Revisiting Drenick's failure law of complex equipment

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Abstract
Though the exponential distribution is not a good failure time model for most component types, under certain conditions it is applicable to complex repairable systems with large numbers of components, as pointed out by Drenick in 1960. The "failure law of complex equipment" proposes that in series systems with many components that are immediately repaired or replaced when they fail, system failures will be (asymptotically) exponentially distributed regardless of the component failure time distributions. We review necessary conditions for this result, and present some simulation studies to assess how well it holds in systems with finite numbers of components. We also discuss its applicability to estimation of the reliability of piping subsystems in commercial nuclear power plants. These subsystems typically contain thousands of individual piping components in series, each with extremely low failure rate, and meet the required conditions of the "law" quite well. Since available data on failures is insufficient to estimate more than the first moment of the failure time distribution of a single element, the ability to characterize the subsystem failure law as exponential has significant value.

Key Words: Failure law, complex systems, exponential distribution, Drenick's law, Nuclear power plant

1. Introduction

A motivating question for this discussion is based on a reliability analysis scenario with the following characteristics: Suppose empirical data is available for components in a complex system, but only failure rates have been collected (number of failures per time unit by component type); a reliability prediction is desired for a system consisting of a large number of components in series (an example will be presented in Section 3). Given this information, how should we predict future reliability of the system?

With no second-moment information, an obvious choice for fitting component failure-time distributions is the exponential, $F_i(t) = 1 - \exp(\lambda_i t)$, where λ_i is the observed failure rate for the ith component.The system failure distribution is then $F(t) = 1 - \exp(\Lambda t)$, where $\Lambda = \sum_i \lambda_i$. Essentially this approach was advocated for electronic systems by a U. S. military handbook, MIL-HDBK-217F (DoD (1991)). Though widely used, this methodology has been widely criticized; see, e.g., Morris and Reilly (1993); Salemi et al. (2008), Sect. 2.2.4. Modern texts favor component failure-time distributions such as the Weibull, which can be based on physical models of failure mechanisms.

Though the exponential distribution is not a good failure time model for most component types, under certain conditions it is applicable to complex repairable systems with large numbers of components, as pointed out by Drenick (1960), whose "failure law[1] of complex equipment" asserts that in series systems with many components with small failure rates, which are immediately repaired or replaced when they fail, system failures will be (asymptotically) exponentially distributed regardless of the component failure time dis-

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[1]"Law" in this context refers to a probability distribution, not a hypothesis that claims to be universally true. We refer to the hypothesis under discussion as "Drenick's theorem."

tributions. Though Drenick's result is implicitly invoked by many partisans of the MIL-HDBK-217F approach, it is seldom mentioned explicitly in modern literature. This is somewhat surprising; as we review in Section 2, there are a remarkable number of proofs of essentially the same result, many of which predate Drenick's paper.

In the remainder of this paper, after establishing notation, Section 2 reviews Drenick's theorem and its various proofs and presents some simulation results showing its applicability to systems with finite numbers of components; Section 3 discusses its application in the context of a real-world problem, the reliability of piping subsystems in commercial nuclear power plants; and Section 4 has discussion and suggestions for future work.

1.1 Definitions and notation

We use the following notation for quantities related to reliability. To simplify the discussion, we gloss over some subtleties; for rigorous definitions see, e.g., Thompson (1981); Barlow and Proschan (1987); Meeker and Escobar (1998). For non-repairable items such as light bulbs, the time until failure is a random variable T. The time-to-failure cumulative distribution function (CDF) $F(t)$ is the probability that the item fails at or before time t. The failure probability density function (PDF) is $f(t) = dF(t)/dt$. The reliability (or survival) function $R(t) = 1 - F(t)$ is the probability that the item fails after time t. An additional commonly used measure is the hazard function $h(t) = f(t)/R(t)$—the probability density of failure at time t, conditional on survival up to t. If $h(t)$ is constant, it is usually called the hazard rate, or sometimes the failure rate (but see below). Among continuous distributions, only the exponential has a constant hazard rate. The mean life of an item (mean time to failure or MTTF) is $\mu = \int_0^\infty tf(t)dt$.

For repairable items, T is the time to the first failure, or to subsequent failures after the item has been perfectly repaired. The average time from repair to failure is referred to as the mean time between failures (MTBF), the mean of the time-to-failure distribution. "Failure rate" in the context of repairable system is ambiguous (Thompson (1981)). In this paper, failure rate is the number of failures per time unit for a system in equilibrium; only in the case of an exponential failure law is the failure rate in this sense equal to the hazard rate.

In this paper, parametric distributions are assumed. For example, the exponential reliability function, commonly used for electronic components, is $R(t|\lambda) = e^{-\lambda t}$, where the parameter $\lambda = 1/\text{MTTF}$ is the hazard rate. A distribution frequently applied to complex systems or items subject to wearout is the Weibull, $R(t|\alpha, \beta) = e^{-(t/\beta)^\alpha}$, with two parameters.

Reliability prediction for complex systems with many repairable or replaceable components requires methods that take into account the system structure along with individual component reliabilities; many texts on reliability analysis discuss the requisite calculations, e.g., Barlow and Proschan (1987). Here we are concerned with systems in which components are in series, i.e., a failure of any one component will cause the system to fail, and $R_{\text{Sys}} = \prod_i R_i$, where R_i are the component reliabilities.

2. Drenick's theorem

Any system can be viewed as a chain of elements in series, where individual elements may have parallel structure; see Figure 1 for an example. Drenick's theorem (Drenick (1960)) applies to systems that are "complex" in the sense that that they contain a large number of repairable elements, each with high reliability (low failure rate), where the repair time is negligible.

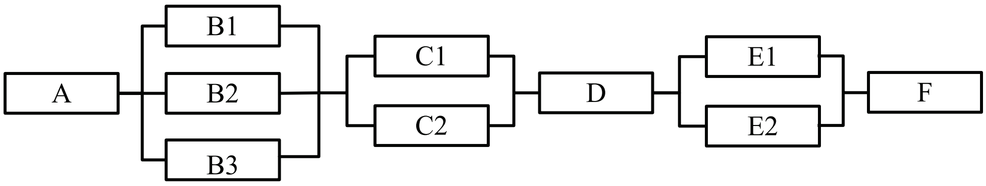

Figure 1: A system structured as elements in series.

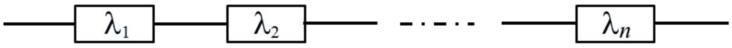

Figure 2: Schematic of the setup for Drenick's theorem.

The theorem applies exactly in the ideal case where the number of components goes to infinity, and repair time goes to zero, or the repair time is absorbed into time between failures. Subject to some conditions stated below, it asserts that for components with failure rates (defined here as 1/mean life, or equivalently the number of failures per unit time) λ_i, the failure law for the system converges to exponential(Λ), where $\Lambda = \sum_{i=1}^{\infty} \lambda_i$, (almost) regardless of the component failure laws. This also characterizes the arrival process for system failures as Poisson, with the probability mass function (PMF) for the number of failures in $(0, t]$ being P$\{N(t) = k\} = \exp(-\Lambda t)(\Lambda^k t)/k!$.

Section 2.3 presents some examples of how the theorem is approximately fulfilled when the number of components is large, but finite. Drenick's result can be useful for a variety of reasons:

- For systems with complex structure, calculating the system failure law from component failure time distributions may be intractable, even if those distributions are known.
- Estimating component failure time distributions may be impractical based on limited available failure data.
- By Occam's razor, we may wish to minimize the assumptions necessary to justify a system failure time distribution. Formally, given only the knowledge that failure times are non-negative, the maximum entropy model is the exponential distribution (Skilling (1989)). This also relates to the previous point, in that failure data may only be sufficient to estimate the first moment of the distribution, which is sufficient to characterize an exponential distribution.
- As described below, proofs of the theorem exist based on several points of view; this offers several alternatives for theoretical justification of an exponential failure law.

The setup for Drenick's proof of the theorem is as follows: The system consists of n independent, repairable components in series (see Figure 2); the time to repair or replace a failed component is negligible, or is absorbed into time between failures; the ith component failure model is a renewal process with rate λ_i (in particular, times between failures are iid with distribution function F_i); individual component processes are independent; and the system has reached its equilibrium state. Then let $\Lambda = \sum_{i=1}^{n} \lambda_i$. If

i) $\lim_{n\to\infty} \sup \frac{\lambda_i}{\Lambda} = 0$

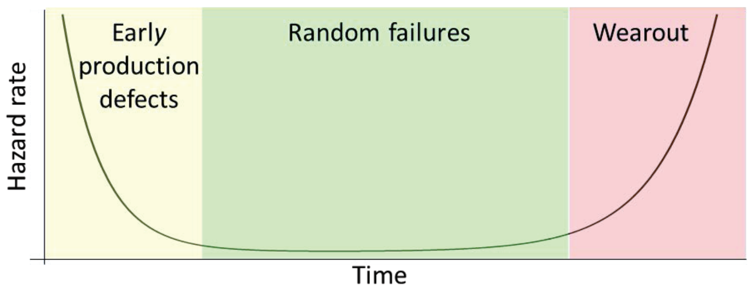

Figure 3: The bathtub curve.

(ii) $F_i(t) = \mathrm{O}(t^\beta) for 0 < \beta < 1$ as $t \to 0$,
then as $n \to \infty$, the system failure law converges in distribution to $F(t) = 1 - \exp(\Lambda t)$. Condition (i) says that all component failure rates are asymptotically negligible; condition (ii) limits how fast the hazard function can increase near the origin, and is satisfied by all failure distributions in common use (e.g., gamma, lognormal, Weibull).

For details of the proof, which is quite technical, see Drenick (1960). For additional details and commentary see Chapter 13 of Kececioglu (1991) and Section 2.3 of Barlow and Proschan (1987).

2.1 The bathtub curve

Figure 3 shows the celebrated "bathtub curve," a standard reliability engineering rule of thumb for complex components and systems. The theory is that systems exhibit an "infant mortality" or burn-in phase when weaker components fail quickly and the failure rate is increasing; a longer period of useful life when failures are due to random events and the failure rate is nearly constant; and an end-of-life wearout phase when physical degradation leads to an increasing failure rate.

Though the bathtub curve has become conventional wisdom, it has also been criticized as unrealistic; see, e.g., Klutke et al. (2003). Discussing this in detail is beyond the scope of this paper, so we simply point out that many engineers consider it a design principle, and strive to make the constant-hazard portion of the curve occupy as much of the system lifetime as possible. To the extent that the curve "holds water" (and it may be true for a large class of complex systems), Drenick's theorem can be invoked to justify it. "Random failures" characterizes the interval of stationary component failure rates, where the failure process is essentially Poisson.

2.2 Failure as a Poisson process

Instead of looking at the distribution of times between failures, we can look at the arrival process for failure events—i.e., if $N(t)$ counts the number of failures in $(0, t]$, what is its probability distribution for any value of t? In the setup for Drenick's theorem (see Figure 2), we can think of each component generating a stream of failures, and all the component streams aggregating to produce a stream of system failures; Figure 4 is a pictorial representation.

The equivalent to Drenick's theorem from this perspective says that, subject to conditions described below, the aggregate stream converges to a Poisson stream, with PMF

$$\mathrm{P}\{N(t) = k\} = \exp(-\Lambda t)\Lambda^k/k!, \tag{1}$$

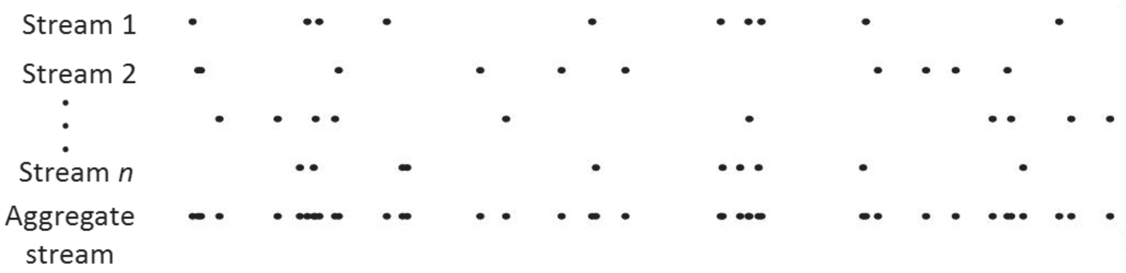

Figure 4: An example of aggregated arrival streams.

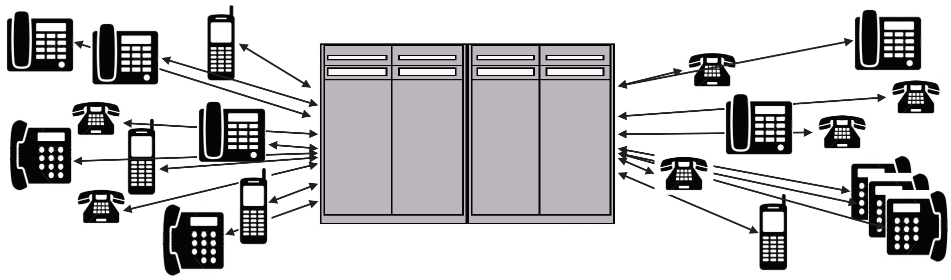

Figure 5: A telephone exchange aggregates many input streams with arbitrary distributions and small mean arrival rates.

where $\Lambda = \sum_i \lambda_i, i = 1, \ldots, n$ and λ_i is the arrival rate for failures of the ith component. This is formally equivalent to P$\{T_{n+1} - T_n \leq t\} = 1 - \exp(-\Lambda t)$, a statement about the interarrival times which is the conclusion of Drenick's result..

On reviewing the many roughly equivalent proofs of convergence to a Poisson process, one easily gets the impression (as pointed out in Section 1.4 of Kingman (1993)), that very general assumptions of randomness imply "the inevitability of the Poisson distribution." It was shown by Fry (1965) (Chapter IX, Section 116) that if points on the real line are placed both individually and collectively at random, their distribution in any finite interval must be Poisson. Of course, verifying that the appropriate randomness assumptions are satisfied in a particular application may be a non-trivial task.

2.2.1 Superposed renewal processes and queueing systems

A problem isomorphic to that of system reliability viewed as an aggregation of failure processes, each with small failure rate, is the following: picture a telephone exchange (as shown in Figure 5) servicing thousands of subscribers; each subscriber produces an input stream of calls with arbitrary distribution and a very small arrival rate. What are the characteristics of the aggregate stream of calls seen by the exchange? Historically, this type of process was studied much earlier than the equivalent reliability problem; work by telephone engineers attempting to predict the queueing behavior of exchanges started more than a century ago, and led to proofs that are equivalent to Drenick's.

Though engineers had long assumed, heuristically, that aggregate arrivals tended to be approximately Poisson distributed, the first proof of this phenomenon was published by Palm in 1943 (Palm (1988)). A more rigorous proof was given by Khintchine (1960, Chapter 5), who showed that the aggregate stream converges to a Poisson process with the PMF given in Equation (1) if

i) The individual streams are independent and stochastically stationary;

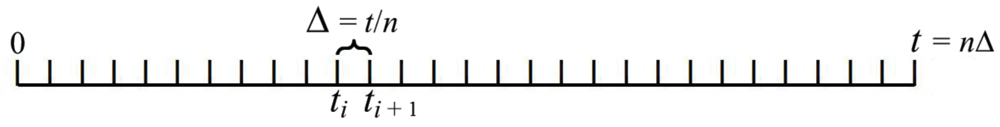

Figure 6: A Bernoulli process where independent failures may occur in any subinterval $(t_i, t_{i+1}]$.

(ii) As $n \to \infty$, each $\lambda_i \to 0$;

(iii) Given an arrival from an individual stream at time t, the probability of an arrival from the same stream in $(t, t + \Delta t]$ is o(Δt).

(Note that Drenick's condition (ii) is sufficient to satisfy (iii) here.) A similar proof, with somewhat weaker conditions, was given by Ososkov (1956). A general proof for streams viewed as renewal processes where each process has the same distribution was given by Cox and Smith (1954). For related proofs based on renewal and point process theory, see Cox (1962), Cox and Isham (1980), Chapter 7 of Thompson (1988), and Ross (1992).

2.2.2 *Binomial limit theorems*

As in Figure 6, consider a time interval $[0, t]$ divided into subintervals $(t_i, t_{i+1}]$, $i = 0, \ldots, n-1$, each of length $\Delta = t/n$, in any of which a failure may occur with probability p. The following theorem, due to Poisson, is well known: If P{failure in $(t_i, t_{i+1}]$} $\equiv p, \forall i$, and $p \to 0$ as $n \to \infty$ subject to $np = \Lambda$ (constant), then $N(t)$, the number of failures in $(0, t]$, is distributed as P$\{N(t) = k\} = \exp(\Lambda)\Lambda^k/k!$ (binomial convergence to Poisson).

It is less well known that the same convergence in distribution occurs if

(i) Failures in separate subintervals are independent,

(ii) P{failure in $(t_i, t_{i+1}]$} $= p_i$, $p_i \neq p_j$ for $i \neq j$,

(iii) $p_i \to 0$ as $n \to \infty$ and

(iv) $\sum_i p_i = \Lambda$.

See Koopman (1950) for a proof. Since $p_i \to 0$ implies P{≥ 2 failures in (t_i, t_{i+1})} $=$ o(Δ), this result is equivalent to Drenick's theorem.

This is an alternative way of conceptualizing the problem which may be difficult to apply in practice, since we may not be able to estimate the p_i. However, see Walsh (1955) for applications. A more abstract view of this convergence can be found in Gnedenko and Kolmogorov (1954) or Chapter VI of Loève (1977).

2.3 Applicability of Drenick's theorem to finite series systems

Figure 7 plots the results of a simulation showing how Drenick's result applies to a system with a finite number n of components. All the component failure laws are gamma(α, β), with α and β randomized over components, subject to having the system MTBF = $1/\Lambda$ and $\lim\sup_{n\to\infty} \frac{\lambda_i}{\Lambda} = 0$, where λ_i are the component failure rates. Even with n as small as 10, the exponential convergence is evident.

A second example simulates a series of components with a mixture of Weibull, gamma, and lognormal failure laws (the general density shapes are plotted in Figure 8). Distribution parameters are again randomized subject to the two conditions for Drenick's theorem. Figure 9 shows the convergence to exponential occurs in the simulation, though more components are required.

Drenick's theorem can also be be a reasonable approximation under weaker conditions than Drenick proposed; see Blumenthal et al. (1971, 1973). In particular, the assumptions

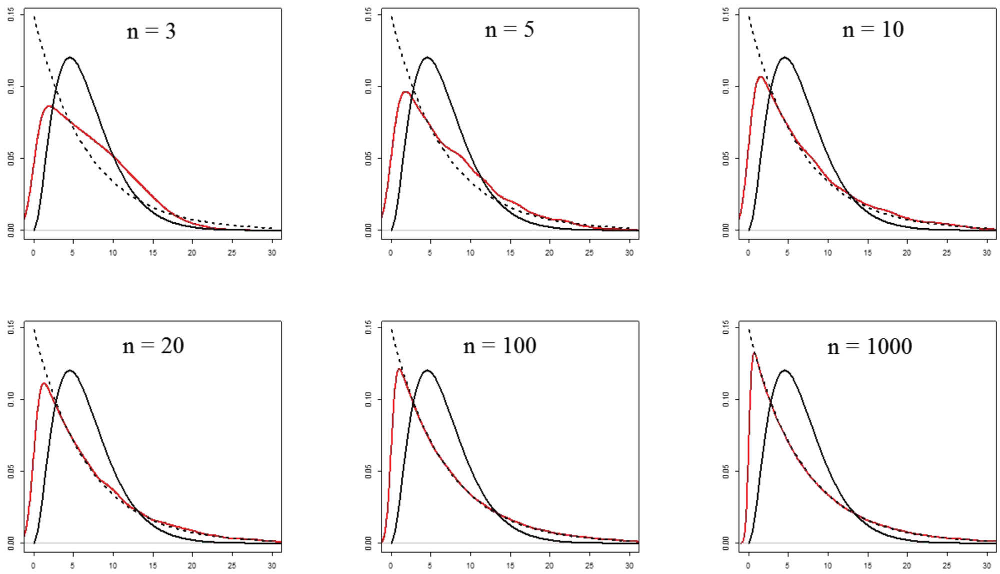

Figure 7: Convergence to an exponential law in a system with a finite number of components with gamma failure-time distributions. Red: kernel density estimate for system failure distribution; black: gamma distribution with the same mean; black dotted: exponential distribution with the same mean.

of independent components and having reached an equilibrium state can be relaxed. With somewhat different conditions, convergence to exponential also occurs for the time to first failure; see Drenick (1960).

2.4 Applicability to more general systems

One can argue that, in practice, well-designed high-reliability systems evolve through the development process based on the following design principles (see, e.g., Barlow and Proschan (1987); Mohamed et al. (1992); Halverson and Ozdes (1992)):

1. Identify components or subsystems that must all function in order for the system to function; i.e., identify a set of components or subsystems that are effectively in series.
2. Remove common-cause failure modes and other sources of dependency between failure times of individual elements.
3. Iteratively improve the reliability of selected elements until the desired reliability is achieved; this involves substituting higher-reliability components, or adding redundancy. (Adding component-level redundancy does not change the general series character of the system; see Figure 1.)

Regarding the last point, in order to optimally improve system reliability by improving component or subsystem reliability, it is easy to see that the system evolves towards equal reliability in each serial element. Suppose the ith element has reliability r_i, $i = 1, 2, \ldots, n$; then system reliability R_{Sys} is given by

$$R_{\text{Sys}} = \prod_{i=1}^{n} r_i$$

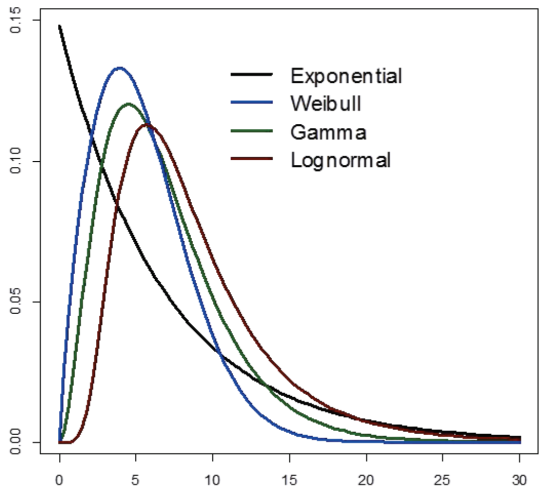

Figure 8: Base failure-time distributions for example 2.

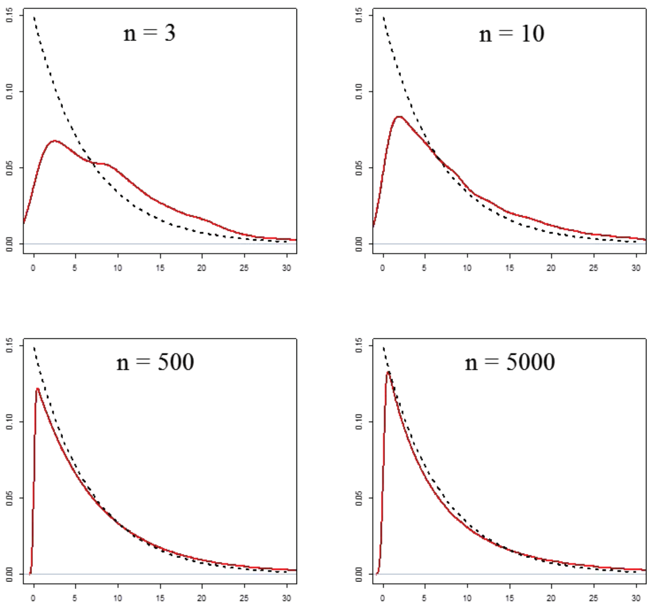

Figure 9: Convergence to an exponential law for example 2. Red: kernel density estimate for system failure distribution; black dotted: exponential distribution with the same mean.

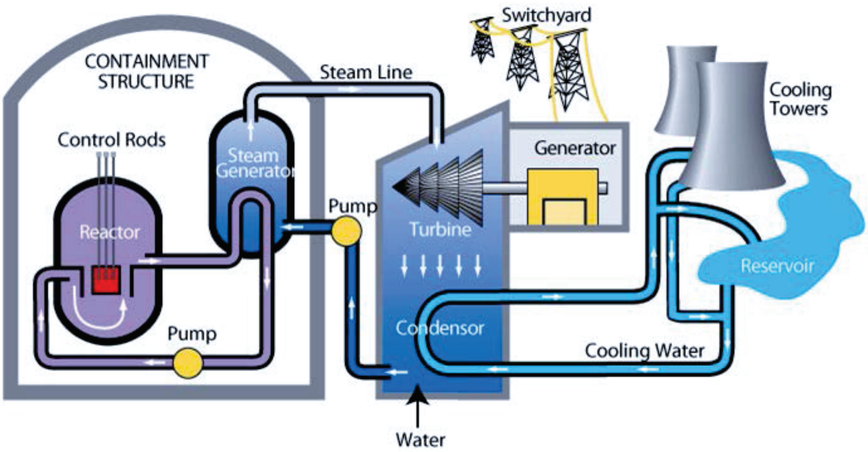

Figure 10: A typical nuclear power plant, showing piping loops.

and the improvement in system reliability by increasing the reliability of the jth element is given by

$$\frac{\partial R_{\text{Sys}}}{\partial r_j} = \prod_{i \neq j} r_i,$$

which is maximized by choosing $r_j = \min_i(r_i)$. Such a system, if it contains a large number of components, will eventually approximately satisfy the conditions of Drenick's theorem.

3. Application: Piping subsystem reliability

As an application example, we discuss the use of Drenick's theorem for estimating the reliability of piping subsystems in commercial nuclear power plants (NPPs) (e.g., see Westinghouse Electric (2003)). Figure 10 is a schematic view of a typical plant. The point of interest for this paper is that an NPP may contain 40+ miles of piping, with thousands of piping elements in series. Each element has a very low failure rate, thus on the face of it, NPP piping subsystems appear to fit the conditions for applicability of Drenick's theorem. Piping reliability issues will be similar for any system with many piping elements in series —e.g., for refineries, oil and natural gas pipelines, municipal water systems, etc.

In systems of this sort, available data on failures typically consists of failure rates per element, collected from many plants with varying total numbers of elements (Fleming and Lydell (2004), Mikschl et al. (1999)). An implicit assumption in (Fleming (2004)), which is not explicitly justified, is that a plant failure rate for a given failure mode can be derived by adding individual failure rates, i.e., essentially the "parts count" approach recommended by MIL-HDBK-217F.

Flaming (2004) proposed a continuous time Markov chain (CTMC) to model individual piping elements, whose state diagram (flowgraph) is shown in Figure 11. The states (nodes in the diagram) represent damage conditions. The forward transitions (links in the diagrams) represent successively worsening damage, with state 4 being catastrophic failure. The backward transitions represent repairs following detection by inspection. Transitions are labeled with the rate (per year) at which the transition occurs. Fleming et al. (1994) derived failure and repair rates from reactor operating logs; the data quality was such that only the first moments could be estimated, which was adequate given the assumption of exponential transition time distributions in Markov process models.

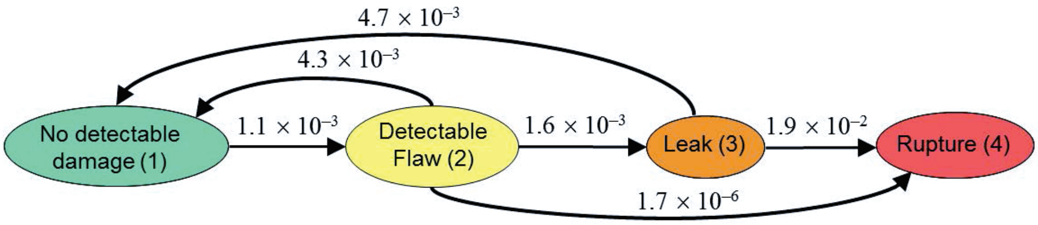

Figure 11: Flowgraph for Fleming's Markov model of pipe failure.

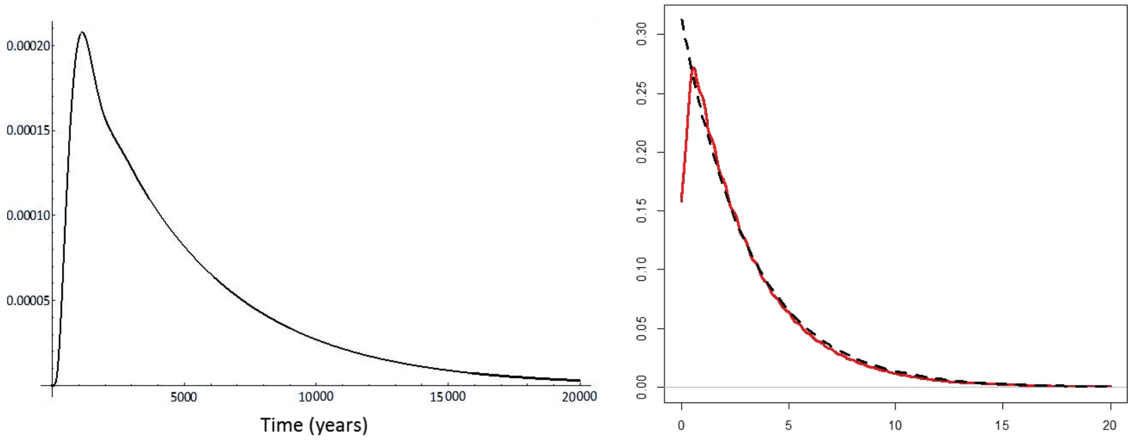

Figure 12: Left: failure density for one pipe element from the semi-Markov model; right: estimated system failure density for 1,000 pipe elements (dashed black is exponential, red is estimated failure density).

The assumption of exponential failure rates for the observed failure modes (e.g., stress corrosion cracking) is inconsistent with the literature on the subject. To improve on the CTMC, We proposed a Semi-Markov process model (SMP) based on Fleming's state transitions (Collins et al. (2013)). The SMP used Fleming's rates, with coefficients of variation and distribution models (lognormal, Weibull, and inverse Gaussian) derived from a literature review. The component failure law in this model is the first passage distribution from state 1 to state 4; its estimated density is shown on the left in Figure 12.

This model, like Fleming's, is for one component. The key question is whether Fleming's assumption of additive failure rates is justified for extrapolation to a plant failure model. To validate the use of Drenick's theorem in the context of the SMP model, we used simulation along the lines of that shown in Figures 7 and 9. The left-hand side of Figure 12, as mentioned, is the failure time density for a single element; the density shown was computed numerically using Laplace transform inversion. The right-hand side of the figure is based on sampling from the element failure density, and assuming 1,000 elements in series with the same failure law. Convergence to an exponential failure time distribution for the plant appears plausible[2].

[2] An artifact of kernel smoothing makes it impossible to accurately represent a density, like the exponential, with a discontinuity at the origin. Without this, the fit to an exponential density would be more obvious

4. Summary and discussion

We reviewed conditions for Drenick's theorem, and several proofs, and saw that its use for certain classes of systems is plausible, and consistent with engineering intuition. Simulation examples confirm this. In particular, piping subsystem reliability appears to be a valid application of the theorem. Despite the modern criticism of a "random failures" model for system reliability, we believe this area deserves further study. Topics for a future research agenda include:

- The extent to which perfect repair is required for validity of Drenick's theorem; see Ascher (1968) for a discussion of "good as new" versus "bad as old" repair, the latter being typical in some systems;
- Statistical tests for the conditions of the theorem, and for the presence of an exponential system failure law;
- The relationship of Drenick's theorem to extreme value theory, since for non-equilibrium conditions we expect the time to first system failure to be governed by the component with the minimum failure time, leading to a Weibull law.

5. Acknowledgments

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