Optimum Constant-stress and Step-stress Accelerated Life Tests under Time and Cost Constraints

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Abstract
By running life tests at higher stress levels than normal operating conditions, accelerated life testing quickly yields information on the lifetime distribution of a test unit. The lifetime at the design stress is then estimated through extrapolation using a regression model. To conduct an accelerated life test efficiently with constrained resources in practice, several decision variables such as the allocation proportions and stress durations should be determined carefully at the design stage. These decision variables affect not only the experimental cost but also the estimate precision of the lifetime parameters of interest. In this work, under the constraint that the total experimental cost does not exceed a pre-specified budget, the optimal decision variables are determined based on C/D/A-optimality criteria. In particular, the constant-stress and step-stress accelerated life tests are considered with the exponential failure data under time constraint as well. We illustrate the proposed methods using a case study, and under a given budget constraint, the efficiencies of these two stress loading schemes are compared in terms of the ratio of optimal objective functions based on the information matrix.

Key Words: Accelerated life test, Constant-stress test, Cost constrained optimization, Step-stress test, Type-I censoring

1. Introduction

The automated manufacturing systems are widely used in industries, and the evolution of flexible manufacturing systems (FMS) offers increasing flexibility and efficiency of production as well as cost effectiveness. With the global competition in manufacturing environments, planning and decision making process in the field of FMS are ever critical in order to meet higher quality, reliability, and responsiveness to customization while decreasing the total costs. With increasing reliability and substantially long life-spans of products, it is often difficult for standard life testing methods under normal operating conditions to obtain sufficient information about the failure time distribution of the products. This difficulty is overcome by accelerated life test (ALT) where the test units are subjected to higher stress levels than normal for rapid failures. By applying more severe stresses, ALT collects information on the parameters of lifetime distributions more quickly. The lifetime at the design stress is then estimated through extrapolation using a regression model.

In order to conduct ALT efficiently with constrained resources in practice, several decision variables such as the allocation proportions and stress durations should be determined carefully at the design stage. These decision variables affect not only the experimental cost but also the estimate precision of the lifetime parameters of interest. For this reason, the optimal ALT design has attracted great attention in the reliability literature. Miller and Nelson [11] initiated research in this area by considering a simple step-stress model with exponential failure time distribution under complete sampling. The fundamental model used was the one proposed by Sedyakin [15], which is known as the *cumulative exposure model*. This model was further discussed and generalized by Bagdonavicius [1] and Nelson [12]. Bai et al. [2] then extended the results of Miller and Nelson [11] to the time-censored situation. Nelson and Kielpinski [13] studied the optimally censored ALT for normal and lognormal distributions while Schneider [14] considered sampling plans for Weibull distribution using the maximum likelihood estimators (MLE). Khamis [7] compared constant-stress ALT and step-stress ALT under Weibull lifetime distribution for units subjected to stress. Meeter and Meeker [10] then developed the statistical models and ALT plans with a non-constant shape parameter. Later, Seo et al. [16] investigated the optimal ALT sampling plans for deciding the lot acceptability under Weibull distribution with a non-constant shape parameter and Type-I/II censorings. Under complete sampling, Hu et al. [6] studied the statistical equivalency of a simple step-stress ALT to other stress loading designs while Han and Ng [5] compared the efficiencies of general k-level constant-stress and step-stress ALT under complete sampling and Type-I censoring.

The focus of this paper is to investigate the optimal ALT plans under the constraint that the total experimental cost does not exceed a pre-specified budget. In particular, the general k-level constant-stress and step-stress ALT are considered with the exponential lifetime distribution for units subjected to stress under Type-I censoring. Assuming a log-linear relationship between the mean lifetime parameter and stress level, with the accelerated failure time (AFT) model for the effect of changing stress in step-stress ALT, the optimal design variables are determined under various optimality criteria. The proposed methods are illustrated using a case study, and under a given budget constraint, the efficiencies of these two stress loading schemes are compared using the ratio of optimal objective functions based on the information matrix as a measure of relative efficiency.

2. Model Description

Let $x(t) \in [0,1]$ be the given standardized stress loading (a deterministic function of time) for ALT. Then, let us define $0 \equiv x_0 \leq x_1 < x_2 < \ldots < x_k \leq 1$ to be the ordered stress levels used in ALT. It is further assumed that under any specific stress level, the exponential distribution describes the failure mechanism of a test unit. Also, it is assumed that under any stress level x_i, the mean time to failure (MTTF) of a test unit, θ_i, is a log-linear function of stress given by

$$\log \theta_i = \alpha + \beta x_i \tag{1}$$

where the regression parameters α and β need to be estimated. Here we consider two popular classes of ALT: constant-stress and step-stress. In constant-stress testing, a unit is tested at a fixed stress level until failure occurs or the life test is terminated, whichever comes first. On the other hand, (step-up) step-stress testing allows the experimenter to gradually increase the stress levels at some pre-fixed time points during the test.

3. Cost Constrained Optimization

In order to conduct an ALT experiment efficiently with constrained resources in practice, several decision variables such as the allocation proportions and stress durations should be determined carefully at the design stage. It is because these decision variables affect the experimental cost as well as the precision of the parameter estimates of interest. There is a body of literature addressing the model optimization related to certain cost functions. Chien et al. [3] proposed a generalized replacement policy of systems subject to shocks from a non-homogeneous Poisson process, and optimized the model by minimizing the cost rate. Liao et al. [8] investigated a condition-based maintenance policy for continuously degrading systems and determined the optimum maintenance threshold while Zhu et al. [17] obtained the optimal maintenance schedule under a repair cost constraint in terms of the degradation threshold and the time to perform preventive maintenance. Using a mixed integer programming, an optimal progressively censored group acceptance sampling plan was developed by Fernandez et al. [4] via minimizing the expected test cost under several constraints.

Under the constraint that the total experimental cost does not exceed a pre-specified budget, a typical decision problem of interest can be formulated as to optimize (minimize or maximize) an objective function of choice subject to $C_T \leq C_B$, where C_B is the total pre-specified budget and C_T is the total cost for running an ALT. In general, for a life test at a constant stress level x with the sample size n under Type-I censoring at the time point τ, the total cost of test can be expressed in a simplified form as

$$C_T = C_{set} + n\, C_{unit} + C_{op}(x) \min\{Y_{n:n}, \tau\} + C_{ins} \left(\sum_{l=1}^{n_T} Y_{l:n} + (n - n_T)\, \tau\right) + n_T\, C_{fail} + (n - n_T)\, C_{unfail} \tag{2}$$

where $Y_{l:n}$ is the l-th ordered failure time of n units from a lifetime distribution at the stress level x, characterized by, say, the PDF $f_x(t)$ and the CDF $F_x(t) = 1 - S_x(t)$. Here, n_T denotes the total number of units failed until time τ while $n - n_T$ denotes the number of units censored at τ.

Among the non-negative cost parameters in (2), C_{set} denotes the fixed cost for setting up an ALT experiment, which includes the costs of facility and testing chambers. C_{unit} is the cost of each test unit, including the costs of manufacturing, purchasing, and/or installation. $C_{op}(x)$ is the operation cost of conducting an ALT per unit time under the given setup which depends on the applied stress level. Although both C_{set} and $C_{op}(x)$ may increase with the scale of ALT (*e.g.*, A larger n requires a larger facility to accommodate), here we assume that the changes in C_{set} and $C_{op}(x)$ are negligible in a neighborhood of n under the optimal condition, keeping these costs constant and uniform. This is a reasonable assumption as the fixed costs accommodate a range of the sample sizes by absorbing the scaling/sizing effects until it is necessary to require additional resources (*i.e.*, step-wise cost increments). C_{ins} is the cost of inspection or measurement per unit time per test unit. The unit time is assumed in a natural time scale of measurement for convenience although different frequencies can be set for interval inspection in other situations. C_{fail} is the loss

incurred by a failed unit in the inspection, which includes the costs of scrapping and waste management, while C_{unfail} is the loss incurred by an unfailed unit in the inspection, which includes the costs of refurbishing or disintegration.
The most conservative way to protect the completion of an ALT experiment against the case of budget shortfall is to set the constraint that the largest possible total cost (*i.e.*, the worst scenario) does not exceed the pre-specified budget (*viz.*, $\max\{C_T\} \leq C_B$). Under the given time constraint τ, a fixed upper bound of the total experimental cost is

$$\max\{C_T\} = C_{set} + n\,(C_{unit} + C_{fail}) + C_{op}(x)\,\tau + n\,C_{ins}\,\tau \tag{3}$$

with $C_{fail} \geq C_{unfail}$.

4. Optimal Design Criteria

Here, we define different optimality criteria for determining the optimal design points under the cost constraint, which then can be used to compare between the multi-level constant-stress test and step-stress test. For the k-level constant-stress testing, the focus is to determine the optimal allocation proportions $\boldsymbol{\pi}^* = (\pi_1^*,\pi_2^*,\ldots,\pi_k^*)$ with $\pi_k^* = 1 - \sum_{i=1}^{k-1} \pi_i^*$ while it is to determine the optimal stress durations $\boldsymbol{\Delta}^* = (\Delta_1^*,\Delta_2^*,\ldots,\Delta_k^*)$ for the k-level step-stress testing. These objective functions are purely based on the Fisher information matrix $\mathbf{I}_n(\alpha, \beta)$.

4.1 C-Optimality

In an ALT experiment, researchers often wish to estimate the parameters of interest with maximum precision and minimum variability possible. In both the constant-stress and step-stress settings under consideration here, such a parameter of interest is the mean lifetime of a unit at the use-condition (*viz.*, θ_0). For this purpose, we consider an objective function given by

$$\varphi(.) = n\ \mathrm{AVar}(\log \hat{\theta}_0) = n\ \mathrm{AVar}(\hat{\alpha}) \tag{4}$$

where AVar stands for asymptotic variance. The C-optimal design points are the ones that minimize (4).

4.2 D-Optimality

Another optimality criterion often used in planning ALT is based on the determinant of the Fisher information matrix, which equals to the reciprocal of the determinant of the asymptotic variance-covariance matrix. Note that the overall volume of the Wald-type joint confidence region of (α, β) is proportional to $|\mathbf{I}_n^{-1}(\alpha, \beta)|^{1/2}$ at a fixed level of confidence. In other words, it is inversely proportional to $|\mathbf{I}_n(\alpha, \beta)|^{1/2}$, the square root of the determinant of $\mathbf{I}_n(\alpha, \beta)$. Consequently, a larger value of $|\mathbf{I}_n(\alpha, \beta)|$ would correspond to a smaller asymptotic joint confidence ellipsoid of (α, β) and thus a higher joint precision of the estimators of α and β. Motivated by this, our second objective function is simply given by

$$\delta(.) = n^{-2}\,|\mathbf{I}_n(\alpha, \beta)|. \tag{5}$$

The *D*-optimal design points are obtained by maximizing (5) for the maximal joint precision of the estimators of (α, β).

4.3 A-Optimality

The last optimality criterion considered in our study is based on the trace of the first-order approximation of the variance-covariance matrix of the MLEs. It is identical to the sum of the diagonal elements of $\mathbf{I}_n^{-1}(\alpha, \beta)$. The *A*-optimality criterion provides an overall measure of the average variance of the parameter estimates and gives the sum of the eigenvalues of the inverse of the Fisher information matrix. The *A*-optimal design points minimize the objective function defined by

$$a(.) = n\ \mathrm{tr}(\mathbf{I}_n^{-1}(\alpha, \beta)). \tag{6}$$

It is of interest to note that for a simple constant-stress test, the *D*-optimal design allocates an equal number of test units at each stress level regardless of the stress levels used, the presence of Type-I censoring nor the time points of censoring at any stress level. All the optimality criteria considered here, as well as some other information-based criteria, have been used extensively in the design selection process for linearly designed experiments. From a practitioner's point of view, the choice of the optimality criterion will be certainly guided by the objective of the experiment. In cases where the planner is more interested in the precise estimation of the MTTF θ_0 at normal use-condition, the *C*-optimality is surely the criterion of choice. On the other hand, if one is more concerned about estimating the regression parameters α and β with high precision, a more reasonable criterion of choice should be the *D*-optimality or *A*-optimality.

5. Case Study: Rear Suspension Aft Lateral Links ALT

Lu et al. [9] described a step-stress ALT to estimate the reliability of a rear suspension aft lateral link. Four stress (load) levels ranging from 1500 lbs to 3000 lbs with an increment of 500 lbs were used to conduct a pilot study with the sample size of $n = 25$. The normal use-stress and the highest allowable stress were set to be 1500 lbs and 4800 lbs, respectively. Initially, a two-parameter Weibull distribution with a constant shape parameter was assumed to model the lifetime of this product at any stress level. However, the pilot data to estimate these Weibull parameters also supported modeling by an exponential distribution at any stress level. Consistent with our model assumptions, the cumulative exposure model was used to represent the effect of changing stress along with the log-linear parameter-stress assumption in (1). Fitting a regression model to the estimate of MTTF θ_i and the standardized log-stress level x_i, the least square estimates of (α, β) were obtained to be (13.4337, –7.6836). Lu et al. [9] then proposed an eight-level step-stress ALT plan for data collection under Type-I censoring with $n = 12$ test units.

Since a simple step-stress ALT is easier to carry out and its test duration could be shorter by exposing test units to a higher stress level than the original test plan, Hu et al. [6] devised a simple (step-up) step-stress testing plan, which is Type-III Statistically Equivalent (SE) to the original eight-level step-stress ALT. Without loss of estimation precision, the simple SE step-stress ALT plan is to use the standardized log-stress levels of $(x_1, x_2) = (0.52, 1)$ with the stress change time point at 8257. Based on the resulting Fisher information matrix with $n = 12$, the objective functions in (4), (5), and (6) yields φ_{ss} =

11.311, δ_{ss} = 0.057, and a_{ss} = 28.829, respectively. It is also assumed that at an appropriate cost measurement unit, $C_{set}^{ss} = C_{set}^{cs} = 10$, $C_{unit} = 1.0$, $C_{fail} = 0.5$, $C_{unfail} = 0.2$, and $C_{ins} = 0.001$. Also, the operation cost is set to be $C_{op}(x_1) = 0.521$ and $C_{op}(x_2) = 1.001$, which forms a linear function of the standardized stress levels. Then, the SE step-stress ALT has the expected total cost of $E[C_T^{ss}] = 5187.29$ and the upper bound of the total cost at $\max\{C_T^{ss}\} = 12810.66$ with the expected termination time of test $T^{€}_{ss} = 9022.01$.

Table 1: Optimal step durations, objective optima, mean experimental costs, and mean termination times of the simple step-stress test under Type-I censoring

$k = 2$	**Unconstrained** ($C_B = \infty$)			**Constrained** ($C_B = 16000$)		
Step Duration	Δ_C* 13223.34	Δ_D* 8566.20	Δ_A* 10041.75	Δ_C* 10317.15	Δ_D* 8566.20	Δ_A* 10041.75
Optima	φ_{ss}* 10.144	δ_{ss}* 0.057	a_{ss}* 28.300	φ_{ss}* 10.486	δ_{ss}* 0.057	a_{ss}* 28.300
$T^{€}_{ss}$	13852.02	9323.28	10760.59	11028.72	9323.28	10760.59
$E[C_T^{ss}]$	7678.17	5342.96	6085.03	6223.36	5342.96	6085.026
$\max\{C_T^{ss}\}$	20499.05	13289.33	15573.63	15999.98	13289.33	15573.63

Now, with the pre-specified budget constraint at $C_B = 16000$, it is desired to determine the optimal design points under the cost and time constraints for planning a simple ALT experiment, and investigate the relative efficiency of step-stress ALT compared to constant-stress ALT. For the simple step-stress ALT, Table 1 presents the values of the optimal step duration Δ_C*, Δ_D*, Δ_A*, and the corresponding optima of each objective function described in Section 4 with/without the cost constraint. The expected total costs $E[C_T^{ss}]$ and the upper bounds of the total costs $\max\{C_T^{ss}\}$ are also presented in Table 1 along with the expected termination time of test $T^{€}_{ss}$, computed by the formulae in Han and Ng [5]. From Table 1, it is observed that $\Delta_C^* > \Delta_A^* > \Delta_D^*$ in the unconstrained, globally optimal situation. The same order is also followed by $T^{€}_{ss}$, $E[C_T^{ss}]$, and $\max\{C_T^{ss}\}$. Under the cost constraint at $C_B = 16000$, the *D*- and *A*-optimality still yield the globally optimal designs since their largest experimental costs did not exceed C_B at the unconstrained optimal conditions. Only Δ_C* got however reduced considerably in order to meet the cost constraint although its fold change in the corresponding optima is almost negligible; see also Table 3.

Table 2: Optimal allocation proportions, objective optima, mean experimental costs, and mean termination times of the simple constant-stress test under Type-I censoring

$k = 2$	**Unconstrained** ($C_B = \infty$)			**Constrained** ($C_B = 16000$)		
Allocation Proportion	$\boldsymbol{\pi}_C$* (0.703, 0.297)	$\boldsymbol{\pi}_D$* (0.500, 0.500)	$\boldsymbol{\pi}_A$* (0.627, 0.373)	$\boldsymbol{\pi}_C$* (0.718, 0.282)	$\boldsymbol{\pi}_D$* (0.500, 0.500)	$\boldsymbol{\pi}_A$* (0.627, 0.373)
Optima	φ_{cs}* 13.499	δ_{cs}* 0.029	a_{cs}* 40.046	φ_{cs}* 15.006	δ_{cs}* 0.029	a_{cs}* 40.046
$T^{€}_{cs}$	13799.95	9311.31	10682.23	10884.55	9311.31	10682.23
$E[C_T^{cs}]$	7628.32	5304.72	5980.85	6057.23	5304.72	5980.85
$\max\{C_T^{cs}\}$	20340.37	13186.54	15453.13	15876.17	13186.54	15453.13

Using the optimal step durations obtained in Table 1 as the censoring time points at each stress level, the allocation proportions $\boldsymbol{\pi} = (\pi_1,\pi_2)$ were then optimized for a simple sequential constant-stress test under Type-I censoring. Table 2 presents the values of these optimal allocation proportions $\boldsymbol{\pi}_C$*, $\boldsymbol{\pi}_D$*, and $\boldsymbol{\pi}_A$* along with the corresponding optima of each objective function described in Section 4 with/without the cost constraint $C_B = 16000$. Like in Table 1, the expected total costs $E[C_T^{cs}]$ and the upper bounds of the total costs $\max\{C_T^{cs}\}$ are also presented in Table 2 with the expected termination time of test T^{ϵ}_{cs}, computed by the formulae in Han and Ng [5]. From Table 2, it is observed that $\pi_{1,C}^* > \pi_{1,A}^* > \pi_{1,D}^* = 0.5$ in the unconstrained, globally optimal situation. As mentioned in Section 4, the *D*-optimality allocates an equal number of test units at two stress levels. Similar to the results for the simple step-stress ALT in Table 1, T^{ϵ}_{cs}, $E[C_T^{cs}]$, and $\max\{C_T^{cs}\}$ all follow the same order. Since $C_B = 16000$ was exceeded by the maximal cost of the *C*-optimal design only at the unconstrained optimal condition, under the cost constraint, $\boldsymbol{\pi}_C$* alone had to be changed, allocating more test units at the first stress level in order to meet the budget constraint. No changes were made for the *D*- and *A*-optimal designs under the cost constraint.

Table 3: Efficiency of the simple constant-stress and step-stress tests under Type-I censoring with/without the cost constraint

	Optimality		
Efficiency	*C*	*D*	*A*
Unconstrained Constant-stress vs. Type-III SE Step-stress	0.84	0.50	0.72
Unconstrained Step-stress vs. Type-III SE Step-stress	1.12	1.01	1.02
Constrained Constant-stress vs. Type-III SE Step-stress	0.75	0.50	0.72
Constrained Step-stress vs. Type-III SE Step-stress	1.08	1.01	1.02
Constrained Constant-stress vs. Unconstrained Constant-stress	0.90	1.00	1.00
Constrained Step-stress vs. Unconstrained Step-stress	0.97	1.00	1.00
Unconstrained Step-stress vs. Unconstrained Constant-stress	1.33	2.00	1.42
Constrained Step-stress vs. Constrained Constant-stress	1.43	2.00	1.42

Using the results obtained in Tables 1 and 2, Table 3 tabulates the efficiency of the constant-stress ALT and the step-stress ALT under the unconstrained/constrained optimal conditions. Without the cost constraint, the optimal step-stress designs achieve higher efficiency than the simple Type-III SE step-stress plan, especially for the *C*-optimality. The constant-stress designs, however, do not even attain the same efficiency to the Type-III SE plan. Hence, under the cost constraint, the constant-stress *C*-optimal design performs even worse although the other two designs attain the same efficiency of the unconstrained Type-III SE plan, owing to their maximal costs strictly less than C_B. The situation is similar for the constrained step-stress designs. Table 3 also shows that only the *C*-optimal design experiences reduction in efficiency due to introduction of the cost constraint, and that reduction in efficiency is less severe for the step-stress ALT than for the constant-stress ALT. Comparing between the constant-stress and step-stress tests, the highest efficiency is again achieved by the *D*-optimality, followed by the *A*-optimality, and then by the *C*-optimality in general. Under the cost constraint, the step-stress *C*-optimal design shows even higher efficiency to the constant-stress one when compared to the unconstrained, globally optimized condition. Overall from Table 3, the step-stress test is again shown to be more efficient compared to the corresponding constant-stress one in all cases under the unconstrained/constrained optimal situations.

6. Summary

In this work, we investigated the constrained optimal ALT plans subject to the total maximal experimental cost not exceeding a pre-specified budget. In particular, the general *k*-level constant-stress and step-stress ALT were considered with exponential failure time distributions under Type-I censoring. Assuming a log-linear relationship between the mean lifetime parameter and stress level, with the AFT model for the effect of changing stress in step-stress ALT, the MLEs of the regression parameters were derived along with the associated Fisher information. Then, the optimal settings of stress durations and allocation proportions were determined according to the *C*/*D*/*A*-optimality criteria based on the information matrix under a given cost constraint. The proposed methods were illustrated using a case study, and under a given budget constraint, the relative efficiencies of the two stress loading schemes under consideration were measured in terms of ratios of the optima in each criterion. Regardless of the stress loadings, the *C*-optimal design was generally found to take the longest to complete the test and hence, cost the most. It was also the most severely affected by a given cost constraint, taking heavy reduction in its efficiency, while the *D*-optimal design was the least affected. The results of a further numerical study quantified the advantage of using step-stress ALT compared to constant-stress one. The step-stress tests were demonstrated to be overall more efficient compared to the corresponding constant-stress tests under the unconstrained/constrained optimal situations.

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