, Elliot et al., 2012). Regardless of the formula chosen, the common practice is to average unweighted price movements to obtain elementary aggregate (e.a.) indexes, and then to aggregate the e.a. indexes using an arithmetic mean with expenditure weights.

For example in Canada, most of the e.a. indexes are calculated using the Jevons index, an unweighted geometric mean of the price ratios of representative products within a basic class for two periods, 0 and t , being compared:

$$I_j^{0:t} = \prod_{i=1}^n \left(\frac{p_i^t}{p_i^0} \right)^{1/n}, \quad (1)$$

where $I_j^{0:t}$ is the index of the basic class j between period 0 and t ; p_i^t is the price of representative item i at time t ; p_i^0 is the price of item i at base time 0 and n is the number of representative items in basic class j (the term item refers to a representative product/outlet combination). Once the basic class indexes are obtained, they are aggregated to intermediate levels, then up to the 8 major components of the CPI, and finally to the all-item national CPI. These aggregations are done using a Laspeyres' type index, defined in its general form as:

$$I^{0:t} = \sum_{j=1}^k I_j^{0:t} w_j^0 \quad (2)$$

where w_j^0 is the basket weight (or expenditure shares), of basic class j at time 0. In Canada, the published index uses a slightly modified version, where the weights are price updated and the index is chain-linked to ensure continuity in the series. For more details see Statistics Canada (1995).

As shown in Table 1, the distribution of price quotes by types of store in the CPI sample does not always fully reflect the market shares by types of stores. The market shares in Table 1 are based on Statistics Canada's Quarterly Retail Commodity Survey (QRCS) data. For this project, we define two types of stores: *specialized stores* and *non-specialized stores*. This second group includes, among others, NAICS 452110 (Department stores), 445110 (Supermarkets) and 444110 (Home centers).

Table 1: Distributions of in-sample price quotes and market shares by type of store

Major classes	% Quotes		% Sales	
	Non special. stores	Specialized stores	Non special. stores	Specialized stores
Food/Non-Alcoholic Bev.	97.03	2.97	82.30	17.70
Clothing/Footwear	49.67	50.33	21.72	78.28

In a previous paper (Toninelli et al., 2013), we have explored the use of a different (and more detailed) weighted aggregation structure of the index to evaluate whether it can improve the quality of the final index. In particular, when the survey frame provides information such as industrial classification (NAICS) and size measures (in terms of sales) per commodity types for each location, we studied whether the estimate can better represent the purchases of the target population by using the types of stores' market shares in the aggregation structure. In this study, we observed that the sub-indexes obtained considering the type of outlets are clearly different. These results seem to confirm the initial hypothesis that the introduction of an additional weighted level of aggregation by redefining the elementary aggregate by types of stores would lead to more representative estimates. This is especially true when the sample distribution between specialized and non-specialized stores does not reflect their respective market shares.

In this paper we go further on this topic studying the behavior of the new aggregation structure by computing the different versions of the index over a longer period. The cumulative effect obtained with the chained version of the indexes is also assessed. The methodology used for the study will be explained in section 2. Section 3 will present and discuss the main results and conclusions and some notes on future research will follow in section 4.

2. Methodology

This study was done using a simulated population based on the distributions of the survey data collected monthly from January 2010 to December 2013. The dataset includes prices of a sample of products and services, representative of Canadians consumption. The products and services included in the study cover roughly 55% of the CPI basket. The primary source for weights (expenditure shares) is Statistics Canada's Survey of Household Spending (SHS), whereas the market shares per type of stores are based on the QRCS as described in section 1.

The use of this simulated dataset has some limitations. The various versions of the index were computed to focus on the impact of the new aggregation method. This means that the indexes were estimated with all other things being equal, i.e., even though all versions were computed considering the same dataset, no additional treatment was done on the data (such as out-of-season imputation). Also, as mentioned above, the results obtained represent 55% of the basket, making comparisons with any official index impossible. Another limitation comes from the fact that due to the nature of the market for some major components, the sample for a type of store can be relatively small, limiting the inference that can be done on some of the results. In particular, the major class "Shelter" has a very small portion of its basket in-scope for this study and a very small sample for the specialized stores.

The first step in evaluating the potential impact of using market shares on the CPI calculation is to determine whether the prices from two defined types of stores move the same way or differently. If the two groups' price movements behave the same way, the additional level of aggregation would have a negligible impact. To test this, various versions of the CPI were compiled using the simulated data: *i) the Classic Index* is computed using the "classic" current methodology (Jevons index at the elementary aggregate level, Laspeyres weighted formula to aggregate indexes all the way up to the all-item index); *ii) the Non-specialized sub-Index* is the sub-index computed (with the classic methodology) on non-specialized stores only; *iii) the Specialized sub-Index*: is the sub-index computed with the classic methodology on specialized stores only.

The first results showed that the use of an intermediate level of aggregation using market shares could have an impact on the final estimates. Two methods can be used to add this intermediate level. First, the elementary aggregate can be redefined as the combination of the same group of homogeneous representative products as in the current definition, but divided by type of store. The index for these two new e.a.s are compiled using the Jevons index (see formula (1)) and aggregated using a Laspeyres' index, using type of stores market shares as weights ($w_{j,spec}^0$ for specialized stores and $w_{j,non-s}^0$ for non-specialized stores) rather than the typical expenditure weights. The e.a. index using the intermediate level of aggregation becomes:

$$I_j^{0:t} = I_{j,spec}^{0:t} \cdot w_{j,spec}^0 + I_{j,non-s.}^{0:t} \cdot w_{j,non-s.}^0 \quad (3)$$

The second studied method uses the Jevons index for the same redefined elementary aggregate, but uses a weighted geometric mean to aggregate the two new e.a.s, where market shares are the weights:

$$I_j^{0:t} = \left[(I_{j,spec}^{0:t})^{w_{j,spec}^0} * (I_{j,non-s.}^{0:t})^{w_{j,non-s.}^0} \right]^{1/(w_{j,spec}^0 + w_{j,non-s.}^0)} \quad (4)$$

The following additional versions of the CPI were computed using these two methods and compared to the indexes described above: a) *Final Arithmetic*: a first weighted aggregation is made at the low level (elementary aggregate level) by type of stores, using an arithmetic weighted formula (weights: market shares by type of store); b) *Final Geometric*: the computation of the index introduces a geometric weighted average (weights: market shares by type of store) at the lower level of aggregation. For concision, only the Final Arithmetic index will be shown in the results presented in this paper, and it will be referred as *Final Index*.

3. Results

3.1 Sub-indexes estimates by type of store

The first question we would like to answer in this work is about the behaviour of the sub-indexes' estimates (i.e., the indexes computed for specialized and non-specialized stores) by major class: can we obtain similar results in computing price indexes for the two types of store? Figure 1 shows the differences observed between the sub-indexes by major class. The green bars represent the specialized sub-index, whereas the purple bars represent the non-specialized sub-index; the percentage differences between the two indexes are shown at the base of the second bar.

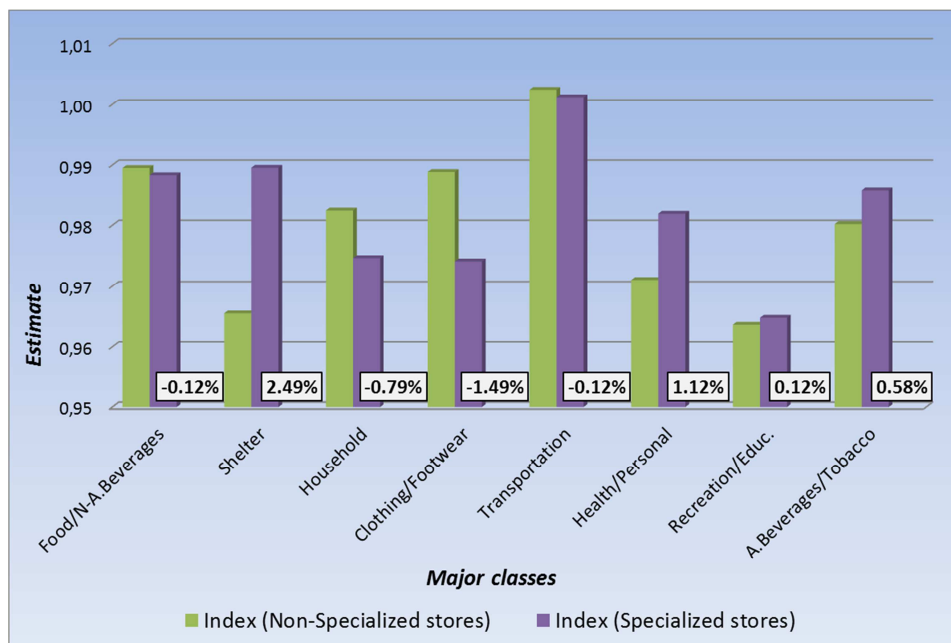


Figure 1: Sub-indexes (Specialized vs Non-specialized) by major classes

The overall analysis of 4 years of data using averages of monthly indexes confirmed what was found considering only one year data (Toninelli et al., 2013): remarkable differences are found between the two sub-indexes by major classes. For some of the major classes the estimates for specialized stores are higher (Shelter, Health/Personal Care, Recreation/Education and Alcoholic Beverages/Tobacco), whereas for the remaining classes the estimates for non-specialized stores are higher (Food/Non-alcoholic Beverages, Household, Clothing/Footwear and Transportation). The biggest differences between the two types of store are observed for Shelter (specialized store estimate is 2.49% higher than the non-specialized store one), Clothing/Footwear (the specialized index is 1.49% smaller) and Health/Personal Care (specialized stores show a higher variation of prices than non-specialized ones: +1.12%).

The same comparison was made in the long run, that is comparing the chained sub-indexes obtained taking into account 4 years of data. This second comparison was done to understand if the sub-indexes' patterns through time are changing randomly (i.e., the differences are balancing out over time), or if the trends are mostly confirmed, reinforcing themselves when considering more than one year data. These results are shown in Figure 2.

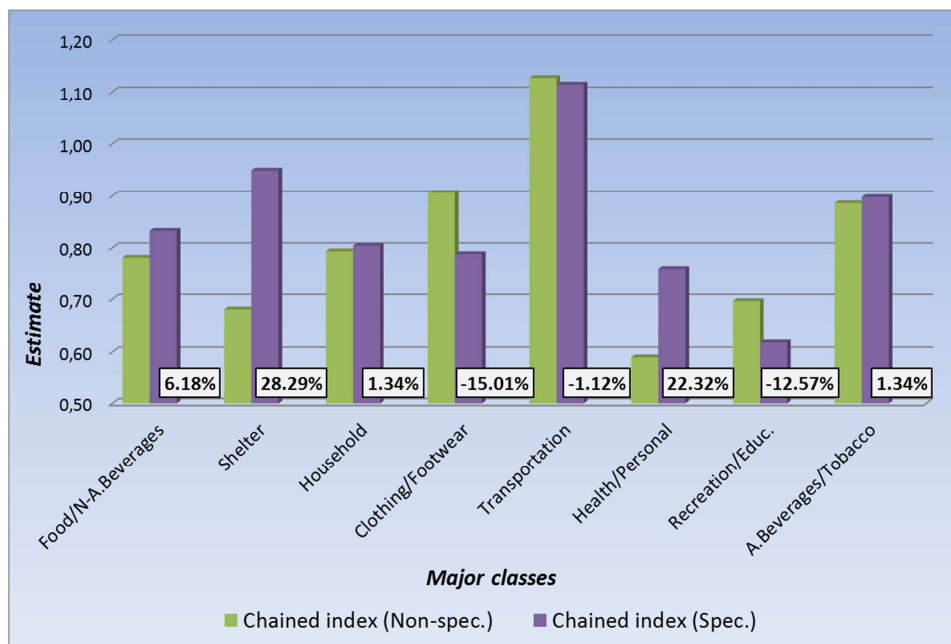


Figure 2: Chained sub-indexes (Specialized and Non-specialized) by major classes

Comparing the percentages of Figure 1 to the ones shown in Figure 2 it becomes clear that, if we consider the chained indexes, the differences between the two sub-indexes are amplified over time. Nevertheless, these differences of Figure 2 do not always confirm the signs of the differences observed for the not-chained indexes (Figure 1), even if this happens for most of the classes (5 out of 8). For Shelter, Clothing/Footwear, Health/Personal Care (and, to a small extent, for Transportation and Alcoholic Beverages/Tobacco) the sign of the differences between the two sub-indexes are confirmed, whereas for Food/Non Alcoholic Beverages, Household and Recreation/Education the signs referred to the chained sub-indexes are inverted. For example, in the Food/Non Alcoholic Beverages major class the chained index show a higher value of the specialized stores index (+6.18%), contrary to what happens with the

“simple” (not-chained) index, where the specialized sub-index is lower (-0.12%). On the contrary, for Clothing/Footwear and, to a certain extent, for Transportation the chained indexes confirm the trends observed on the simple indexes (the specialized indexes are lower than the non-specialized ones).

Overall, the biggest differences between the two types of chained sub-indexes are observed for Shelter and Health/Personal Care (same classes with big differences observed in the not-chained estimates). The specialized stores estimates are higher than the non-specialized ones (the differences are bigger than 22%); on the other side, the specialized index is largely smaller than the non-specialized one for Clothing/Footwear (-15.01%) and for Recreation/Education (-12.57%).

Even if the relative differences between the sub-indexes by major class are not always confirmed over time, it is clear that, considering both the not-chained and the chained indexes, there are differences that can become noticeable between the estimates referred to the specialized and to the non-specialized stores. This further confirms the findings of our previous paper (Toninelli et al., 2013): a different aggregation structure, taking into account the relative importance of the two types of store (specialized/non-specialized), can potentially bring to more representative estimates.

3.2 Classic and Final estimates vs Sub-indexes

In the following step of our analysis, we compare the Classic index (computed without the weighting by type of store) and the Final index (compiled with the additional level of aggregation, i.e. weighting by type of store) estimates with the sub-indexes by type of store. This analysis mainly aims at testing two main prerequisites that both indexes should have. The first one is that even though it is possible to have the aggregated index outside of the limits set by the sub-indexes due to the non-linear nature of the Jevons index, one would expect that a price index stays between the two empirical “boundaries” represented by the sub-indexes by type of stores (specialized and non-specialized). In addition to this, it is expected that the chained version of an index should be coherent with the chained sub-indexes, over time. The second hypothesis pertains the relative importance of the two types of store: an “ideal” index should reflect more closely the pattern of the “prevalent” sub-index, that is the one computed for the type of stores (specialized rather than non-specialized) that is more important within a certain class of goods. These two hypotheses will guide us in understanding if the Classic index and the Final index can be considered “good” estimates of a CPI.

As regards the first prerequisite, in Table 2 the percentage differences between the Classic/Final indexes and the sub-indexes (by kind of store) are shown (considering the estimates by major class).

Table 2: Chained indexes comparison (Classic and Final indexes vs Sub-indexes)

<i>Major classes</i>	<i>Classic index vs</i>		<i>Final (Arithmetic) index vs</i>	
	<i>Non special. ind.</i>	<i>Specialized ind.</i>	<i>Non special. ind.</i>	<i>Specialized ind.</i>
Food/N-A.B.	1.77	-4.53	-4.42	-10.33
Shelter	22.43	-12.20	11.40	-20.12
Household	-1.70	-3.02	-6.30	-7.56
Clothing/F.	-9.38	4.22	-15.23	-2.50
Transport.	-1.30	-0.20	5.89	7.08
Health/P.c.	9.96	-14.58	2.14	-20.65
Recr./Educ.	-10.67	0.56	-19.07	-8.89
A.Bev./Tob.	-2.45	-3.75	-5.20	-6.47

Percentage differences (index vs Sub-indexes)

Observing the table, we notice that the observed differences vary a lot (in both size and direction) at the major class level. In evaluating different alternative index formulas, this is what we really would like to obtain: our ideal index, at the major class level, should be closer to the sub-index of the type of store that is prevalent for that class. Thus, what happens if we compare the Final and the Classic indexes to the sub-indexes? In our following analysis we will take into account the two most relevant classes for this study: Food/Non-Alcoholic Beverages (simply Food, in the following) and Clothing/Footwear (simply Clothing, in the following).

For the Food major class, where the non-specialized type of store is the prevalent category, both the Classic and Final index are closer to the non-specialized sub-index, than to the specialized one, as we were expecting. In fact, the Classic index is only a little bit higher than the specialized one (+1.77%), whereas the difference in comparison to the non-specialized index is more noticeable (-4.53%). On the other side, the Final index is also closer to the specialized index (-4.42%) than to the non-specialized one (-10.33%).

On the other hand, for the Clothing major class, where the specialized type of store is the prevalent category, both the Classic and Final estimates are closer to the specialized sub-index. The Classic index, for example, is closer to the specialized sub-index (+4.22%) than to the non-specialized one (-9.38%). The Final index also reflects better the specialized index (-2.50%) than the non-specialized one (-15.23%).

These results highlight that both the Classic and the Final indexes seem to reflect more closely the most representative sub-index of the different major classes. What is surprising is that the Final index is not always the closest one to the prevalent type of store index (it happens for Clothing, but not for Food, for example); this is what we were expecting, seen the way the Final index is computed.

3.3 Comparison of estimates over time (chained indexes)

Taking into account the big differences observed by major classes, we finally analyze the indexes' patterns through time, considering the two main major classes (Clothing and Food). In this section the Classic and the Final estimates are compared, over time, with the sub-indexes expected "boundaries" taking into consideration the data observed from January 2010 to December 2012.

The results for the Clothing major class, in particular, are shown in Figure 3. The comparison is made by means of chained indexes obtained with what we defined Method 1 (see Toninelli et al., 2013 and Toninelli and Beaulieu, 2013), that is first obtaining the chained time series, and then aggregating them up to obtain the aggregated estimates. Observing the patterns of the four compared chained indexes shown in Figure 3 we can first evaluate if the Classic and Final estimates are able to stay within the borders represented by the two sub-indexes. Analyzing this, we should take into account that for the Clothing major class, the specialized type of store is the prevalent category; thus, we expect to obtain an index that could be closer to the pattern of the Specialized sub-Index, than to the behaviour of the Non-specialized one.

The Classic index seems to be extremely coherent with the sub-indexes boundaries: it almost always falls within the two sub-indexes, and it's just slightly more influenced by the specialized one, at least for the first observations.

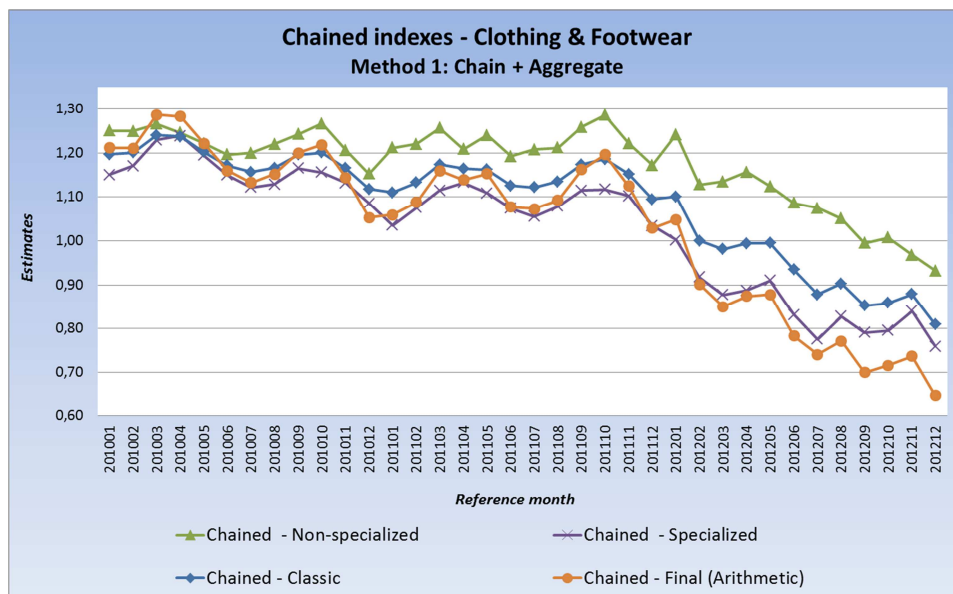


Figure 3: Comparison of chained indexes over time (major class: Clothing/Footwear)

The Final index in general seems to follow closer than the Classic index the pattern of the specialized sub-index until August 2012 (that is for 2 years and 8 months), if we exclude the first five observations and few other exceptions (October 2010 and October 2011, suggesting seasonal kind of outliers). Nevertheless, the movements of the prevalent index seems to be amplified by the Final index, leading to less coherence with the two sub-indexes boundaries: these are already overtaken at the third and fourth observations, and then again on December 2010 and on December 2011 (even if this could be attributed to the closeness to the Specialized sub-index); moreover the Final index breaks the lower “specialized boundary” starting from February 2012, and then never comes back in the interval between the two sub-indexes. In fact, even if the movement from the most prevalent sub-index is stable, the Final index seems to over-amplify any movement in one of its components.

If the Final index seems to be more closely linked to the Specialized sub-index (and sometimes to excessively amplify its pattern), at least in the first part of the observed period, on the other hand the Classic index shows a pattern that is more a balance between the Non-specialized and the Specialized ones; nevertheless, in the last part of the time series, starting from September 2012, it follows more closely the pattern of the reference category sub-index.

These results contradict at least in part the findings of the work discussed in Toninelli et al. (2013), where the Final index showed a pattern more able of staying between the sub-indexes boundaries, whereas the Classic index seemed to be weaker, from this point of view. This all suggests that a deeper study is needed, in order to understand the reasons for this incoherence in the results. At this point it also becomes clear that a comparison with a superlative index is necessary, in order to identify which index can perform better, between the Classic and the Final one.

In Figure 4 the same comparison of Figure 3 is made, focusing on the Food major class (note that the y-axis is set to a different scale in the two figures, for clarity purposes).

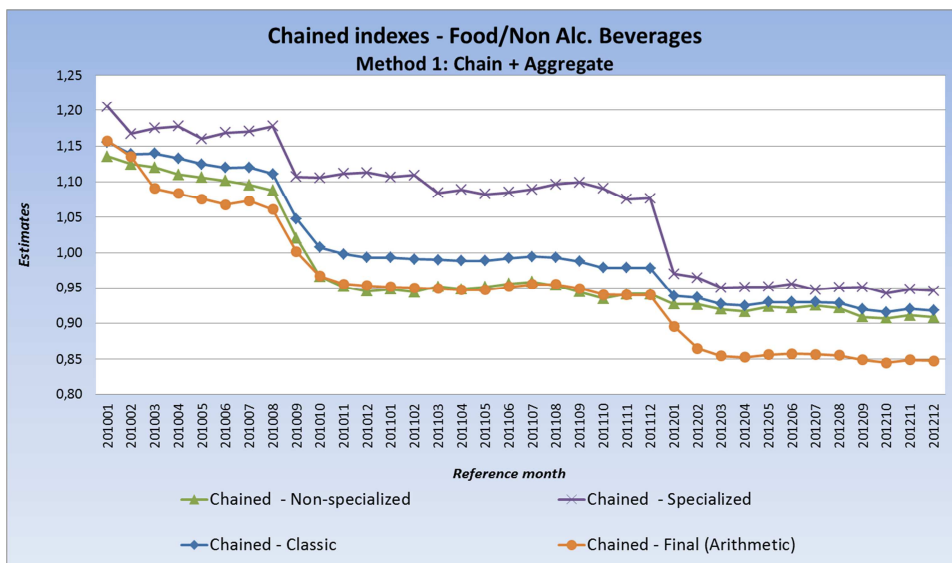


Figure 4: Comparison of chained indexes over time (major class: Food/Non Alc. Bev.)

The evaluation criteria for data shown in Figure 4 are the same highlighted before. We expect that a representative index stays between the two sub-indexes boundaries and that it reflects more closely the pattern of the sub-index representing the prevalent type of store. For Food, the non-specialized stores is the prevalent category, thus our “ideal” index should follow closer this sub-index’s pattern.

From the first point of view, the Final index does not seem to fit the criterion: after the two first observations it goes immediately “out-of-the-boundaries”; then it follows very closely the non-specialized index (from October 2010 to December 2011), and then again it shows very low values that overtake the sub-indexes’ limits.

Considering the second criterion (closeness to the reference sub-index), the Final index works better from September 2010 to December 2011 (when, for most of the observations, the index is overlapping the Non-specialized sub-Index), but then moves away from the sub-indexes from January 2012 to December 2012.

Better results are obtained for both the criteria by the Classic index: it follows more closely the sub-index of the reference category for most of the observations (until August 2010, and then from January 2012 to the end of the series), and it is more able to stay within the two sub-indexes’ boundaries for the whole considered time.

From what is seen in this section, there are clear signs that the new Final index (computed with the Arithmetic weighted average), seems to over-amplify the movements in sub-indexes, in comparison to the Classic index. The multiplicative effect of the chaining seems to make the Final index move further away from the others over a long period of time. Nevertheless the situation changes a lot not only class by class, but also considering the different parts of the analyzed time series: for some of them the Classic index seems to be better, for some others, the Final index can be considered the best choice. However, in general both the indexes seem usually to follow closer the prevalent sub-index; nevertheless, also this behaviour varies a lot class by class and according to the considered time. With these contradicting results, we can only conclude that the Final index, despite expected to be more representative, may not be appropriate for all

components of a CPI, depending on the nature of the products, of the market and on the quality of the auxiliary data available.

4. Conclusions and further research

This paper extends the analysis started by Toninelli et al. (2013) and further discussed in Toninelli and Beaulieu (2013), taking into consideration a longer time interval of data (from January 2010 to December 2013).

The first part of the analysis confirms the previous results: the Specialized and the Non-specialized sub-indexes move very differently. At an average level, higher values are observed for specialized stores, but the situation changes a lot considering the different major classes. In fact, taking into account the “simple” estimate (not-chained indexes), the observed absolute differences ranges from a minimum of 0.12 to a maximum of 2.49%.

The discrepancies between the two sub-indexes are even more emphasized if we consider the chained indexes (computed on all 4 years aggregating the chained time series). It is not clear if the discrepancy between the two sub-indexes is a random phenomenon that over compensate itself over time: in 5 out of 8 major classes the difference (and the direction of the discrepancy) is emphasized by the computation of the chained indexes, whereas in the remaining 3 classes, this doesn't happen. What emerges clearly is, again, that different behaviour and big rather than narrow differences are observed according to the considered classes. This all confirms again, also in the long run, that an index with a new aggregation structure, that considers the weighted average of two sub-indexes by type of store, should increase the representativeness of the estimates of prices' movements over time, taking into account the different behaviour of the two sub-indexes.

In this paper we also compared the Classic and Final index with the reference sub-index, that is with the index that represents the prevalent category in a certain major class. In particular, the major classes of Food (where the non-specialized type of store emerge as prevalent) and Clothing (for which the specialized store is the most important category) were considered. For these classes, both the Classic and the Final index follow more closely the pattern of the prevalent sub-index, than the pattern of the other sub-index. But the relative distances between the two indexes and the reference sub-index and the closeness of one rather than of the other to this reference sub-index vary a lot both by major classes and considering different times in the series. Thus, at this point it's hard to decide which choice is better.

In the last part of the paper we focused the analysis on the two already mentioned main major classes, Food and Clothing. The aim of this part was comparing over time, considering the chained indexes, the Classic and Final indexes with the two sub-indexes referred to the specialized and to the non-specialized stores. In this part, the two main criteria to evaluate the relative strength of the indexes were: 1) how they are able to stay within the “boundaries” represented by the two sub-indexes; 2) how much they are able to follow more closely the reference sub-index, i.e. the sub-index that represents the prevalent type of store in a certain major class. The findings of this analysis are not really conclusive. First, they seem to contradict the previous results obtained with a different dataset (and by means of an analysis focused on a shorter time; see Toninelli et al., 2013, and Toninelli and Beaulieu, 2013). Second, the evaluation of the relative capability of the two indexes in staying between the two sub-indexes vary, not only by major class, but it

is also very different within the same time series: for some part of it, an index can be better, whereas for some other, the other index can perform better. Third, we checked which of the two compared indexes can follow more closely the reference sub-index. Also in this case, the relative closeness to the sub-index of the prevalent type of store varies a lot between the two considered major classes: overall, for Clothing the Final index seems to perform better, at least in the first part of the time series, whereas for Food the two indexes alternate themselves in being closer to the reference category sub-index, and the Classic index seems to perform better in the long run. The main conclusion of this part is that the Final index seems to amplify any movement in the sub-components in the aggregated index.

The findings of this work show that a further, deeper study is needed. In fact, the analysis should be carried on at least at the major class level, because an overall analysis brings to inconclusive results, that can be only seen as the compensation of opposite trends, or that cannot be completely generalized. A bigger dataset, with more data (covering a longer period), would be helpful in obtaining more precise results.

Furthermore, we have to understand the reasons that cause the incoherencies with the first findings of our research, which highlighted better performances of the Final, rather of the Classic indexes: contrary to what we found before, the latter in the long run now seems to be more coherent with the limits defined by the sub-indexes.

Finally, we could reach more general and valid conclusions if we compared the Classic index and the new Final index (with the additional level of aggregation) with a superlative index (i.e., the Fisher or the Törnqvist index). This phase of the analysis is already under development. The comparison with the Fisher “ideal” index can be done considering two strategies: using the Fisher index computed as the geometric average of Laspeyres and Paasche indexes, in their classic versions, or computing these two indexes with an additional level of aggregation, based on the type of stores (specialized/non-specialized). The comparison with the Törnqvist index, in particular, will probably bring to more general and robust results, seen that it averages (by means of a geometric average) the price relatives, weighting them by means of the arithmetic average of the value shares for the two considered periods.

Another interesting further research could be developed taking into account how and how much the different studied indexes can be influenced by the change of the basket effect. Also this part of the analysis is already under development, comparing the 2009 and the 2011 basket structures.

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Disclaimer

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References

- Elliott, D., R. O'Neill, J. Ralph, and R. Sanderson. 2012. Stochastic and Sampling Approaches to the Choice of Elementary Aggregate Formula. Discussion Paper. Office for National Statistics.
- International Labour Office (ILO), IMF, OECD, Eurostat, UNECE, and the World Bank. 2004. Consumer Price Index Manual: Theory and Practice. Geneva, Switzerland: International Labour Office.
- Statistics Canada. 1995. The Consumer Price Index Reference Paper Update based on 1992 Expenditures. Catalogue 62-553. Ottawa, ON: Statistics Canada.
- Toninelli, D., M. Beaulieu. 2013. Using Frame Information to Enhance the Quality of a Price Index. In 2013 International Methodology Symposium proceedings. Ottawa, ON: Statistics Canada.
- Toninelli, D., Z. Patak, and M. Beaulieu. 2013. Enhancing the Quality of Price Index Estimates Combining Updated Weights, a More Representative Sample Design and a Different Aggregation Structure. In JSM Proceedings, Statistical Computing Section. Alexandria, VA: American Statistical Association. 2038-2052.