Using Conjecturing Tasks to Support In-service Teachers' Conceptual Statistical Knowledge

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Abstract

The evolution of mathematics and science education standards has established a cross-curricular emphasis on data driven decision making and has, in turn, created a demand for teachers to deepen their conceptual statistical knowledge (Franklin et al., 2007; National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010; National Research Council, 1996; NCTM, 1989, 2000; NGSS Lead States, 2013). With many teachers' limited education in statistics and latent misconceptions of core statistical concepts, addressing this demand can be a challenge. In this paper, we describe how middle and high school math and science teachers used a conjecturing task in a professional development setting to deepen their conceptual statistical knowledge. First, we describe the nature of conjecturing tasks and review how they have been used during instruction in mathematics education studies. Then, we provide a detailed analysis of how in-service teachers completed the *Exploring Data Sets* conjecturing task. Specifically, we examine alternative statistical conceptions that teacher participants held, illustrate the ways the conjecturing task transformed teacher participants' statistical understandings, and depict the ways teacher participants used technology to support and revise their statistical understandings as they completed the conjecturing task.

Key Words: Teacher professional development, conjecturing tasks, misconceptions, statistical conceptual knowledge

1. Introduction

As schools across the United States implement the Common Core State standards, middle and high school mathematics and science teachers are charged with the task of helping students develop statistical thinking. As a result, mathematics and science educators are reconsidering what in-service math and science teachers need to know regarding the role of statistics in math and science curricula (Kader & Jacobbe, 2013; Peck, Gould, & Miller, 2013; Sotos, Vanhoof, Van den Noortgate, & Onghena, 2007). We formed the Statistical Reasoning and Thinking (StaRT) Project in an effort to help seventh through twelfth grades in-service mathematics and science teachers deepen their conceptual statistical knowledge. In this paper, we describe how our team of mathematics and science educators used conjecturing tasks to to accomplish this goal. StaRT teacher participants completed conjecturing tasks during a two-week summer professional development institute. The StaRT Project team led the institute activities and consists of a mathematics educator in the Department of Mathematical Sciences, an elementary and middle school math and science educator in the Department of Elementary Education, and three graduate assistants pursuing their Ph.D.'s in Mathematics and Science Education at a regional university in Middle Tennessee. StaRT teacher participants came from both middle and high school teaching backgrounds and possessed a wide range of teaching and statistics experience. During the 10-day summer institute, participants completed what we termed *Doing Statistics Tasks* and *Conjecturing Tasks*. *Doing Statistics Tasks*, required teacher participants to fully engage in the process of formulating questions, collecting data, analyzing data, and interpreting results – all four components of statistical investigations identified in the GAISE framework (Franklin et al., 2007). *Conjecturing Tasks* required teacher participants to formulate and revise their own conjectures regarding fundamental statistical concepts. In the next section, we identify the features of conjecturing tasks in greater detail.

2. Conjecturing Tasks

A conjecture is a plausible proposition of which the truth has not yet been established (Cheng & Lin, 2007). The StaRT Project team devised conjecturing tasks to give teacher participants an opportunity to: 1) make a plausible proposition regarding their conceptual understanding of a statistical concept and 2) test that conjecture against novel problematic situations. Conjecturing tasks have been used in the classroom both to develop conceptual understanding (Norton, 2008) and to support formal mathematical proof (Herbst, 2006). We used conjecturing tasks to develop conceptual understanding, and we designed them in accordance with the following conjecturing task design principles delineated by Lin, Yang, Lee, Tabach, & Stylianides (2012).

First, a conjecturing task must provide opportunities for systematic observation of some phenomenon. Second, the task should provide an opportunity for the construction of knowledge based on the observations made. Third, a conjecturing task should bring prior knowledge to the fore and provide opportunities to transform that knowledge into more sophisticated knowledge and understanding. Finally, a conjecturing task should provide ample opportunities to reflect on the truth of the formulated conjectures based on readily available evidence. It is certainly possible to form incorrect and meaningless conjectures when following the first three principles above, so including a time for reflection on the veracity of the formulated conjectures is essential when successfully implementing conjecturing tasks.

Norton (2008) and Liu & Ho (2008) provide examples of investigations that have used conjecturing tasks to study mathematics learning. Drawing on the work of Piaget and von Glassersfeld, Norton analyzed one student's construction of schemes for understanding fractions. Liu and Ho examined elementary school students' learning of statistics terms through the use of making and testing conjectures. At the heart of both of these studies is the idea that people learn when they observe a problematic situation, use their current knowledge to form a conjecture for the potential solution of the problem, and then reflect on and revise those conjectures as they test them against available data.

In preparation for the StaRT Institute, we developed eleven conjecturing tasks based on the above principles (Lin et al. 2012) and our own teaching experiences. We implemented nine of the eleven tasks during the course of the summer institute. As we planned for the implementation of these conjecturing tasks, we sought to provide opportunities for the StaRT teacher participants 1) to make critical decisions in the conjecturing process and 2) to play a role in determining what they would investigate and how they would conduct the investigation. These efforts were made in the hopes of producing richer and more varied conjectures among the teacher participants (McGraw & Grant, 2005).

So far, we have described the structure of conjecturing tasks and their place in the StaRT professional development project. In the remainder of this paper, we describe research we conducted during the implementation of one conjecturing task (*Exploring Data Sets*) in order to illuminate the ways the task contributed to the development of teacher participants' conceptual statistical knowledge.

3. The Research Study

The StaRT team developed conjecturing tasks for the purpose of helping teacher participants deepen their conceptual understanding of statistics. As we implemented these tasks with teacher participants, we collected data to gain insight into the following guiding research questions:

1. What alternative conceptions regarding statistics do our participants hold?
2. Can conjecturing tasks be used to successfully change participants' naïve understandings of statistics to a more formal understanding?
3. How did participants use technology to support their development of and revision of statistical conjectures?

3.1 Participants

Thirty-five middle school and high school mathematics and science teachers from across Middle Tennessee participated in the StaRT Project. Teachers ranged from having twenty plus years of experience in the classroom to teachers completing their first year. Of the 35 teacher participants, 22 were able to complete both the pre-test and post-test (described below). Eight of the 22 teachers were males and 14 were females; 18 taught math and four taught science. Each participant completed tasks using the TI-Nspire calculator during the professional development. At the conclusion of the institute, they were given Nspire teacher software and 10 Nspire handhelds to use in their classrooms as they implement Common Core aligned mathematics and science statistics-related lessons with their students.

3.2 Data Collection and Analysis

During the StaRT Project, the StaRT team collected qualitative and quantitative data to inform our guiding research questions. Pre- and post-test data were collected at the beginning and end of the ten day summer institute. These tests consisted of 28 questions taken from the NSF-supported ARTIST database (Garfield, 2006) that covered a variety of statistical concepts such as samples and sampling, measures of spread, confidence intervals, central limit theorem, and data representation. This pre- and post-test data allowed the StaRT team to determine whether teacher participants' knowledge had changed over the course of the professional development sessions. Test items were grouped according to the key statistical concepts they addressed so that we could analyze pre- and post-test data in order to track the participants' growth and areas of strength or weakness. Possible gains in content knowledge were calculated by finding the difference between post-test and pre-test scores. A paired *t* test was used to identify significance of findings.

In addition to the quantitative data, the StaRT team also collected participants' responses to the conjecturing tasks via TI-Nspire documents and through answers to polling questions collected using TI Navigator software. We gathered additional qualitative data via conversations with the participants, photographs of participants' data representations, and field notes of questions asked and answered during the conjecturing tasks. We analyzed the qualitative data using a theme analysis to provide insights into various conceptions and misconceptions the participants held.

3.3 Description of Exploring Data Sets Task

The *Exploring Data Sets* task is representative of many of the conjecturing tasks we implemented with StaRT teacher participants during the professional development. We will first describe this task and then present the findings of our data analysis and the conclusions we draw from that data that speak to our guiding research questions.

The *Exploring Data Sets* task asked participants to create three different data sets of 10 integers from 0 to 10 that all have the same mean but different amounts of variability. Relying on their intuition, participants made conjectures as to which data sets had more variability. After making their conjectures, participants entered the data sets into a TI-Nspire spreadsheet. This spreadsheet was dynamically linked to other Nspire pages that displayed dotplots of participants' data sets and reported the mean and standard deviation (described as the average distance from the mean) for the data sets. After students saw the representations of their data sets, they were asked if they thought their initial conjectures were correct. Participants were also asked to report the factors they believed influenced their initial thinking and the ways in which their initial thinking had changed. After collecting this information using the polling feature of the TI-Navigator software, the StaRT team identified several possible alternative conceptions held by the participants. These conceptions and the role they played in teacher participant learning are detailed in the next section.

3.4 Findings

A theme analysis of StaRT participant's Nspire documents, poll responses, and class discussions revealed several alternative conceptions participants held regarding variability in a data set. The first of these alternative conceptions was that greater range implies greater variability in a data set. To illustrate this, we present data sets that Beth (all names are pseudonyms) created containing three different ranges such that data set 1 had the highest range and data set 3 had the lowest (see Figure 1). Based on this ordering and her comments during moments of class reflection and sharing, we believe Beth thought that data set 1 had the most variability and data set 3 had the least. We indicate these conjectures in Figure 1 with the text at the top of each data set, and we show in red text whether these conjectures are correct or incorrect based on the average distance from the mean measure reported at the bottom of each of the three screenshots. When Beth was confronted with evidence that conflicted with her conjectures (i.e. that data set 1 had the middle measure of Variability and that data set 2 had the highest), she may have realized her conceptual understanding of "variability as range" conflicted with the measure being used to describe variability. Beth's comments collected from the class poll indicate that she "had mistaken variability for range".

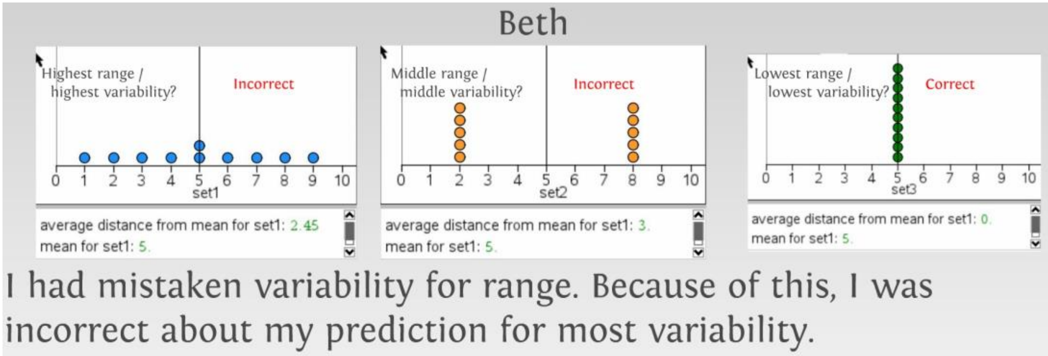

Figure 1: Beth's data sets and comments.

Wendy's data sets and comments also exemplify the "variability as range" conception (see Figure 2). However, all three data sets that Wendy created actually supported her alternative conception rather than providing conflicting evidence. During the reflection portion of this conjecturing task, we asked teacher participants to discuss their findings in small groups. Wendy was paired with Beth, and it was Beth's data that led Wendy to alter her initial conjectures regarding her conceptual understanding of variability. Wendy says, "I thought variance and range were related but realized there can be greater range but little variability. (Thanks Beth!)"

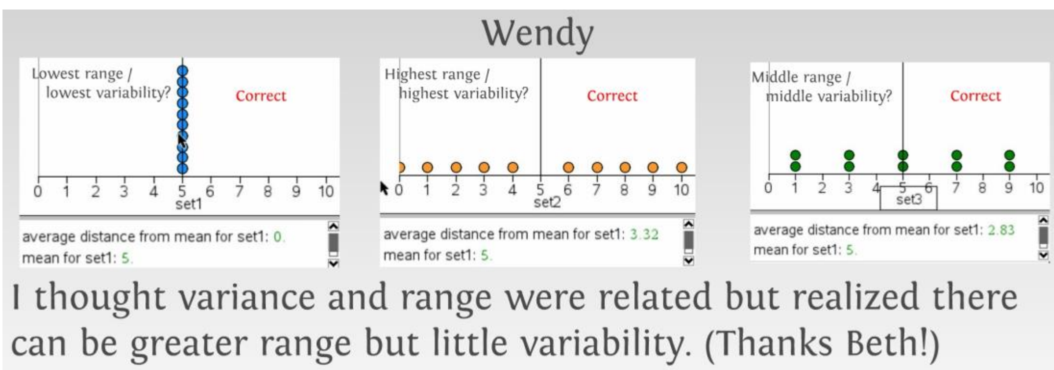

Figure 2: Wendy's data sets and comments.

A second alternative conception we garnered from the teacher participant data was that data sets with high variety (i.e., sets that contain many different numbers with few repetitions) also possess high variability. For example, Yuri created data set 1 with one repeated number, data set 2 with five repeats (8 was repeated three times and 2 was repeated twice), and data set 3 with three repeats (see Figure 3). Based on this data and the ensuing class discussion we gathered that Yuri suspected data set 1 would have the highest variability and data set 2 would have the lowest (since it had the most repeated values). Yuri's data confronted this alternative conception. Unfortunately, since none of his data sets fit his conjecture, Yuri seemed to have difficulty finding a pathway for adapting his conjecture. We sense Yuri's frustration when he merely says, "the data showed otherwise" without providing thoughts as to how he might adapt his conjecture to be more conceptually sound.

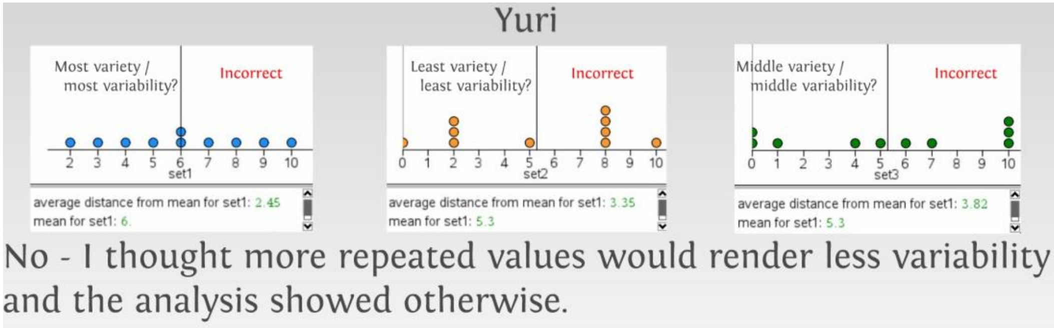

Figure3: Yuri's data sets and comments.

Like Yuri, Weslie's data sets and comments (see Figure 4) suggested that she had focused on variety. Her data sets progressed more systematically from no variety (many repeated values) to greater variety (less repeated values) with data set 1 having the least variety and data set 2 having the most. The displays of Weslie's data sets, however, more clearly illustrate the increasing spread of the data and perhaps suggested a pathway for adapting her conjecture. When Weslie said, "I looked at variability with respect to the numbers used," we believe she was saying that since she "used" only one number in data set 1 that it should have the least variability, and that since she "used" seven numbers (1, 2, 3, 4, 5, 6, and 7) in data set 2 it should have the most variability. When the measure of variability in the Nspire pages presented evidence that conflicted with her conjecture, she was able to identify that variability is a more nuanced concept than just how many different numbers are used in a data set. Indeed, when Weslie shared her data sets with the entire StaRT group during an all class discussion, we first began using the word "variety" to describe Weslie's conception of variability. She confirmed that this was the notion she was relying on, and the group began to establish that "distance from the mean" and "variety" in a data set are not as closely linked as one may think.

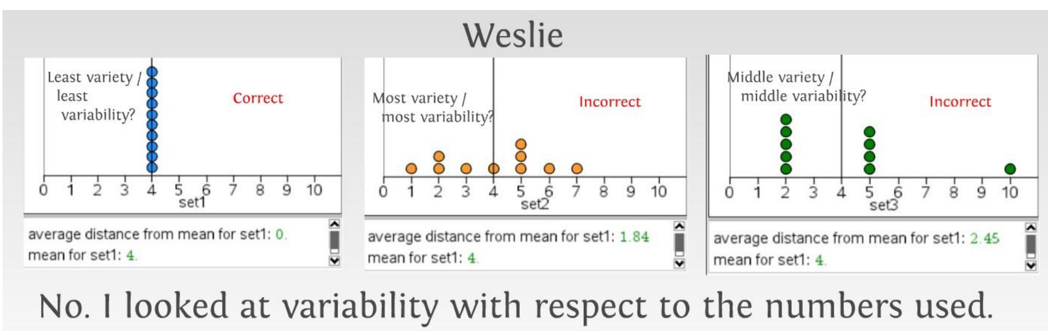

Figure 4: Weslie's data sets and comments.

A third misconception that became apparent upon examining participants' descriptions of how their understanding of variability changed as a result of completing this conjecturing task is that several of the participants viewed variability as a measure, not a concept. Based on a summary of the comments we received (see Figure 5), the word *variability* acted as a cue for some participants to perform a calculation, often the range. We seek to highlight this misconception because there is little reason to think that the data produced during this task would cause students to confront this misconception. The data did not specifically address terminology. As teacher participants' shared their reflections on the task and as we commenced all-class discussions throughout the completion of the task, opportunities emerged to address misconceptions that had not been addressed by the design of the task. Thus, classroom discussions served the critical dual purpose of 1) providing participants more data (from other participants) to help refine their conjectures and 2) giving participants a chance to confront other latent misconceptions.

Variability as a measure

- I had mistaken variability for range.
- [T]he variability for one set was 0 and the other set had a variability of 3.
- I incorrectly thought that variability meant different range of numbers rather than the average distance from the mean.
- Variability is the distance between points in the data.
- Variability seems to be defined as the average distance of all the data points from the mean.

Variability as a concept

- Variability means how different the numbers are.
- Vary means change.
- The greater the distance between the numbers, the more the variability.

Figure 5: Participant's summaries of what variability is.

The data analysis not only revealed how conjecturing tasks uncovered teacher participants' alternative conceptions of statistics, it also showed how conjecturing tasks successfully helped participants change their naïve understandings of statistics to more formal ones. We analyzed pre-test and post-test scores from this 28 question assessment using a paired *t*-test. Significant gains were found in both the overall score ($p < 0.001$) as well as the subset of nine questions targeted by the content in this particular task ($p < 0.05$). The number correct on the full test increased by an average of 3.29 questions, and the number correct on the subset of nine increased by .79 of a question.

The third question guiding this study was "How did participants use the technology to support their development of and revision of statistical conjectures?" From this work, we were able to glean several implications for future use of conjecturing tasks.

When implementing a conjecturing task, it is critical that students have plenty of opportunities to collect new data and to discuss findings with their peers and with their instructor. It is also recommended that students use tools, such as the TI-Nspire, that will assist them in making and investigating conjectures by allowing them to work in a dynamic environment that removes distractors such as tedious calculations. For the conjecturing task described earlier, it was essential for students to see the dotplots and the variation measures quickly after they made their conjecture and to flip effortlessly between the dotplots to make comparisons. Additionally, it is recommended that adequate time be allotted for the instructor to examine the student responses to the question and the calculator files that they have created. Taking time to examine these responses allowed the StaRT team to find evidence of conceptions and misconceptions and to carefully select and to sequence students to share their findings during the class-wide discussion. This careful sequencing allowed for the instructors to show a progression of conceptions, moving from naïve to statistically accurate, hopefully encouraging conceptual change among the participants.

4. Discussion

Conjecturing tasks afford instructors and students a venue for drawing out latent student misconceptions. Correcting misconceptions requires students to test their existing conceptions against new evidence. Experiencing conflict generated when their conceptions do not match the evidence promotes conceptual change. By requiring students to use only their existing knowledge to formulate conjecture, they are forced to rely on the concepts they have developed preceding the lesson, including these latent misconceptions that might have otherwise remained unearthed. Not all students have the same alternative conceptions, thus conjecturing tasks can be unpredictable and time consuming. As a team, the researchers were able to anticipate many of the responses prior to the task, but not all of them and certainly not every misconception. Thus, we had to rely on our abilities to quickly diagnose these issues as the students were submitting their responses. This might be even more challenging for newer statistics instructors that are less familiar with the misunderstandings that students bring to the classroom. In these cases it is suggested that the instructor familiarize themselves with the alternative conceptions identified in the literature as well as have the students submit their conjectures at the end of the day or even prior to the next class, thus giving the instructors plenty of time to study the responses.

During the course of this paper, the researchers defined a conjecturing task as one that requires students to test existing conceptual understandings against data in novel problematic situations. The focus was limited to part of one conjecturing task given during the StaRT Project institute. Student-created dotplots and quick poll responses were used to illustrate three misconceptions that were elicited during this task. Furthermore, it became evident how a combination of individual data, group data, and class discussions may have provided these students with pathways for refining their conjectures by either adapting to the conflicting data or accommodating the supporting data. There are two follow-up meetings scheduled with these teachers in the fall of this year where the researchers will be able to see if some participants' misconceptions returned (a common occurrence with misconceptions) (Duit & Treagust, 2003) by collecting a second round of post-test data.

Acknowledgement

The StaRT Project was supported by a STEM PD grant of $199,988 in U.S. federal funds administered by the state of Tennessee.

References

Cheng, Y., & Lin, F. (2007). The effectiveness and limitation of reading and coloring strategy in learning geometry proof. In J. Woo, H. Lew, K. Park, & D. Seo (Eds.), *Proceedings of the 31st annual conference of the International Group for the Psychology of Mathematics Education*. Soul, Korea: PME31.

Duit, R., & Treagust, D. F. (2003). Conceptual change: A powerful framework for improving science teaching and learning. *International Journal of Science Education*, *25*(6), 671–688.

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2007). *Guidelines for assessment and instruction in statistics education (GAISE) report: A pre-K-12 curriculum framework*. American Statistical Association.

Garfield, J. (2006). ARTIST home page. Retrieved from https://apps3.cehd.umn.edu/artist/publications.html

Herbst, P. G. (2006). Teaching Geometry with Problems: Negotiating Instructional Situations and Mathematical Tasks. *Journal for Research in Mathematics Education*, *37*(4), 313–347. doi:10.2307/30034853

Kader, G., & Jacobbe, T. (2013). *Developing essential understanding of statistics for teaching mathematics in grades 6-8*. (P. Wilson, Ed.) (1st edition.). Reston, VA: National Council of Teachers of Mathematics.

Lin, F.-L., Yang, K.-L., Lee, K.-H., Tabach, M., & Stylianides, G. (2012). Principles of task design for conjecturing and proving. In G. Hanna & M. de Villiers (Eds.), *Proof and proving in mathematics education* (pp. 305–325). Springer Netherlands. Retrieved from http://link.springer.com/chapter/10.1007/978-94-007-2129-6_13

Liu, S., & Ho, F. (2008). Conjecture activities for comprehending statistics terms through speculations on the functions of imaginary spectrometers. *Australian Mathematics Teacher, 64*(3), 17–24.

McGraw, R., & Grant, M. (2005). Investigating mathematics with technology: Lesson structures that support a range of methods and solutions. In W. J. Masalski & P. C. Elliott (Eds.), *Technology-supported mathematics learning environments: Sixty-seventh yearbook* (pp. 303–317). Reston, VA: NCTM.

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.

National Research Council. (1996). *National science education standards*. Washington, D.C.: National Academy Press.

NCTM. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

NCTM. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

NGSS Lead States. (2013). *Next generation science standards: For states, by states*. Washington, D.C.: The National Academies Press.

Norton, A. (2008). Josh’s operational conjectures: Abductions of a splitting operation and the construction of new fractional schemes. *Journal for Research in Mathematics Education*, *39*(4), 401–430. doi:10.2307/40539305

Peck, R., Gould, R., & Miller, S. J. (2013). *Developing essential understanding of statistics for teaching mathematics in grades 9-12*. (P. S. Wilson, Ed.). Reston, VA: National Council of Teachers of Mathematics.

Sotos, A. E. C., Vanhoof, S., Van den Noortgate, W., & Onghena, P. (2007). Students’ misconceptions of statistical inference: A review of the empirical evidence from research on statistics education. *Educational Research Review*, *2*(2), 98–113.