Novel Switch Detection Algorithm in Logic-Based Guidance

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Abstract

A planar interception problem of maneuvering targets is considered. Recently, a logic-based guidance algorithm (LBA) was developed, assuming that the target performs a randomly switched bang-bang maneuver. Assuming perfect information, the differential game based guidance law, incorporated in the LBA, guarantees hit-to-kill accuracy. Since the only direct information, available to the interceptor, is the noisy line-of-sight angle measurements, all state-variables should be estimated. In the LBA, the estimators are tuned assuming the knowledge of the target switch moments. In this paper, a novel switch detection algorithm, based on a sequentially tested hypothesis, is proposed. Algorithm efficiency is demonstrated by numerical simulation.

Key Words: guidance, random switch, statistical hypothesis test, switch detection, convex optimization

1. Introduction

The homing guidance of an interceptor missile can be modeled as a zero sum pursuit-evasion differential game (Shinar, 1981; Shima and Shinar, 2002; Turetsky and Shinar, 2003), where the cost functional is the miss distance. The game solution is a triplet, namely the optimal interceptor guidance law, the optimal evasive maneuver of the target and the guaranteed miss distance. The solution depends on the interceptor/target maneuverability ratio, as well as the dynamics of the interceptor missile and the target. The worst case for the interceptor, not being able to measure the actual target acceleration, is to assume ideal target dynamics. In this case there is no need to estimate the target acceleration, because it is not included in the game optimal guidance law. If the guaranteed miss distance, based on assuming perfect state information is small enough compared to the lethality radius of the interceptor's warhead, target destruction is robustly achieved against any feasible target maneuver. However, if the interceptor/target maneuverability ratio is not sufficient for such robust target destruction, another solution method, which includes the estimated actual target acceleration, is needed.

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In realistic interception scenarios with noise-corrupted measurements, an estimator has become an indispensable element of the guidance system and the homing performance of the interceptor missile has been limited by the estimation accuracy. Although, for realistic interceptor guidance scenarios with noise-corrupted measurements, bounded controls, and saturated state variables, as well as non-Gaussian random disturbances, the validity of the separation theorem of Wonham (1968) (stating that the estimation and control processes can be separately optimized) has never been proved, it has been of common practice to design the estimators and missile guidance laws independently. The estimators were simple Wiener or Kalman filters and the guidance laws were derived using simplified (linearized and planar) deterministic models. In most cases, such convenient design approach had been acceptable, because it succeeded in satisfying the performance requirements, due to the substantial maneuverability advantage of guided missiles over their manned aircraft targets. However, applying this suboptimal approach, for example, to the interception of antisurface missiles with high maneuverability, results in unsatisfactory homing performance (Shinar and Shima, 2002).

In (Shinar et al., 2007), an innovative guidance strategy was introduced, based on integrating the design of a multiple model adaptive estimator and a differential game based guidance law, leading to a potential breakthrough in the interception of randomly maneuvering targets in critical scenarios, such as the Ballistic Missile Defense. It was assumed that the target performs a randomly switched bang-bang maneuver. In this, logic-based, algorithm, the interception endgame was divided into two time intervals. At the first interval (from the beginning and to some critical time), a single relatively slow estimator was used. At the final interval, a set estimators were employed, which were tuned to the moment of the target maneuver switch. Thus, the detection of the switch moment becomes the crucial component of this approach, significantly affecting the interceptor's homing performance.

In (Shinar et al., 2007), it was assumed that the maneuver is detected with a constant delay of 0.1 s. In (Shinar and Turetsky, 2009), the logic based approach was successfully applied in the three-dimensional interception problem. In (Shinar and Turetsky, 2014), the logic based guidance/estimation algorithm was improved by reducing the critical time and by more accurate parameter tuning. These improvements allowed to reduce the 95% value of the accumulated miss distance distribution to the half as reported by Shinar et al. (2007). However, no target maneuver detection algorithm was designed, and the results were obtained assuming an arbitrary small constant detection delay.

In this paper, the missing link of the logic based algorithm, namely, a statistical switch detection algorithm, is presented. It is based on sequential testing of statistical hypotheses. It is shown that the proposed detection algorithm produces a time-varying detection delay which does deteriorate the homing performance.

2. Previous Results and Problem Statement

2.1 Engagement model

The planar engagement between two moving objects - an interceptor (*pursuer*) and a target (*evader*) - is considered. In Fig. 1, a schematic view of the interception geometry is shown. The x axis of the coordinate system is aligned with the initial line of sight. The origin is collocated with the initial pursuer position. The points (x_P, y_P), (x_E, y_E) are the current coordinates of the objects; V_P and V_E are the velocities of the pursuer and evader; a_P, a_E are the lateral accelerations of the pursuer and evader, respectively; ϕ_P, ϕ_E are the respective angles between the velocity vectors and the reference line of sight; and $y = y_E - y_P$ is the separation normal to the initial line of sight.

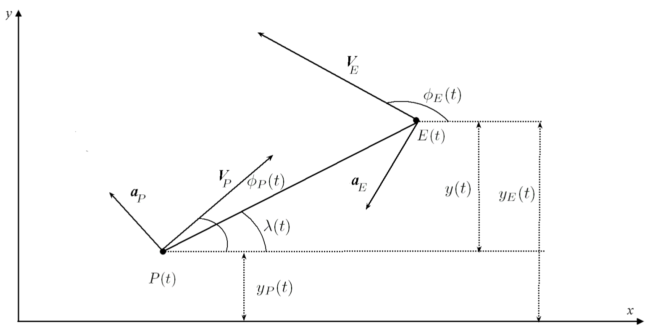

Figure 1: Interception geometry

Due to the simplifying assumptions (constant velocities V_P and V_E, constant maximal lateral accelerations $a_P^{\max}$ and $a_E^{\max}$, first-order dynamics with time constants τ_P and τ_E, small aspect angles ϕ_P and ϕ_E of the pursuer and evader, respectively), the original nonlinear model can be linearized and the respective final time t_f can be calculated. The linearized engagement model is described (Turetsky and Shinar, 2003) by the set of differential equations

$$\begin{array}{rcll} \dot{x}_1 & = & x_2 & , x_1(0) = 0, \\ \dot{x}_2 & = & x_3 - x_4 & , x_2(0) = x_{20}, \\ \dot{x}_3 & = & (a_E^{\max} v - x_3)/\tau_E & , x_3(0) = 0, \\ \dot{x}_4 & = & (a_P^{\max} u - x_4)/\tau_P & , x_4(0) = 0, \end{array} \tag{1}$$

where $x_1 = y_E - y_P$ is the relative separation normal to the initial line of sight; x_2 is the relative normal velocity; x_3 and x_4 are the lateral accelerations of the evader and the pursuer, respectively, both normal to the initial line of sight; τ_E, τ_P are the respective time constants; $a_E^{\max}$, $a_P^{\max}$ are the respective maximal absolute values of the lateral accelerations;

$$x_{20} = V_E \varphi_E^0 - V_P \varphi_P^0, \tag{2}$$

φ_E^0 and φ_P^0 are initial values of φ_E and φ_P, respectively. The state vector of the system (1) is $x = (x_1, x_2, x_3, x_4)^T$. The controls u and v of the pursuer and evader are their acceleration commands, normalized by $a_P^{\max}$ and $a_E^{\max}$, and satisfying for $0 \le t \le t_f$ the constraints

$$|u(t)| \le 1, \ \ |v(t)| \le 1. \tag{3}$$

The objective of the pursuer is to minimize the miss distance $x_1(t_f)$, by using a feedback strategy $u(t, x)$ against any admissible evader control $v(t)$.

2.2 Differential game based strategy

The interception problem can be modeled as a zero-sum differential game for the system (1) with the cost functional

$$J_x = |x_1(t_f)|, \tag{4}$$

and the control constraints (3). The solution of this differential game (Shinar, 1981) is obtained by scalarization. Let $\Phi(t, \tau)$, $0 \le \tau \le t \le t_f$, be the fundamental matrix of the homogeneous system corresponding to (1), $d = (1, 0, 0, 0)$. By the terminal projective transformation (Krasovskii and Subbotin, 1988)

$$z = d^T \Phi(t_f, t)x, \tag{5}$$

where T denotes the transposition, the system (1) is reduced (Turetsky and Shinar, 2003) to the scalar equation

$$dz/d\vartheta = h_1(\vartheta)u - h_2(\vartheta)v, \tag{6}$$

where the new independent variable is the time-to-go

$$\vartheta = t_f - t, \tag{7}$$

the coefficient functions are

$$h_1(\vartheta) = a_P^{\max}\tau_P\psi(\vartheta/\tau_P), \;\; h_2(t) = a_E^{\max}\tau_E\psi(\vartheta/\tau_E), \tag{8}$$

$$\psi(\xi) \triangleq \exp(-\xi) + \xi - 1. \tag{9}$$

The controls $u(\vartheta)$ and $v(\vartheta)$, denoting in fact $u(t_f - \vartheta)$ and $v(t_f - \vartheta)$, satisfy the constraints

$$|u(\vartheta)| \le 1, \;\; |v(\vartheta)| \le 1, \;\; t_f \ge \vartheta \ge 0. \tag{10}$$

Since $x_1(t_f) = 0$ is equivalent to $z(0) = 0$, the original differential game is equivalent to the scalar differential game for the system (6), the cost functional

$$J_z = |z(0)|, \tag{11}$$

and the control constraints (10).

If the strategy $u^0(\vartheta, z)$ is optimal in the scalar differential game, then the strategy $u(t, x) = u^0(t_f - t, d\Phi(t_f, t)x)$ is optimal in the original game.

The solution of the game (6), (11) – (10) is based (Shinar, 1981) on the decomposition of the game space (ϑ, z) into two regions of different strategies. In the first (*singular*) region D_0 the optimal control strategies $u^0(\vartheta, z)$ and $v^0(\vartheta, z)$ are *arbitrary* subject to (10), and the value of the game is constant (zero or positive). In the second (*regular*) region $D_1 = R^2 \backslash D_0$ the optimal strategies have a "bang-bang" structure:

$$u^0(\vartheta, z) = v^0(\vartheta, z) =$$

$$\begin{cases} \text{arbitrary s.t. (10)}, & (\vartheta, z) \in D_0, \\ \\ \operatorname{sign} z(\vartheta), & (\vartheta, z) \in D_1. \end{cases} \tag{12}$$

and the value of the game is non-zero, depending on the initial conditions. In the case, where the pursuer has advantage both in maneuverability

$$a_p^{\max} > a_e^{\max}, \tag{13}$$

and advantage in agility, defined as

$$\frac{a_p^{\max}}{\tau_p} \ge \frac{a_e^{\max}}{\tau_e}, \tag{14}$$

the singular region is

$$D_0 = \{(\vartheta, z) : \;\; \vartheta > 0, \; |z| < z^*(\vartheta)\}, \tag{15}$$

where

$$z^*(\vartheta) = \int_{\bar{\vartheta}}^{\vartheta} [h_1(\xi) - h_2(\xi)])d\xi. \tag{16}$$

For any initial position (ϑ_0, z_0) the value of the perfect information game is given by

$$J^* = J^*(\vartheta_0, z_0) = \begin{cases} 0, & (\vartheta_0, z_0) \in D_0, \\ |z_0| - z^*(\vartheta_0), & (\vartheta_0, z_0) \notin D_0. \end{cases} \tag{17}$$

Thus, in this case, the closure $\mathrm{clo}(D_0)$ of the singular region becomes the *robust capture zone*, i.e. the set of all initial positions, from which the pursuer can guarantee zero miss distance against any admissible evader strategy. The optimal strategy (guidance law) $u^0(\vartheta, z)$ is known in the literature as DGL/1 (Shima and Shinar, 2002).

2.3 Practical implementation of optimal strategy

By (5), the scalar state variable z is given explicitly as

$$z(\vartheta) = x_1 + \theta x_2 + \tau_E^2 \psi(\theta/\tau_E) x_4 - \tau_P^2 \psi(\theta/\tau_P) x_3, \tag{18}$$

i.e. in order to implement the optimal strategy $u^0(\vartheta, z)$, given in (12), one needs to know all the components of the state vector x. In real-life scenario, the exact values of x_1, x_2 and x_4 are not available and should be reconstructed by an estimator, incorporated into the control loop, based on the noisy measurements of the line-of-sight angle

$$\hat{\lambda}(\vartheta) = \lambda(\vartheta) + \eta(\vartheta), \tag{19}$$

where $\eta(\vartheta)$ is a measurement error. Thus, the "practical" optimal pursuer strategy becomes

$$\hat{u} = u^0(\vartheta, \hat{z}), \tag{20}$$

where

$$\hat{z} = \hat{x}_1 + \theta \hat{x}_2 + \tau_E^2 \psi(\theta/\tau_E) \hat{x}_4 - \tau_P^2 \psi(\theta/\tau_P) x_3, \tag{21}$$

and $\hat{x}_1$, $\hat{x}_2$ and $\hat{x}_4$ being the estimator outputs.

2.4 Logic based guidance

The logic based approach is based on the following observation (Shinar et al., 2007) derived from the results of a very large set of Monte Carlo simulations of a planar interception endgame scenario. The estimator in these simulations was a Kalman filter augmented with a shaping filter using an exponentially correlated acceleration (ECA) model (Singer, 1970). Such a shaping filter has first order dynamics with two tuning parameters, the correlation time of the maneuver τ_s and the (assumed) level of the process noise, expressed by its standard deviation $\sigma_s = a_E^{\max}/C_s$. The simulations used a differential game based guidance law DGL/1 and a set of parameters that guarantee, in the ideal case of perfect information and without an estimator in the guidance loop, zero miss distances against all admissible bounded target maneuvers. However, the simulation results for a large set of interceptions, where the target performs in a short duration (4 sec) interception endgame randomly changing bang-bang type evasive maneuvers, demonstrate a very different outcome.

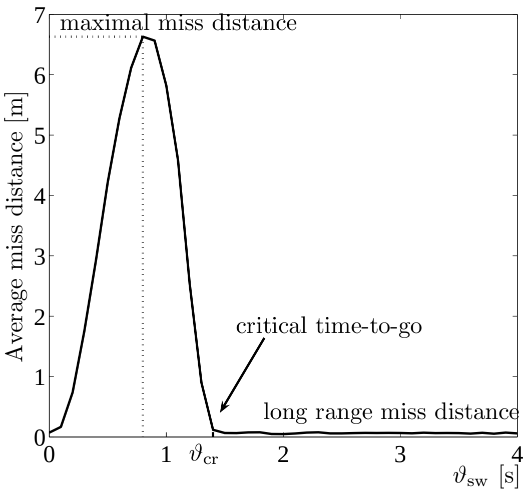

Figure 2: Average miss distance vs. switch moment

In Fig. 2, the homing performance, expressed by the average miss distance of a large number of Monte Carlo simulations, is depicted as the function of the moment $\vartheta_{\rm sw}$ of the target maneuver direction change. In this example, $t_f = 4$ s, $\tau_P = \tau_E = 0.2$ s, $a_P^{\max} = 200$ m/s^2, $a_E^{\max} = 100$ m/s^2, $\tau_s = 0.4$ s, $C_s = 3$. This figure shows that the interception endgame can be divided into two regions of different homing performance by a critical time-to-go $\vartheta_{\rm cr}$ that serves as the boundary between the regions of small and large miss distances. In this example, $\vartheta_{\rm cr} = 1.4$ s. It is seen that that small miss distances can be achieved only if the direction change of the target acceleration occurs in the early part of the endgame (for $\vartheta_{\rm sw} > \vartheta_{\rm cr}$). In this case, sufficient time remains for the estimated acceleration to converge to its true quasi-steady value. The Kalman filter design minimizes the variance of the converged estimation error and the guidance law receives soon enough almost correct values of the zero-effort miss distance for achieving good homing precision. If the target acceleration change occurs later, the combination of the same estimator with the same guidance law fails to provide satisfactory results because of the estimation delay.

This observation suggests the following estimation scheme which (i) assumes that the direction switch in the target maneuver can be detected sufficiently fast and (ii) utilizes the idea of a ”tuned” estimator Shinar et al. (2007), expecting that the switch occurs at some prescribed moment. The bank of the estimators, working in parallel, consists of an untuned filter $\mathcal{F}_0$ and a set of filters $\mathcal{F}_1, ..., \mathcal{F}_k$, tuned to the moments $\vartheta_1, ..., \vartheta_k \in [0, \vartheta_{\rm cr}]$, respectively. If the direction switch in the target maneuver was detected on at $\vartheta_d \in [\vartheta_{\rm cr}, t_f]$, then the estimates $\hat{x}_1$, $\hat{x}_2$ and $\hat{x}_4$ in (20) – (21) are obtained from $\mathcal{F}_0$. If the switch was detected at $\vartheta_d \in [0, \vartheta_{\rm cr}]$, these estimates are taken from the estimator $\mathcal{F}_i$ for $i = \min\{m : \vartheta_m \geq \vartheta_d\}$.

For the sake of completeness, we reproduce the results, obtained by using the logic based algorithm, which were reported by Shinar et al. (2007) and by Shinar and Turetsky (2014). The best results (the 95% value of the accumulated miss distance distribution equals 35.5 cm (Shinar and Turetsky, 2014)) were obtained for the system parameters of Fig. 2 and for $\vartheta_{\rm cr} = 1.1$ s, one ($k = 1$) filter, tuned for $\vartheta_1 = 0.8$ s, $\tau_s = 0.4$ s, $C_s = 0.5$ for both untuned and tuned filters $\mathcal{F}_0$ and $\mathcal{F}_1$.

In conclusion of this section, we need to emphasize that the successful performance of the logic based algorithm, reported previously, is subject to a sufficiently fast switch detection. The objective of this paper is to present a novel statistical switch detection

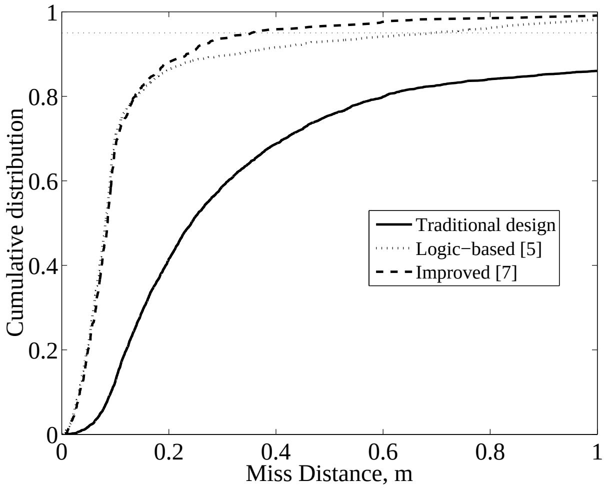

Figure 3: Miss distance cumulative distribution

algorithm.

3. Switch detection algorithm

3.1 Discrete time model

We consider a discrete analog of the engagement model described in Section 2.1. Let us divide the interval $[0, t_f]$ into N equal subintervals by the points $t_0, t_1, \ldots, t_N$, where $t_n = n\Delta t$, $n = 0, \ldots, N$. Consider the discrete-time dynamical system

$$X_{n+1} = AX_n + bu_n + cv_n \tag{22}$$

$$y_n = D_n X_n + \sigma\xi_n, \tag{23}$$

where $X_n = (x_1(t_n), \ldots, x_4(t_n))^T$ is the state vector, $u_n = u(t_n) \in \mathrm{R}$ and $v_n = v(t_n) \in \mathrm{R}$ are the pursuer and evader controls, $y_n = \lambda(t_n) \in \mathrm{R}$ is the observed output, σ is a standard deviation of the line-of-sight angle measurement error. We assume that ξ_n, $n = 1, 2, \ldots, N$, is a sequence of independent standard normal random variables. Due to the small angles assumption,

$$x_1(t) \approx \lambda(t)r(t), \tag{24}$$

where $r(t)$ is the current distance between the pursuer and the evader. Thus, by using the Euler discretization of (1) and due to (19) and (24), the matrices and vectors in (22) – (23) are

$$A = \begin{bmatrix} 1 & \Delta t & 0 & 0 \\ 0 & 1 & \Delta t & -\Delta t \\ 0 & 0 & 1 - \Delta t/\tau_e & 0 \\ 0 & 0 & 0 & 1 - \Delta t/\tau_p \end{bmatrix}, \tag{25}$$

$$b = (0, 0, 0, \Delta t a_p^{\max}/\tau_p)^T, \tag{26}$$

$$c = (0, 0, \Delta t a_e^{\max}/\tau_e, 0)^T, \tag{27}$$

$$D_n = (1/r(t_n), 0, 0, 0). \tag{28}$$

Our goal is to detect the presence of a switch in the evader control sequence v_n as quickly as possible. More formally, if the evader control is given by the bang-bang single switch function

$$v_n = \begin{cases} 1, & n \le n_{\mathrm{sw}}, \\ \\ -1, & n > n_{\mathrm{sw}}, \end{cases} \tag{29}$$

for some unknown switch moment $n_{\mathrm{sw}} \in \{1, 2, \ldots, N\}$ then we want to detect the switch n_{sw} as quickly as possible on the basis of observations $y_1, y_2, \ldots, y_N$ from (23), where N is a fixed observation horizon.

3.2 Detection problem statement

We rewrite the model in the following equivalent form. Iterating (22) – (23) we have for any $n = 1, \ldots, N$

$$\begin{aligned} y_n = D_n A^n X_0 + D_n \sum_{m=0}^{n-1} A^{n-m-1} c v_m \\ + D_n \sum_{m=0}^{n-1} A^{n-m-1} b u_m + \sigma \xi_n. \end{aligned} \tag{30}$$

Denote

$$\begin{aligned} \zeta_n &\triangleq y_n - D_n A^n X_0 - D_n \sum_{m=0}^{n-1} A^{n-m-1} b u_m \\ h_{m,n} &\triangleq D_n A^{n-m-1} c; \end{aligned}$$

then (30) takes form

$$\zeta_n = \sum_{m=0}^{n-1} h_{m,n} v_m + \sigma \xi_n, \quad n = 1, \ldots, N. \tag{31}$$

Since pursuer control u_n and all parameters involved are known, ζ_n can be computed from the observations $y_1, \ldots, y_n$.

Let us define $\zeta = (\zeta_1, \ldots, \zeta_N)^T$, $v = (v_0, \ldots, v_{N-1})^T$, $\xi = (\xi_1, \ldots, \xi_N)^T$ and

$$H = \begin{bmatrix} h_{0,1} & 0 & 0 & \cdots & 0 \\ h_{0,2} & h_{1,2} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_{0,N} & h_{1,N} & h_{2,N} & \cdots & h_{N-1,N} \end{bmatrix}.$$

With this notation (31) takes the form

$$\zeta = Hv + \sigma \xi, \quad \xi \sim \mathcal{N}_N(0, I), \tag{32}$$

where $\mathcal{N}_k(\mu, \Sigma)$ denotes a multivariate normal distribution with k-dimensional mean vector and $k \times k$ covariance matrix Σ. For any $n \in \{1, \ldots, N\}$, let us denote $\zeta(n) = (\zeta_1, \ldots, \zeta_n)^T$, $\xi(n) = (\xi_1, \ldots, \xi_n)^T \sim \mathcal{N}_n(0, I)$, $v(n) \triangleq (v_0, \ldots, v_{n-1})^T$, and let $H(n)$ be the $n \times N$ matrix comprised of the first n rows of matrix H. Then, due to (32), at time instance n we observe

$$\zeta(n) = H(n)v + \sigma \xi(n), \quad n = 1, \ldots, N. \tag{33}$$

Note that by definition of H, at time instance n only n first coordinates of v affect the output even though vector v is N–dimensional.

From now on we focus on the model (33). In terms of this model, we observe sequentially vectors $\zeta(n) \in \mathrm{R}^n$, $n = 1, \ldots, N$. For $k = 1, \ldots, N$ denote

$$g^{(k)} \triangleq (\underbrace{1, \ldots, 1}_{k}, -1, \ldots, -1) \in \mathrm{R}^N. \tag{34}$$

Then the absence of the switch corresponds to $v = g^{(N)}$ while the presence of the switch at time instance k means that $v = g^{(k)}$. Thus, the switch detection problem can be formulated as the problem of sequential testing the hypotheses

$$\Pi_0 : v = g^{(N)} \quad \text{against} \quad \Pi_1 : v = g^{(k)},\ k \in \{1, \ldots, N-1\}$$

using observations $\zeta(1), \zeta(2), \ldots, \zeta(n)$ from (33).

By a *sequential switch detector* we mean a pair (ψ, ν) consisting of

- a sequence of decision functions $\psi = \{\psi_n\}$, where $\psi_n \in \{0, 1\}$ is a random variable depending on $\{\zeta(1), \ldots, \zeta(n)\}$ only, $n = 1, \ldots, N$;
- the corresponding stopping time

$$\nu = \min\{n \in (1, \ldots, N) : \psi_n = 1\}, \tag{35}$$

where the event $\{\nu = n\} = \{\psi_1 = 0, \ldots, \psi_{n-1} = 0, \psi_n = 1\}$ corresponds to detection of the switch after observing $\zeta(1), \ldots, \zeta(n)$.

The performance of a detection procedure is determined by the detection delay subject to a constraint on the probability of false alarm. In particular, we require that for every n the probability that decision function ψ_n raises false alarm is less than α, i.e.

$$\max_{n=1,\ldots,N} P_0\{\hat{\psi}_n = 1\} \leq \alpha, \tag{36}$$

where P_0 stands for the probability measure of the observations $\zeta(1), \zeta(2), \ldots$ under the hypothesis that there is no switch. In the area of sequential change–point detection the constraint (36) was advocated by Brodsky and Darkhovsky (1993). This is also closely related to the probability of "false alarm per time unit" considered in Lai (1995) and Lai (1998). If the switch occurs at time instance k, the detection delay of procedure (ψ, ν) is $(\nu - k)_+$. We will be interested in distribution of this random variable under the probability measure P_k of observations when the switch occurs at k.

3.3 Detection procedure and its properties

The switch detection algorithm, developed in this section, is based on the idea of combining pairwise tests of hypothesis $\Pi_0 : v = g^{(N)}$ (absence of switch) against alternatives $\Pi_{1,k} : v = g^{(k)}$, $k = 1, \ldots, N-1$ (switch occurs at time instance k). We refer to Goldenshluger et al. (2013) for a general convex–optimization–based framework for solution of such testing problems.

Note that for any $n = 1, \ldots, N$ the observation vector $\zeta(n)$ is multivariate normal with mean $z_0(n) := H(n)g^{(N)}$ and covariance matrix $\sigma^2 I_n$ under Π_0, and multivariate normal with mean $z_{1,k}(n) = H(n)g^{(k)}$ and covariance matrix $\sigma^2 I_n$ under the alternative hypothesis $\Pi_{1,k}$. The likelihood ratio test of Π_0 against $\Pi_{1,k}$ on the basis of observation $\zeta(n)$ rejects Π_0 for large values of statistic

$$T_k(n) = \frac{[z_0(n) - z_{1,k}(n)]^T [z_0(n) - \zeta(n)]}{\rho_k(n)},$$

where $\rho_k(n) = \|z_0(n) - z_{1,k}(n)\|$. Specifically, according to the Neyman–Pearson lemma, the test of size $\alpha \in (0,1)$ with the minimal second type error probability rejects Π_0 if

$$T_k(n) \geq \sigma Q(\alpha), \tag{37}$$

where $Q(\cdot)$ is the Gaussian quantile function defined by relation

$$\frac{1}{\sqrt{2\pi}} \int_{Q(\alpha)} e^{-t^2/2} dt = \alpha, \quad \alpha \in (0,1).$$

In our detection procedure, we combine tests of type (37) for possible switch locations, which results in the sequence of the decision functions

$$\hat{\psi}_n = \begin{cases} 1, & \max\limits_{k=1,\dots,n} T_k(n) \geq \sigma Q(\alpha/N), \\ \\ 0, & \text{otherwise} \end{cases} \tag{38}$$

together with the associated stopping time

$$\hat{\nu} = \min\{n \in (1,\dots,N) : \hat{\psi}_n = 1\}. \tag{39}$$

The choice of the threshold $\sigma Q(\alpha/N)$ in (38) guarantees that the procedure $(\hat{\psi}, \hat{\nu})$ satisfies (36). In addition, the following "oracle" property on the detection delay of $(\hat{\psi}, \hat{\nu})$ holds. Let $\beta \in (0,1)$ be fixed and consider all detectors with the probability of the false alarm per time unit $\leq \alpha$, see (36). If the switch occurs at time instance k then in order to detect this change with the second type error probability $\leq \beta$ any procedure $(\psi, \hat{n}_{\rm sw})$ requires at least

$$n_k^*(\sigma, \beta) \triangleq \min\big\{n : \rho_k(n) \geq \sigma[Q(\alpha) + Q(\beta)]\big\}, \tag{40}$$

observations, yielding *the oracle detection delay* $n_k^*(\sigma, \beta)$. In other words, for any detection procedure (ψ, ν) satisfying (36)

$$P_k\big\{(\nu - k)_+ \geq n_k^*(\sigma, \beta)\big\} \geq \beta, \quad \forall k, \tag{41}$$

where P_k stands for the probability measure of observations when the switch occurs at k.

On the other hand, for the detection procedure (38)–(39), it is shown in Goldenshluger et al. (2013) that

$$P_k\big\{(\hat{\nu} - k)_+ \geq n_k^*(\gamma_N \sigma, \beta)\big\} \leq \beta, \ \ \forall k, \tag{42}$$

where $\gamma_N \triangleq Q(\alpha/N)/Q(\alpha)$.

Remark 1 *The inequalities (41) and (42) can be interpreted as follows: if the noise level σ were inflated by the factor $\gamma_N = Q(\alpha/N)/Q(\alpha) \asymp \sqrt{\ln N}$ then there would be no procedure with better detection abilities than those of (38)–(39). The factor γ_N characterizes non–optimality of this detection procedure.*

Remark 2 *The inequalities (41) and (42) provide upper and lower bounds on the $(1-\beta)$–quantile of the distribution of the detection delay. Specifically, let $q_k(1-\beta)$ stand for the $(1-\beta)$–quantile of the distribution of $(\hat{\nu}-k)_+$ under P_k; then (41) and (42) imply that*

$$n_k^*(\sigma,\beta) \le q_k(1-\beta) \le n_k^*(\gamma_N\sigma,\beta), \quad \forall k. \tag{43}$$

3.4 Numerical example

In this example, the system parameters are as in Fig. 3; in the detection algorithm, $N = 500$. In the inverse time, we denote $\Delta\vartheta = \Delta t = t_f/N$, the actual switch moment $\vartheta_{\rm sw} = t_f - n_{\rm sw}\Delta t$. In the simulation, for each switch moment $\vartheta_{\rm sw}^i = i\Delta\vartheta$ ($i = 0,\ldots,N$, $\Delta\vartheta = t_f/N = 0.04$ s), $N_{\rm MC} = 100$ Monte Carlo runs are carried out for different realizations of the measurement noise ξ. The pursuer's control is $u = \mathrm{sign}(z)$. For each $\vartheta_{\rm sw}^i$ in each simulation run $j = 1,\ldots,N_{\rm MC}$ we apply the detection procedure described in Sections 3.1–3.3. If a run does not result in false alarm we record the stopping times (switch moment estimates) $\hat{\vartheta}_{\rm sw}^{ji}$, and compute corresponding detection delays

$$\Delta t_d(\hat{\vartheta}_{sw}^{ji}) = (\hat{\vartheta}_{\rm sw}^{ji} - \vartheta_{\rm sw}^i)_+.$$

In Fig. 4, the empirical 80%–percentile Δt_d^{80} of $\Delta t_d(\hat{\vartheta}_{sw})$ for $\alpha = 0.01$, $\beta = 0.2$,

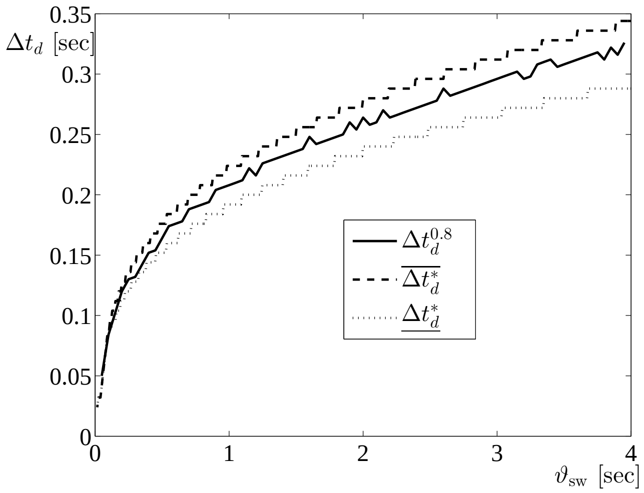

Figure 4: Detection delay

along with its lower bounds $\underline{\Delta t_d^*}(\vartheta_{\rm sw}^i) \triangleq n_i^*(\sigma,\beta)\Delta t$ and upper bounds $\overline{\Delta t_d^*}(\vartheta_{\rm sw}^i) \triangleq n_i^*(\gamma_N\sigma,\beta)\Delta t$, are shown as functions of an actual switch moment $\vartheta_{\rm sw}^i$. This figure illustrates the inequality (43). In Fig. 5, the detection delay, averaged over all runs with no false alarm, is shown as a function of an actual switch moment $\vartheta_{\rm sw}$ for two values of α. It is seen that five times decreasing α does not lead to a dramatic growth of the average detection delay.

The actual false alarms rate, found in Monte Carlo simulations, i.e. the percentage of the cases where $\hat{\vartheta}_{\rm sw}^{ji} < \vartheta_{\rm sw}^i$, is shown in Fig. 6. It is seen that the actual false alarms

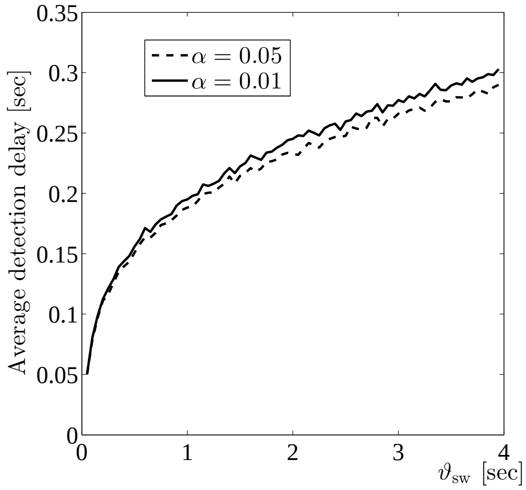

Figure 5: Detection delay

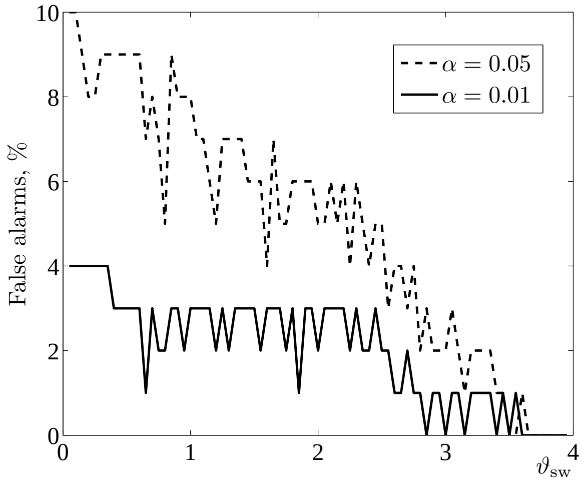

Figure 6: False alarms

rate decreases by decreasing false alarm probability α per time unit, used in the algorithm, while the average detection delay increases slightly (see Fig. 5).

The varying detection delay was approximated by a smooth curve and used in the simulation in order to obtain the miss distance distribution. In Fig. 7, this distribution is compared with that shown in Fig. 3 in dashed line. It is seen that using the varying detection delay, corresponding to the proposed detection algorithm, does not deteriorate the results, in comparison with a small constant detection delay, which is not feasible by any detection procedure.

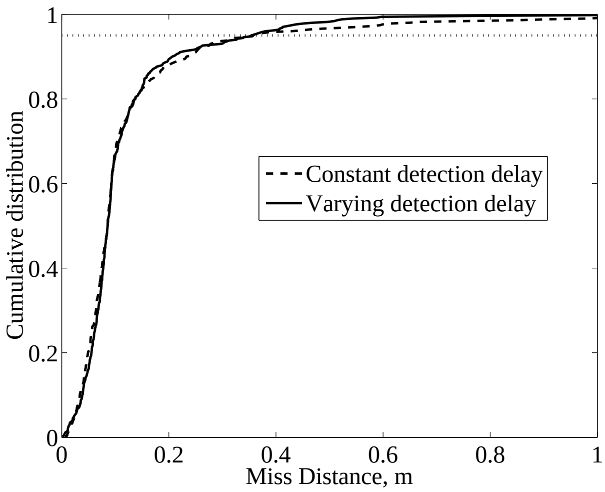

Figure 7: Miss distance: constant vs. varying detection delay

4. Conclusions

In the paper, a novel switch detection algorithm is presented. In general, this algorithm, constructed for a linear dynamics/measurement model, is based on a statistical hypothesis test and can be transformed to a convex optimization problem. The algorithm guarantees that the false alarm probability per time unit is not larger than a given parameter α. Simulations show the actual false alarm rate (the probability of ”detecting” the switch before the actual switch moment) is proportional to α. The average detection delay increases for decreasing α.

In this paper, this statistical algorithm is applied to detecting the moment of a target maneuver direction change. It is shown that the proposed algorithm can be successfully incorporated into the logic based guidance scheme, where the control input is obtained as the output of the estimator, chosen based on the target switch moment. Previously, the advantage of the logic based approach was demonstrated by assuming a constant detection delay which is not feasible by any detection procedure. The simulation shows that by approximating the actual delay of the algorithm, the interception results are not deteriorated.

Acknowledgment

The research of A. G. was partially supported by Israel Science Foundation grant No. 104/11. The research of J. S. was partially supported by Israel Science Foundation grant No. 139/11.

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