A New Modified Trimmed Mean for Estimating Confidence Interval of Mean for Skewed Populations

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Abstract

In this paper, we propose a new modified trimmed mean for the point and confidence interval (CI) estimate of the mean of skewed populations. The performances of the proposed estimates have been compared using real life examples and a simulation study with the Student's *t*, mad *t*, median *t* and trimmed *t* for varying levels of skewness in the populations. From the results of examples and simulation study, it appears that with skewed distribution, the proposed trimmed *t* and modified trimmed *t* CIs are as good as mad *t* or median *t* CIs in coverage probability consideration. With lower percent trimmed, trimmed and modified trimmed *t* CIs are identical or close to the Student's *t* CI, and with increased percentage trimmed, they are identical or close to the median *t* CI.

Key Words: Student's *t*, mad *t*, median *t*, modified trimmed mean**,** confidence interval, coverage probability.

1. Introduction

Let $X_1, X_2, \dots, X_n$ be a random sample from any skewed distribution with mean μ and standard deviation σ. Given the sample, we wish to find the confidence interval (CI) for μ when the population standard deviation σ is unknown. The sample mean $\bar{X}$ of a random sample for any population with mean μ and standard deviation σ is approximately distributed as normal with a mean μ and standard deviation $\sigma/\sqrt{n}$, provided n is large. Therefore, when σ is known, the statistic $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ follows a standard normal distribution. As such, a $100(1-\alpha)\%$ CI for μ is given by

$$\left[\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$$

where, $z_{\alpha/2}$ is the upper $(\alpha/2)th$ percentile of the standard normal distribution.

In real life, however, it is unlikely that σ is known. Then, an estimate of σ given by the sample standard deviation $s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2}$ is used to compute various t confidence intervals. Among various modifications, student's *t* (Student, 1908) CI is the most efficient and useful at normal models. Johnson (1978), proposed a modification of the Student's *t* CI for skewed distributions. Since Johnson (1978), Kleijnen et al. (1986), Meeden (1999), Willink (2005), Kibria (2006), Shi and Kibria (2007) are a few to mention who proposed several modifications. In this article, we proposed two methods of CIs for the mean of skewed populations.

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The organization of the remaining paper is as follows. Student's t and various modifications appear in section 2. The proposed new methods of CIs are given in section 3. A simulation study has been carried out in section 4 in order to compare performance of commonly used CIs along with the proposed methods. Some real life examples have been provided in section 5 to demonstrate application of the new method in relation to the other methods. Finally, a conclusion is given in section 6.

2. Various *t* CIs

In this section we consider various versions of t CIs that are in practice when the population standard deviation σ is unknown.

2.1 Student's *t* CI

When the sample size n is small, the $100(1-\alpha)\%$ CI for μ is due to Student (1908) and is given by

$$\left[\bar{X} - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}, \bar{X} + t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}\right]$$

where $t_{\alpha/2,n-1}$ is the upper $\alpha/2$ percentage point of the Student's t distribution with $(n-1)$ degrees of freedom. This CI is the most popular CI in literature and is omnipresent in statistical practice for making inference due to the efficiency of the method at normal models. However, it is well known that when the population the sample comes from is skewed, Student's t CI has poor coverage probability.

2.2 Johnson's *t* CI

When the sample size n is small and population distribution is non-normal or skewed, the Student's t CI has poor coverage probability. Johnson (1978) proposed the following CI for mean μ for a skewed distribution:

$$[\bar{X} + (\hat{\mu}_3/6s^2n)] \mp t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}$$

where $\hat{\mu}_3 = \frac{n}{(n-1)(n-2)}\sum_{i=1}^{n}(X_i - \bar{X})^3$ is the unbiased estimator of the third central moment μ_3.

It appears in literature (see for example, Kibria, 2006) that the width of Student's t and Johnson's t are same.

2.3 Median *t* CI

It is well known that $\bar{X}$ is preferable to other estimators of center for a distribution that is symmetric or relatively homogeneous. When the distribution is skewed or non-normal, the sample median describes the center of the distribution better than that of the mean. Therefore, for a skewed distribution, it is reasonable to define the standard deviation in terms of the median than the mean (Kibria, 2006). Then, a CI for μ has been computed by

$$\left[\bar{X} - t_{\alpha/2,n-1}\frac{\tilde{s}_1}{\sqrt{n}}, \bar{X} + t_{\alpha/2,n-1}\frac{\tilde{s}_1}{\sqrt{n}}\right]$$

where

$\tilde{s}_1 = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \tilde{x})^2}$ and

$\tilde{x}$ is the sample median. This CI is referred to as median t CI.

2.4 Mad t CI

Kibria (2006) proposed another t CI which has been referred to as mad t CI.

A $100(1-\alpha)\%$ mad t CI for μ is given by

$$\left[\bar{X} - t_{\alpha/2,n-1}\frac{\tilde{s}_2}{\sqrt{n}}, \bar{X} + t_{\alpha/2,n-1}\frac{\tilde{s}_2}{\sqrt{n}}\right]$$

where

$\tilde{s}_2 = \frac{1}{n}\sum_{i=1}^{n}|X_i - \tilde{x}|$ is the sample mean absolute deviation (MAD).

The Median t and Mad t CIs are ad-hoc types of CIs of μ for skewed distribution, which have also been considered by Shi and Kibria (2007). They explained the merits of these CIs in comparison with Johnson's t interval by simulation study and examples.

3. New proposed t CIs

In between mean and median, the trimmed mean is a more robust measure for describing the center than the mean and more efficient than the median. For a skewed distribution with a longer left or right tail, it seems reasonable to define the standard deviation in terms of the trimmed mean rather than mean or median. Therefore, we propose a modification of the Students' t CI given by

$$\left[\bar{X} - t_{\alpha/2,n-1}\frac{s_1}{\sqrt{n}}, \bar{X} + t_{\alpha/2,n-1}\frac{s_1}{\sqrt{n}}\right] \quad (1)$$

where $s_1 = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}_{(p)}\right)^2}$ and $\bar{X}_{(p)}$ is the trimmed mean with $p\%$ data values in both tails trimmed.

Another $100(1-\alpha)\%$ t CI for μ is given by

$$\left[\bar{X} - t_{\alpha/2,n-1}\frac{s_2}{\sqrt{n}}, \bar{X} + t_{\alpha/2,n-1}\frac{s_2}{\sqrt{n}}\right] \quad (2)$$

where

$$s_2 = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \hat{\mu})^2}$$

$$\hat{\mu} = \begin{cases} \bar{X} \ if \ X_{[np]} < \bar{X} < X_{[n(1-p)]} \\ \bar{X}_{(p)} \ other\ wise \end{cases}$$

We refer the two CIs in (1) and (2) as trimmed t and modified trimmed t confidence intervals, respectively. These are ad-hoc types of CIs of μ for skewed distribution. In this article, we assess their performance by example and simulation.

4. Simulation and Result Discussion

In this section, we carry out a simulation study to compare the finite sample performance of the various CIs described in this article. All simulations are performed using the statistical software R (2009). The sample X is simulated from $G(\alpha, \beta)$ population, where α is the shape parameter and β is the scale parameter. Note that the skewness of $G(\alpha, \beta)$ distribution is $\gamma_1 = 2/\sqrt{\alpha}$. In simulations, we choose different values of the parameter α to allow varying levels of skewness of the simulated samples, and the population mean is fixed at 1. In all simulations, the Monte Carlo size is 2,500, chosen arbitrarily. The coverage probability of various CIs is estimated from the proportion of CIs containing the true mean 1 over all MC simulations. Table 1 below provides the characteristics of various population models used in the simulation study.

Table 1 Values of α, β and γ_1 used in simulations of X

Models	α	β	γ_1	mean
M1	16	0.0625	0.5	1
M2	4	0.25	1	1
M3	1	1	2	1
M4	0.25	4	4	1

The performances of the simulations in terms of coverage probability are reported in Tables 3-6. The simulation results suggest that when the skewness is 0.5, all methods perform reasonably well in that the coverage probability is very close to 0.95. As reported in Table 3, the mad *t* CI has the lowest coverage probability with coverage probability of 95% CI ranging from 0.86 to 0.90. The coverage probability of 95% CI ranges from 0.93 to 0.95 for all other CIs.

As reported in Table 6, again, the mad *t* CI has the lowest coverage probability with coverage probability of 95% CI ranging from 0.68 to 0.78. The coverage probabilities of 95% CIs range from 0.73 to 0.91 for *t* CI, 0.75 to 0.92 for med *t* CI, 0.72 to 0.94 for trimmed *t* CI and 0.74 to 0.94 for modified trimmed *t* CI. Clearly, these results suggest that trimmed *t* and modified *t* CI retain the efficiency of Students' *t* CI and the robustness of median *t* CI. The coverage probability is sensitive to the sample size in that as sample size increases, the coverage probability of all methods increases. For larger sample size ($n \geq 30$), the minimum and maximum performances of all the CIs derived from the simulation results of Tables 3-6 with varying skewness and % trimming is presented in Table 7. From the results of Table 7, it follows that higher skewness results in lower coverage probability for all CI methods.

5. Examples

In this section, we provide some real life examples in order to illustrate and compare performance of the proposed CIs in relation to the existing popular alternatives when the samples are assumed to come from negative and positive skewed distributions.

Example 1: Individuals with phenylketonuria (PKU) disorder are unable to metabolize the protein phenylalanine. In medical research, it has been suggested that an elevated level of serum phenylalanine increases a child's likelihood of mental deficiency. The normalized mental age (nMA) score (in months) of a sample of 18 children is considered below from a population of children with high exposure of PKU disorder in order to assess the extent of their mental deficiency (see Wrona, R.M., 1979).

28, 35, 37, 37, 43.5, 44, 45.5, 46, 48, 48.3, 48.7, 51, 52, 53, 53, 54, 54, 55

We are interested to determine the 95% CI of mean normalized mental age score of children with high form of phenylketonuria.

From the histogram and boxplot in Figure 1 of the sample nMA score, it appears the population the sample comes from is a negatively skewed population. The sample mean and the sample skewness of this data are 46.3 and -0.98, respectively. From the *t* test (t = 0.1536, df = 17, p-value = 0.8797) and Wilcoxon signed rank test (w = 83.5, p-value = 0.7581), it is evident that the population data has the mean $\mu = 46$ months.

The 95% CIs together with the length of the corresponding CIs for this example are reported in Table 1.

Figure 1: Histogram and boxplot of the normalized mental age (nMA) score (in months) for the sample of children with higher form of phenylketonuria.

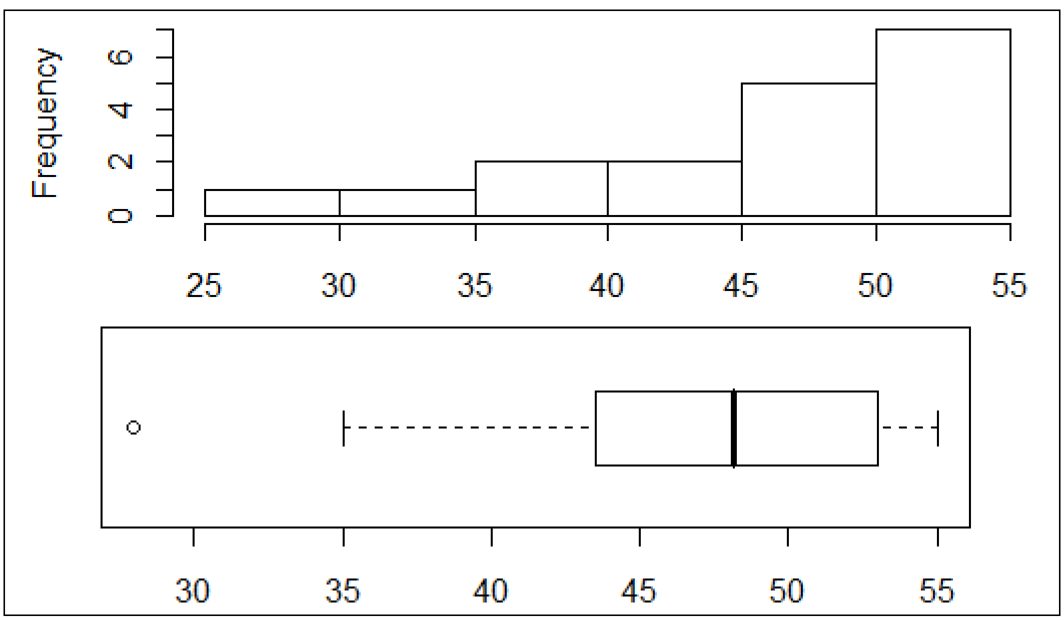

Table 1: The 95% CIs with corresponding lengths for Example 1

% trimmed	Methods	CI	Length
	Student's *t*	(42.46,50.09)	7.63
	Median *t*	(42.34,50.21)	7.87
	Mad *t*	(43.28,49.27)	5.99
5% trimmed	Trimmed *t*	(42.46,50.09)	7.63
	Modified trimmed *t*	(42.46,50.09)	7.63
10% trimmed	Trimmed *t*	(42.45,50.11)	7.66
	Modified trimmed *t*	(42.46,50.09)	7.63
20% trimmed	Trimmed *t*	(42.41,50.14)	7.73
	Modified trimmed *t*	(42.46,50.09)	7.63
25% trimmed	Trimmed *t*	(42.36,50.19)	7.83
	Modified trimmed *t*	(42.46,50.09)	7.63
50% trimmed	Trimmed *t*	(42.34,50.21)	7.83
	Modified trimmed *t*	(42.34,50.21)	7.63

As we see from the 95% CIs with 10% trimmed, all methods have captured the hypothesized mean $\mu = 46$. Lengthwise, mad *t* has the shortest length. The student's *t* and modified trimmed *t* have the second shortest length (7.63), following trimmed *t* and the median *t*, in order, respectively. By increasing the % trimmed, trimmed and modified trimmed *t* CIs approach to med *t* CI. Modified trimmed *t* CI retains the efficiency of Student's *t* and robustness of median *t* CIs.

Example 2: A sample of size 20 is considered from the population of the number of days past presidents of the United States served in the office for the 43 Presidents as of 4 February 2004 (see Hayden, 2005). So the population has 43 data points with mean $\mu = 1824$ days and skewness=0.55. Therefore, the population is positively skewed. The sample data points are as follows:

2921, 1036, 2921, 1460, 1460, 2810, 1460, 881, 1418, 2810,
1460, 1460, 199, 1503, 1110, 1418, 1461, 2921, 1460, 2039

From the sample, the point estimates of mean and skewness are 1710 days and 0.42, respectively. The histogram and boxplot in Figure 2 suggest that the sample comes from the population that is positively skewed.

The 95% CIs together with the length of the corresponding CIs for this example are reported in Table 2.

Figure 2: Histogram and boxplot of the number of days US presidents served in the office in the sample.

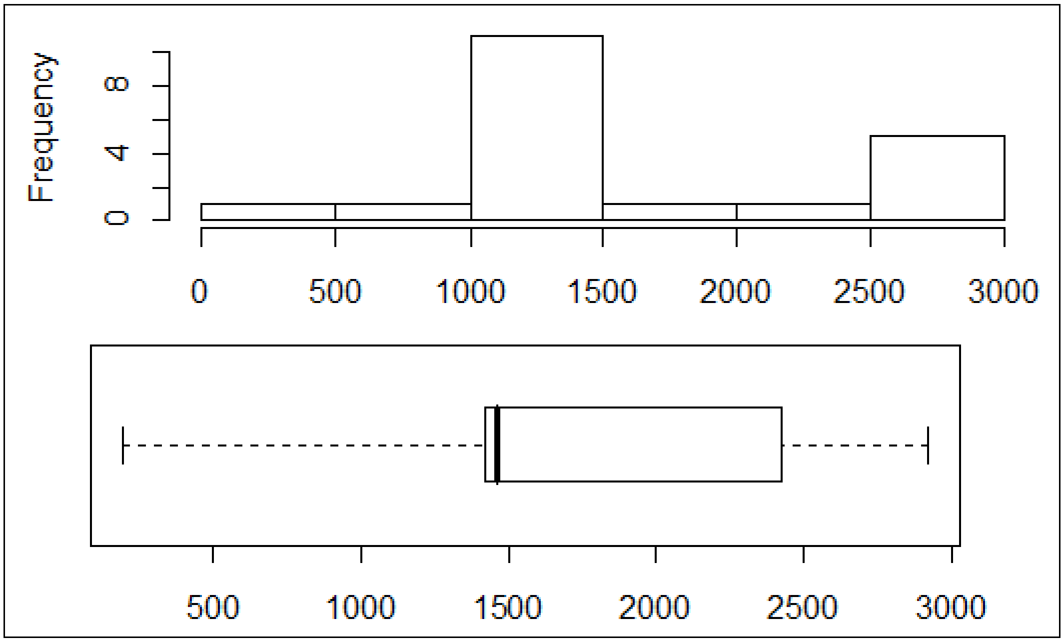

Table 2: The 95% CIs with corresponding lengths for Example 2

% trimmed	Methods	CI	Length
	Student's *t*	(1348, 2072)	724
	Median *t*	(1329, 2092)	763
	Mad *t*	(1422, 1999)	577
5% trimmed	Trimmed *t*	(1348, 2072)	724
	Modified trimmed *t*	(1348, 2072)	724
10% trimmed	Trimmed *t*	(1348, 2072)	724
	Modified trimmed *t*	(1348, 2072)	724
20% trimmed	Trimmed *t*	(1346, 2075)	729
	Modified trimmed *t*	(1348, 2072)	724
25% trimmed	Trimmed *t*	(1337, 2084)	747
	Modified trimmed *t*	(1348, 2072)	724
50% trimmed	Trimmed *t*	(1329, 2092)	763
	Modified trimmed *t*	(1329, 2092)	763

On the basis of estimates, 95% CIs of all methods have captured the population mean $\mu = 1824$ days. Lengthwise, mad *t* has the shortest length. With lower % trimmed (5% and 10%), the trimmed t and modified trimmed *t* CIs are identical to the Student's *t* CI. By increasing the % trimmed from 25% to 50%, trimmed and modified trimmed *t* CIs approach the med *t* CI. Overall, the modified trimmed *t* CI retains the efficiency of Student's *t* and robustness of median *t* CIs.

6. Conclusion

If population distribution the sample comes from is normal, then Student's *t* CI is the most preferable to other alternative CIs in coverage probability consideration. With skewed distribution, the proposed trimmed *t* and modified trimmed *t* CIs are as good as Mad *t* or Median *t* CIs in coverage probability consideration. With lower % trimmed, trimmed and modified trimmed *t* CI are identical or close to the Student's *t* CI, and with increased % trimmed, they are identical or close to the median *t* CI. It also follows that as the level of skewness increases, the coverage probability seems to be under estimated with all the methods with trimmed *t*, modified trimmed *t* and median *t* performing equivalently. It follows from the simulation and example results that the modified trimmed *t* CI retains the efficiency of Student's *t* CI and robustness of the median *t* CI as long as coverage probability is concerned. Therefore, a modified trimmed *t* CI may be recommended to use when there is any doubt of skewness in the population the sample comes from.

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Table 3: Coverage probability of 95% CIs when skewness=0.50

	5% trimmed*					10% trimmed				
n	tci	medtci	madtci	tmci	mtmci	tci	medtci	madtci	tmci	mtmci
5	0.94	0.94	0.89	0.94	0.94	0.94	0.94	0.88	0.94	0.94
6	0.93	0.93	0.88	0.93	0.93	0.93	0.93	0.87	0.93	0.93
7	0.93	0.93	0.86	0.93	0.93	0.93	0.94	0.86	0.93	0.93
8	0.93	0.94	0.87	0.93	0.93	0.94	0.95	0.88	0.94	0.94
9	0.93	0.94	0.86	0.93	0.93	0.93	0.94	0.87	0.93	0.93
10	0.93	0.94	0.87	0.93	0.93	0.94	0.94	0.87	0.94	0.94
11	0.93	0.94	0.86	0.93	0.93	0.94	0.94	0.87	0.94	0.94
12	0.94	0.95	0.88	0.94	0.94	0.94	0.94	0.88	0.94	0.94
13	0.93	0.94	0.87	0.93	0.93	0.94	0.95	0.88	0.94	0.94
14	0.94	0.95	0.86	0.94	0.94	0.94	0.94	0.87	0.94	0.94
15	0.95	0.95	0.88	0.95	0.95	0.94	0.95	0.88	0.94	0.94
20	0.94	0.94	0.87	0.94	0.94	0.94	0.95	0.87	0.94	0.94
25	0.95	0.95	0.88	0.95	0.95	0.95	0.95	0.89	0.95	0.95
30	0.95	0.95	0.88	0.95	0.95	0.95	0.95	0.88	0.95	0.95
35	0.94	0.95	0.88	0.94	0.94	0.94	0.95	0.88	0.95	0.94
40	0.95	0.95	0.88	0.95	0.95	0.94	0.95	0.87	0.94	0.94
45	0.95	0.95	0.87	0.95	0.95	0.95	0.95	0.88	0.95	0.95
50	0.94	0.95	0.88	0.95	0.94	0.95	0.95	0.88	0.95	0.95
	20% trimmed					25% trimmed				
5	0.93	0.93	0.87	0.93	0.93	0.95	0.95	0.90	0.95	0.95
6	0.94	0.94	0.88	0.94	0.94	0.95	0.95	0.89	0.95	0.95
7	0.94	0.95	0.88	0.94	0.94	0.95	0.95	0.89	0.95	0.95
8	0.94	0.94	0.88	0.94	0.94	0.95	0.96	0.89	0.95	0.95
9	0.94	0.94	0.87	0.94	0.94	0.95	0.96	0.89	0.95	0.95
10	0.95	0.95	0.89	0.95	0.95	0.95	0.95	0.89	0.95	0.95
11	0.94	0.95	0.87	0.94	0.94	0.94	0.94	0.87	0.94	0.94
12	0.94	0.94	0.87	0.94	0.94	0.94	0.94	0.88	0.94	0.94
13	0.94	0.94	0.88	0.94	0.94	0.95	0.95	0.88	0.95	0.95
14	0.93	0.94	0.87	0.93	0.93	0.94	0.95	0.88	0.95	0.94
15	0.94	0.95	0.87	0.94	0.94	0.95	0.95	0.90	0.95	0.95
20	0.94	0.95	0.87	0.95	0.94	0.95	0.95	0.88	0.95	0.95
25	0.93	0.94	0.87	0.94	0.93	0.95	0.95	0.88	0.95	0.95
30	0.95	0.95	0.87	0.95	0.95	0.95	0.95	0.88	0.95	0.95
35	0.94	0.94	0.87	0.94	0.94	0.94	0.94	0.87	0.94	0.94
40	0.95	0.95	0.88	0.95	0.95	0.95	0.95	0.88	0.95	0.95
45	0.94	0.94	0.86	0.94	0.94	0.95	0.95	0.87	0.95	0.95
50	0.94	0.95	0.88	0.95	0.94	0.94	0.95	0.88	0.95	0.94

* In all tables, where applicable, tci, madtci, medtci, tmci and mtmci refer to Students' *t*, Mad *t*, Median *t*, trimmed *t* and modified trimmed *t* CIs, respectively.

Table 4: Coverage probability of 95% CIs when skewness=1

	5% trimmed					10% trimmed				
n	tci	medtci	madtci	tmci	mtmci	tci	medtci	madtci	tmci	mtmci
5	0.93	0.94	0.88	0.93	0.93	0.93	0.94	0.87	0.93	0.93
6	0.94	0.94	0.88	0.94	0.94	0.93	0.94	0.87	0.93	0.93
7	0.93	0.94	0.87	0.93	0.93	0.94	0.94	0.88	0.94	0.94
8	0.93	0.93	0.87	0.93	0.93	0.94	0.94	0.87	0.94	0.94
9	0.94	0.94	0.88	0.94	0.94	0.94	0.94	0.87	0.94	0.94
10	0.94	0.95	0.88	0.94	0.94	0.94	0.94	0.87	0.94	0.94
11	0.93	0.93	0.88	0.93	0.93	0.95	0.95	0.88	0.95	0.95
12	0.94	0.94	0.88	0.94	0.94	0.94	0.95	0.87	0.94	0.94
13	0.95	0.95	0.89	0.95	0.95	0.94	0.94	0.86	0.94	0.94
14	0.93	0.94	0.87	0.93	0.93	0.94	0.95	0.87	0.94	0.94
15	0.94	0.95	0.88	0.94	0.94	0.95	0.95	0.89	0.95	0.95
20	0.94	0.95	0.88	0.94	0.94	0.94	0.95	0.87	0.95	0.94
25	0.94	0.94	0.88	0.94	0.94	0.95	0.95	0.87	0.95	0.95
30	0.94	0.95	0.88	0.94	0.94	0.94	0.95	0.87	0.94	0.94
35	0.94	0.95	0.87	0.94	0.94	0.94	0.95	0.87	0.95	0.94
40	0.94	0.94	0.87	0.94	0.94	0.95	0.95	0.87	0.95	0.95
45	0.94	0.94	0.86	0.94	0.94	0.95	0.95	0.87	0.95	0.95
50	0.95	0.95	0.88	0.95	0.95	0.95	0.96	0.88	0.95	0.95
	20% trimmed					25% trimmed				
5	0.93	0.94	0.88	0.94	0.93	0.94	0.95	0.88	0.94	0.94
6	0.93	0.94	0.88	0.94	0.93	0.94	0.95	0.89	0.94	0.94
7	0.93	0.93	0.87	0.93	0.93	0.93	0.94	0.87	0.93	0.93
8	0.93	0.94	0.88	0.94	0.93	0.93	0.94	0.88	0.94	0.93
9	0.94	0.95	0.88	0.94	0.94	0.93	0.93	0.86	0.93	0.93
10	0.94	0.95	0.89	0.95	0.94	0.94	0.94	0.88	0.94	0.94
11	0.93	0.94	0.88	0.94	0.93	0.94	0.95	0.87	0.94	0.94
12	0.94	0.94	0.88	0.94	0.94	0.94	0.94	0.87	0.94	0.94
13	0.95	0.96	0.88	0.95	0.95	0.94	0.95	0.87	0.95	0.94
14	0.94	0.94	0.87	0.94	0.94	0.94	0.94	0.87	0.94	0.94
15	0.93	0.94	0.88	0.94	0.93	0.94	0.94	0.87	0.94	0.94
20	0.93	0.94	0.87	0.93	0.93	0.95	0.95	0.88	0.95	0.95
25	0.95	0.95	0.88	0.95	0.95	0.95	0.95	0.88	0.95	0.95
30	0.94	0.95	0.87	0.94	0.94	0.94	0.94	0.86	0.94	0.94
35	0.95	0.95	0.87	0.95	0.95	0.95	0.95	0.88	0.95	0.95
40	0.95	0.96	0.89	0.95	0.95	0.95	0.95	0.88	0.95	0.95
45	0.95	0.95	0.87	0.95	0.95	0.95	0.95	0.88	0.95	0.95
50	0.94	0.95	0.87	0.95	0.94	0.95	0.95	0.88	0.95	0.95

Table 5: Coverage probability of 95% CIs when skewness=2

	5% trimmed					10% trimmed				
n	tci	medtci	madtci	tmci	mtmci	tci	medtci	madtci	tmci	mtmci
5	0.89	0.90	0.84	0.89	0.89	0.88	0.89	0.83	0.88	0.88
6	0.89	0.90	0.83	0.89	0.89	0.89	0.90	0.83	0.89	0.89
7	0.88	0.89	0.82	0.88	0.88	0.89	0.90	0.82	0.89	0.89
8	0.89	0.90	0.83	0.89	0.89	0.89	0.90	0.83	0.89	0.89
9	0.89	0.90	0.82	0.89	0.89	0.90	0.90	0.83	0.90	0.90
10	0.90	0.90	0.84	0.90	0.90	0.90	0.91	0.83	0.90	0.90
11	0.90	0.91	0.84	0.90	0.90	0.90	0.91	0.84	0.90	0.90
12	0.90	0.91	0.83	0.90	0.90	0.90	0.91	0.84	0.90	0.90
13	0.91	0.92	0.84	0.91	0.91	0.91	0.92	0.85	0.91	0.91
14	0.91	0.92	0.85	0.91	0.91	0.90	0.91	0.84	0.90	0.90
15	0.91	0.92	0.85	0.91	0.91	0.91	0.91	0.84	0.91	0.91
20	0.92	0.93	0.85	0.92	0.92	0.92	0.92	0.84	0.92	0.92
25	0.92	0.93	0.83	0.92	0.92	0.93	0.94	0.86	0.94	0.93
30	0.93	0.94	0.85	0.93	0.93	0.93	0.94	0.84	0.93	0.93
35	0.93	0.94	0.85	0.93	0.93	0.93	0.94	0.84	0.93	0.93
40	0.94	0.94	0.85	0.94	0.94	0.93	0.94	0.84	0.93	0.93
45	0.94	0.94	0.85	0.94	0.94	0.94	0.95	0.86	0.94	0.94
50	0.94	0.95	0.84	0.94	0.94	0.93	0.94	0.85	0.94	0.93
	20% trimmed					25% trimmed				
5	0.88	0.89	0.83	0.88	0.88	0.88	0.89	0.83	0.88	0.88
6	0.88	0.89	0.82	0.89	0.89	0.89	0.89	0.84	0.89	0.89
7	0.90	0.91	0.85	0.90	0.90	0.90	0.90	0.84	0.90	0.90
8	0.88	0.89	0.83	0.89	0.88	0.89	0.90	0.83	0.89	0.89
9	0.91	0.91	0.85	0.91	0.91	0.90	0.91	0.84	0.91	0.90
10	0.89	0.90	0.83	0.90	0.89	0.90	0.90	0.84	0.90	0.90
11	0.91	0.92	0.84	0.91	0.91	0.90	0.91	0.84	0.90	0.90
12	0.91	0.92	0.85	0.92	0.91	0.91	0.92	0.84	0.91	0.91
13	0.90	0.91	0.84	0.91	0.90	0.90	0.91	0.85	0.91	0.91
14	0.92	0.93	0.85	0.92	0.92	0.92	0.92	0.84	0.92	0.92
15	0.91	0.92	0.84	0.91	0.91	0.92	0.93	0.85	0.92	0.92
20	0.92	0.93	0.85	0.92	0.92	0.93	0.93	0.85	0.93	0.93
25	0.93	0.94	0.85	0.93	0.93	0.92	0.93	0.86	0.93	0.92
30	0.93	0.94	0.85	0.94	0.93	0.93	0.94	0.85	0.94	0.93
35	0.93	0.94	0.85	0.94	0.93	0.93	0.93	0.83	0.93	0.93
40	0.93	0.93	0.84	0.93	0.93	0.93	0.94	0.85	0.94	0.93
45	0.93	0.94	0.85	0.94	0.93	0.94	0.94	0.84	0.94	0.94
50	0.94	0.94	0.85	0.94	0.94	0.93	0.93	0.86	0.93	0.93

Table 6: Coverage probability of 95% CIs when skewness=4

	5% trimmed					10% trimmed				
n	tci	medtci	madtci	tmci	mtmci	tci	medtci	madtci	tmci	mtmci
5	0.74	0.77	0.69	0.74	0.74	0.75	0.77	0.71	0.75	0.75
6	0.75	0.76	0.70	0.75	0.75	0.75	0.77	0.69	0.75	0.75
7	0.74	0.76	0.68	0.74	0.74	0.76	0.78	0.71	0.76	0.76
8	0.79	0.80	0.73	0.79	0.79	0.79	0.80	0.72	0.79	0.79
9	0.78	0.81	0.72	0.78	0.78	0.79	0.80	0.72	0.79	0.79
10	0.80	0.82	0.73	0.80	0.80	0.81	0.83	0.74	0.81	0.81
11	0.80	0.82	0.73	0.80	0.80	0.81	0.83	0.74	0.81	0.81
12	0.81	0.83	0.74	0.81	0.81	0.81	0.83	0.74	0.82	0.81
13	0.83	0.84	0.75	0.83	0.83	0.82	0.83	0.74	0.82	0.82
14	0.83	0.85	0.76	0.83	0.83	0.81	0.83	0.73	0.81	0.81
15	0.82	0.84	0.73	0.82	0.82	0.83	0.84	0.74	0.83	0.83
20	0.87	0.88	0.77	0.87	0.87	0.84	0.86	0.75	0.85	0.84
25	0.86	0.88	0.75	0.86	0.86	0.87	0.89	0.76	0.87	0.87
30	0.88	0.90	0.76	0.88	0.88	0.88	0.89	0.77	0.88	0.88
35	0.89	0.91	0.77	0.89	0.89	0.88	0.90	0.75	0.89	0.88
40	0.89	0.90	0.77	0.89	0.89	0.90	0.92	0.78	0.91	0.90
45	0.91	0.92	0.78	0.91	0.91	0.90	0.92	0.78	0.91	0.90
50	0.91	0.92	0.78	0.91	0.91	0.91	0.92	0.77	0.91	0.91
	20% trimmed					25% trimmed				
5	0.74	0.76	0.70	0.76	0.74	0.73	0.75	0.68	0.74	0.74
6	0.74	0.76	0.68	0.75	0.74	0.76	0.77	0.71	0.77	0.77
7	0.76	0.78	0.71	0.77	0.76	0.78	0.80	0.72	0.79	0.78
8	0.79	0.81	0.73	0.80	0.79	0.78	0.79	0.71	0.79	0.78
9	0.78	0.79	0.71	0.78	0.78	0.80	0.81	0.74	0.81	0.80
10	0.82	0.84	0.76	0.83	0.82	0.81	0.82	0.73	0.81	0.81
11	0.81	0.83	0.73	0.82	0.81	0.81	0.83	0.75	0.82	0.82
12	0.81	0.83	0.74	0.82	0.81	0.81	0.83	0.74	0.83	0.82
13	0.81	0.83	0.72	0.82	0.81	0.83	0.84	0.76	0.84	0.84
14	0.82	0.83	0.74	0.83	0.82	0.83	0.85	0.75	0.84	0.83
15	0.84	0.86	0.76	0.85	0.84	0.84	0.86	0.76	0.85	0.85
20	0.85	0.87	0.75	0.86	0.85	0.84	0.86	0.75	0.86	0.85
25	0.86	0.88	0.75	0.88	0.86	0.86	0.88	0.76	0.88	0.87
30	0.86	0.88	0.75	0.87	0.86	0.88	0.90	0.77	0.89	0.88
35	0.88	0.90	0.77	0.90	0.88	0.89	0.90	0.77	0.90	0.89
40	0.89	0.91	0.78	0.90	0.89	0.90	0.91	0.78	0.91	0.90
45	0.89	0.91	0.77	0.91	0.89	0.90	0.92	0.78	0.92	0.91
50	0.90	0.92	0.76	0.91	0.90	0.91	0.92	0.78	0.92	0.91

Table 7: Minimum (min) and maximum (max) coverage probability of various CIs for varying values of skewness and % trimming

skewness		5% trimmed					10% trimmed				
		tci	medtci	madtci	tmci	mtmci	tci	medtci	madtci	tmci	mtmci
	min	0.94	0.95	0.87	0.94	0.94	0.94	0.95	0.87	0.94	0.94
	max	0.95	0.95	0.88	0.95	0.95	0.95	0.95	0.88	0.95	0.95
0.5		20% trimmed					25% trimmed				
	min	0.94	0.94	0.86	0.94	0.94	0.94	0.94	0.87	0.94	0.94
	max	0.95	0.95	0.88	0.95	0.95	0.95	0.95	0.88	0.95	0.95
		5% trimmed					10% trimmed				
	min	0.94	0.94	0.86	0.94	0.94	0.94	0.95	0.87	0.94	0.94
	max	0.95	0.95	0.88	0.95	0.95	0.95	0.96	0.88	0.95	0.95
1		20% trimmed					25% trimmed				
	min	0.94	0.95	0.87	0.94	0.94	0.94	0.94	0.86	0.94	0.94
	max	0.95	0.96	0.89	0.95	0.95	0.95	0.95	0.88	0.95	0.95
		5% trimmed					5% trimmed				
	min	0.93	0.94	0.84	0.93	0.93	0.93	0.94	0.84	0.93	0.93
	max	0.94	0.95	0.85	0.94	0.94	0.94	0.95	0.86	0.94	0.94
2		20% trimmed					25% trimmed				
	min	0.93	0.93	0.84	0.93	0.93	0.93	0.93	0.83	0.93	0.93
	max	0.94	0.94	0.85	0.94	0.94	0.94	0.94	0.86	0.94	0.94
		5% trimmed					10% trimmed				
	min	0.88	0.90	0.76	0.88	0.88	0.88	0.89	0.75	0.88	0.88
	max	0.91	0.92	0.78	0.91	0.91	0.91	0.92	0.78	0.91	0.91
4		20% trimmed					25% trimmed				
	min	0.86	0.88	0.75	0.87	0.86	0.88	0.90	0.77	0.89	0.88
	max	0.90	0.92	0.78	0.91	0.90	0.91	0.92	0.78	0.92	0.91