Inference Procedures for Bivariate Exponential Model of Gumbel in Reliability Theory

Paul R Savariappan[1] S.Thobias[2], P.Chandrasekhar[2], M. Syluvai Anthony[2]
paul@luther.edu drchandrasekharin@yahoo.co.in
1. Department of Mathematics, Luther College, Decorah, Iowa – 52101, USA
2. Department of Statistics, Loyola College, Chennai - 600 034, India.

Abstract

Assuming that the joint distribution of lifetime and repair time of a failed unit in a two unit cold standby system is bivariate exponential (see Gumbel(1960)), measures of system performance such as system reliability, MTBF, point availability and steady state availability are obtained. Further, 100 (1-α) % asymptotic confidence limits for steady state availability of the system and an estimator of system reliability based on moments are obtained. Numerical work is carried out to illustrate the behaviour of steady state availability as well as the system reliability based on moments by simulating samples from bivariate exponential distribution due to Gumbel (1960).

Keywords and Phrases: Multivariate Central Limit theorem, Slutsky theorem, Standby system, Steady state availability.
AMS 1991 Subject Classification: 60 K 10, 60 F 05.

1. Introduction

Introduction of redundancy, repair maintenance and preventive maintenance are some of the well-known methods by which the reliability of a system can be improved. Several authors have extensively studied two unit standby redundant systems in the past. Osaki and Nakagawa (1976) give a bibliography of the work on two unit systems. Most of the studies on two unit warm standby systems are confined to obtaining expressions for various measures of system performance and do not consider the associated statistical inference problems. Chandrasekhar and Natarajan (1994) have considered a two unit cold standby system and obtained the exact confidence limits for the steady state availability of the system under the assumption that the lifetime of online unit and the repair time of a failed unit are independent.

It is well known that the steady state availability is a satisfactory measure for systems, which are operated continuously (e.g., a detection radar system). A point estimate of steady state availability is usually the only statistic calculated, although decisions about the true steady state availability of the system should take uncertainty into account. Since A_∞ = MTBF/(MTBF+MTTR) is estimated by $\hat{A}_\infty = M\hat{T}BF/(M\hat{T}BF+M\hat{T}TR)$, the uncertainties in the values of $M\hat{T}BF$ and $M\hat{T}TR$ lead to an uncertainty in the estimated value of the steady state availability. By treating these estimated parameters as random variables, we can obtain the distribution of the estimated point steady state availability $\hat{A}_\infty$ by combining the distribution of operation and repair times. Hence we can construct estimators and confidence intervals for the steady state availability A_∞.

The exponential distribution, which is analytically very simple to understand, plays a prominent role in Statistics, since it enjoys lack of memory property (LMP). Further it has served as a tool to modeling in “Life testing and Reliability”. In fact, exponential distribution can be used as the starting point for the theory of extreme values. Hence it is of interest to study bivariate distributions, whose marginal are exponential. Many bivariate extensions of exponential distribution are proposed in the literature in order to model the dependence of life times of units in a system. In the case of bivariate exponential model of Marshall and Olkin(1967), because of a singular part, its use is not appropriate in situations, where simultaneous failures of units are quite unlikely to occur.

In Reliability theory, for a two unit or multiunit systems, the failure rate of one unit or more might change upon the failure of other unit or units respectively. Common cause failure or similar environmental conditions may also lead to the dependence of units in a system. Paul Rajamanickam and Chandrasekar(1997) have obtained measures of system performance for two unit systems with a repair facility assuming that the life times and repair time follow a trivariate exponential distribution of Marshall and Olkin(1967). Further, Paul Rajamanickam and Chandrasekar(1998) have considered a system with a repair facility, in which the lifetime and repair time are not necessarily independent and obtained a CAN estimator for the steady state availability of the system and presented the techniques for determining the asymptotic confidence limits for the same. Recently, Chandrasekhar et al (2011) have studied in detail the applications of bivariate and trivariate exponential distributions for a two unit cold and hot standby systems respectively and obtained the measures of system performance. For a flexible model which can accommodate both independent and dependent cases, we use a bivariate exponential distribution due to Gumbel(1960) to model the dependence of life time of online unit and repair time of failed unit in a two unit cold standby system. Hence an attempt is made in this paper to derive a 100(1-α)% confidence interval for the steady state availability of a two unit cold standby system under the assumption that the joint distribution of lifetime of online unit and the repair time of a failed unit in the system is bivariate exponential proposed by Gumbel (1960). The model and the assumptions are discussed in detail in the following section.

Standby Systems

2. Model (Cold standby system)

2.1 The model and assumptions

The system under consideration is a two unit cold standby system with a single repair facility. We have precisely the following assumptions.

(i) The units are similar and statistically not independent. One unit is operating online and other unit is kept as a cold standby. i.e., A unit in standby will not fail. Each unit while online has a constant failure rate say λ_1 and constant repair rate λ_2.

(ii) There is only one repair facility.

(iii) Let T and R denote the lifetime of online unit and repair time of a failed unit respectively in the system. Assuming that the lifetime and the repair time of a unit in the system are statistically dependent, it is only appropriate to consider the following bivariate exponential (BVE) distribution for T and R with the joint survival function and the joint density function respectively given by

$$\overline{F}(y_1, y_2) = e^{-(\lambda_1 y_1 + \lambda_2 y_2 + \theta\lambda_1 y_1 \lambda_2 y_2)} \qquad (2.1)$$

$$f(y_1, y_2) = \lambda_1 \lambda_2 e^{-(\lambda_1 y_1 + \lambda_2 y_2 + \theta\lambda_1 y_1 \lambda_2 y_2)} [(1+\theta\lambda_1 y_1)(1+\theta\lambda_2 y_2) - \theta],$$

$$y_1, y_2 \geq 0; \lambda_1, \lambda_2 > 0; 0 \leq \theta \leq 1. \quad (2.2)$$

see Gumbel (1960).

(iv) Each unit is new after repair and
(v) Switch is perfect and the switchover is instantaneous.

Note: It may be observed that

a. The lifetime T and repair time R are exponentially distributed random variables with the expected values $\frac{1}{\lambda_1}$ and $\frac{1}{\lambda_2}$ respectively.

b. The covariance between T and R is given by

$$\text{Cov(T,R)} = \frac{1}{\lambda_1\lambda_2}\rho \quad \text{where } \rho = \left[\frac{1}{\theta}e^{1/\theta}\ Ei\left(\frac{1}{\theta}\right) - 1\right]$$

and $Ei(x) = \int_x^\infty \frac{e^{-z}}{z} dz$

c. The random variables T and R are independent if $\theta = 0$.

2.2 Analysis of the system

To analyse the behaviour of the system, we note that at any time t, the system will be found in any one of the following mutually exclusive and exhaustive states:
a = (0,0): one unit is operating online and the other is kept in standby.
b = (0,1): one unit is operating online and the other is under repair.
c = (1,1): one unit is under repair and the other is waiting for repair, where
the symbols 0 and 1 represent the operable and failed states of a unit respectively.

Let X(t) and Y(t) denote the state of online unit and the state of the other unit respectively at time t . The vector process Z(t) = {(X(t), Y(t)),t≥0} with the state space E given by

$$E = \{(0,0), (0,1), (1,1)\} \quad (2.3)$$

denotes the state of system at time t.

Since bivariate exponential distribution proposed by Gumbel (1960) has exponential marginals and exponential distribution satisfies LMP, it follows that the stochastic process describing the behaviour of the system is a Markov Process with the infinitesimal generator given by

$$Q = \begin{array}{c} (0,0) \\ (0,1) \\ (1,1) \end{array} \begin{bmatrix} -\lambda_1 & \lambda_1 & 0 \\ \lambda_2 & -(\lambda_1+\lambda_2) & \lambda_1 \\ 0 & \lambda_2 & -\lambda_2 \end{bmatrix} \quad (2.4)$$

It may be noted that the system upstates are (0,0) and (0,1) ,while the state (1,1) is the system downstate. Let $p_{ij}(t)$ = Pr[Z(t)=(i,j)] $\forall$ (i,j) $\in$ E represent the probability that the system is in state (i,j) at time t with the initial condition $p_{00}(0)=1$.We assume that initially both the units are operable and obtain the measures of system performance as follows:

2.2.1 System Reliability

The system reliability R(t) is the probability of failure free operation of the system in (0,t]. To derive an expression for the reliability of the system, we restrict the transitions of the Markov Process to the upstates namely a and b. Using the infinitesimal generator of the process in (2.4), pertaining to these upstates, we derive the following differential – difference equations:

$$\frac{dp_a(t)}{dt} = -\lambda_1 p_a(t) + \lambda_2 p_b(t) \qquad (2.5)$$

$$\frac{dp_b(t)}{dt} = \lambda_1 p_a(t) - (\lambda_1+\lambda_2) p_b(t) \qquad (2.6)$$

Let $L_i(s)$ be the Laplace transform of $p_i(t)$, i = a, b. Taking Laplace transforms on both the sides of the differencial-difference equations given in (2.5) and (2.6), solving for $L_i(s)$, i =a, b and inverting, we get $p_a(t)$ and $p_b(t)$.Thus the system reliability is given by

$$R(t) = \frac{[(\alpha_1+2\lambda_1+\lambda_2)e^{\alpha_1 t} - (\alpha_2+2\lambda_1+\lambda_2)\, e^{\alpha_2 t}]}{(\alpha_1-\alpha_2)}, \qquad (2.7)$$

where α_1 and α_2 are the roots of $s^2+(2\lambda_1+\lambda_2)s+\lambda_1^2 = 0$

2.2.2 Mean time before failure (MTBF)

The system mean time before failure is given by

$$\text{MTBF} = L_a(0)+ L_b(0)$$

$$= \frac{(2\lambda_1+\lambda_2)}{\lambda_1^2} \qquad (2.8)$$

2.2.3 System Availability

The system availability A(t) is the probability that the system operates within the tolerances at a given instant of time t and is obtained as follows:

From the infinitesimal generator given in (2.4), we obtain the following system of differential – difference equations.

$$\frac{dp_a(t)}{dt} = -\lambda_1 p_a(t)+ \lambda_2 p_b(t) \qquad (2.9)$$

$$\frac{dp_b(t)}{dt} = \lambda_1 p_a(t)- (\lambda_1+\lambda_2) p_b(t)+ \lambda_2 p_c(t) \qquad (2.10)$$

$$\frac{dp_c(t)}{dt} = \lambda_1 p_b(t)- \lambda_2 p_c(t) \qquad (2.11)$$

Sovling the system of equations (2.9) — (2.11) and using the fact that
$\sum_{i = a, b, c} p_i (t) = 1$, we obtain the solution as follows:

$$p_a(t) = \frac{\lambda_2^2}{\alpha_1\alpha_2} + \lambda_1 \lambda_2 \left[\frac{(\alpha_1 +\lambda_2)}{\alpha_1 (\alpha_1-\alpha_2)(\alpha_1 +\lambda_1)} e^{\alpha_1 t} + \frac{(\alpha_2 +\lambda_2)}{\alpha_2 (\alpha_2-\alpha_1)(\alpha_2 +\lambda_1)} e^{\alpha_2 t} \right] \qquad (2.12)$$

$$p_b(t) = \frac{\lambda_1 \lambda_2}{\alpha_1\alpha_2} + \lambda_1 \left[\frac{(\alpha_1 + \lambda_2)}{\alpha_1(\alpha_1 - \alpha_2)} e^{\alpha_1 t} + \frac{(\alpha_2 + \lambda_2)}{\alpha_2(\alpha_2 - \alpha_1)} e^{\alpha_2 t} \right] \quad (2.13)$$

$$p_c(t) = \frac{\lambda_1^2}{\alpha_1\alpha_2} + \frac{\lambda_1^2}{\alpha_1(\alpha_1 - \alpha_2)} e^{\alpha_1 t} + \frac{\lambda_1^2}{\alpha_2(\alpha_2 - \alpha_1)} e^{\alpha_2 t}, \quad (2.14)$$

where α_1 and α_2 are the roots of

$$s^2+2(\lambda_1+\lambda_2)s+[\lambda_1(\lambda_1+\lambda_2)+\lambda_2^2] = 0$$

Hence, the system availability is given by

$$A(t) = p_a(t) + p_b(t)$$

$$= \frac{\lambda_2(\lambda_1+\lambda_2)}{\alpha_1\alpha_2} + \lambda_1^2 \left[\frac{e^{\alpha_1 t}}{\alpha_1(\alpha_2-\alpha_1)} + \frac{e^{\alpha_2 t}}{\alpha_2(\alpha_1-\alpha_2)} \right] \quad (2.15)$$

2.2.4 Steady state availability

The system steady state availability is given by

$$A_\infty = \lim_{t \to \infty} A(t)$$

$$= \frac{\lambda_2(\lambda_1+\lambda_2)}{[\lambda_1(\lambda_1+\lambda_2)+\lambda_2^2]} \quad (2.16)$$

It may be noted that (2.7),(2.8),(2.15) and (2.16) are in agreement with John G. Rau (1970). In the following section, we obtain moment estimator, CAN estimator and 100(1- α)% asymptotic confidence interval for the steady state availability of two unit cold standby and hot standy system.

3. Confidence interval for steady state availability of the system

3.1 Moment and CAN estimators for the steady state availability of the system

Let (X_1, Y_1), (X_2, Y_2),…,(X_n,Y_n) be a random sample of size n drawn from a bivariate exponential failure time and repair time population with the survival function given by (2.1). It is clear that $\overline{X}$ and $\overline{Y}$ are the moment estimators of $\frac{1}{\lambda_1}$ and $\frac{1}{\lambda_2}$ respectively, where $\overline{X}$ and $\overline{Y}$ are the sample means of failure times and repair times respectively. Let $\theta_1 = \frac{1}{\lambda_1}$ and $\theta_2 = \frac{1}{\lambda_2}$ respectively.

Clearly, the steady state availability of the system given in (2.16) is simplified to

$$A_\infty = \frac{\theta_1(\theta_1+\theta_2)}{(\theta_1^2+\theta_1\theta_2+\theta_2^2)} \tag{3.1}$$

and hence an estimator of A_∞ based on moments is given by

$$\hat{A}_\infty = \frac{\overline{X}\,(\overline{X}+\overline{Y})}{(\overline{X}^2+\overline{X}\;\overline{Y}+\overline{Y}^2)} \tag{3.2}$$

It may be noted that $\hat{A}_\infty$ given in (3.2) is a real valued function in $\overline{X}$ and $\overline{Y}$, which is also differentiable. Consider the following multivariate central limit theorem. see Radhakrishna Rao(1974).

3.1.1 Multivariate Central Limit Theorem

Suppose $T_1', T_2', T_3', \ldots$ are independent and identically distributed k-dimensional random variables such that $T_n' = (T_{1n}, T_{2n}, T_{3n}, \ldots, T_{kn}), n = 1,2,3,\ldots$ having the first and second order moments $E(T_n) = \mu$ and $Var(T_n) = \Sigma$. Define the sequence of random variable $\overline{T}_n' = (\overline{T}_{1n}, \overline{T}_{2n}, \overline{T}_{3n}, \ldots, \overline{T}_{kn}), n = 1,2,3,\ldots$ where $\overline{T}_{in} = \frac{1}{n}\sum_{j=1}^{n} T_{ij},\ i = 1,2,3,\ldots,k$. Then $\sqrt{n}(\overline{T}_n - \mu) \xrightarrow{d} N_k[0,\Sigma]$ as $n \to \infty$

3.1.2 CAN Estimator

By applying the multivariate central limit theorem given in section 3.1.1, it is readily seen that $\sqrt{n}[(\overline{X},\overline{Y}) - (\theta_1,\theta_2)] \xrightarrow{d} N_2(0, \Sigma)$ as $n \to \infty$, where the dispersion matrix $\Sigma = ((\sigma_{ij}))$ is given by

$$\Sigma = \begin{matrix} \overline{X} \\ \\ \overline{Y} \end{matrix} \begin{bmatrix} \theta_1^2 & \theta_1\theta_2\rho \\ & \\ \theta_1\theta_2\rho & \theta_2^2 \end{bmatrix} \tag{3.3}$$

Again from Radhakrishna Rao(1974), we have

$\sqrt{n}(\hat{A}_\infty - A_\infty) \xrightarrow{d} N(0, \sigma^2(\theta))$ as $n \longrightarrow \infty$, where $\theta = (\theta_1, \theta_2)$ and

$$\sigma^2(\theta) = \theta_1^2\left(\frac{\partial A_\infty}{\partial\theta_1}\right)^2 + \theta_2^2\left(\frac{\partial A_\infty}{\partial\theta_2}\right)^2 + 2\theta_1\theta_2\,\rho\left(\frac{\partial A_\infty}{\partial\theta_1}\right)\left(\frac{\partial A_\infty}{\partial\theta_2}\right)$$

$$= \frac{2\,\theta_1^2\,\theta_2^4\,(2\theta_1+\theta_2)^2\,(1-\rho)}{(\theta_1^2+\theta_1\theta_2+\theta_2^2)^4} \tag{3.4}$$

Thus $\hat{A}_\infty$ is a CAN estimator of A_∞. There are several methods for generating CAN estimators and the Method of Moments and the Method of maximum likelihood are commonly used to generate such estimators. see Sinha (1986).

3.2 Confidence Interval for steady state availability of the system

Let $\sigma^2(\hat{\theta})$ be an estimator of $\sigma^2(\theta)$ obtained by replacing θ by a consistent estimator $\hat{\theta}$ namely $\hat{\theta} = (\overline{X}, \overline{Y})$. Let $\hat{\sigma}^2 = \sigma^2(\hat{\theta})$. Since $\sigma^2(\theta)$ is a continuous function of θ, $\hat{\sigma}^2$ is a consistent estimator of $\sigma^2(\theta)$. i.e., $\hat{\sigma}^2 \xrightarrow{P} \sigma^2(\theta)$ as $n \to \infty$. By Slutsky theorem, we have

$$\frac{\sqrt{n}(\hat{A}_\infty - A_\infty)}{\hat{\sigma}} \xrightarrow{d} N(0,1)$$

$$\text{i.e., } \Pr\left(-k_{\frac{\alpha}{2}} < \frac{\sqrt{n}(\hat{A}_\infty - A_\infty)}{\hat{\sigma}} < k_{\frac{\alpha}{2}}\right) = (1-\alpha),$$

where $k_{\frac{\alpha}{2}}$ is obtained from normal tables. Hence a 100(1-α)% confidence interval for A_∞ is given by $\hat{A}_\infty \pm k_{\frac{\alpha}{2}} \frac{\hat{\sigma}}{\sqrt{n}}$, where $\hat{\sigma}$ is obtained from (3.4).

3.3 An estimator of system Reliability based on moments

We have seen that $\overline{X}$ and $\overline{Y}$ are the moment estimators of $\frac{1}{\lambda_1}$ and $\frac{1}{\lambda_2}$ respectively, where $\overline{X}$ and $\overline{Y}$ are the sample means of failure times of online unit and the repair times of a failed unit respectively. Hence, an estimator of system Reliability based on moments is given by

$$\hat{R}(t) = \frac{[(\overline{X} + 2\overline{Y} + \hat{\alpha}_1 \overline{X}\,\overline{Y})e^{\hat{\alpha}_1 t} - (\overline{X} + 2\overline{Y} + \hat{\alpha}_2 \overline{X}\,\overline{Y})e^{\hat{\alpha}_2 t}]}{\sqrt{\overline{X}(\overline{X} + 4\overline{Y})}}, \quad (3.5)$$

Where

$$\hat{\alpha}_1 = \frac{[\sqrt{\overline{X}(\overline{X} + 4\overline{Y})} - (\overline{X} + 2\overline{Y})]}{2\overline{X}\,\overline{Y}} \quad \text{and}$$

$$\hat{\alpha}_2 = -\frac{[(\overline{X} + 2\overline{Y}) + \sqrt{\overline{X}(\overline{X} + 4\overline{Y})}]}{2\overline{X}\,\overline{Y}}.$$

4. Numerical Illustration

In this section, numerical illustration of the behavior of the steady state availability of the system and its corresponding 95% confidence interval are provided by drawing 36 sets of random samples each of size n=1000 from Gumbel bivariate exponential distribution with survival function as given in 2.1. The random samples are generated by writing programs using R statistical software and Matlab. The values of the parameters λ_1, λ_2 and θ are fixed as follows. The value of θ is fixed as 0.5, while the values of λ_1 and λ_2 are varied over the interval [5,7.5] and [2,4.5] respectively with an increment of 0.5. Table 1 and Table 2 present the estimated values of the steady state availability ($\hat{A}_\infty$) and their corresponding 95% confidence intervals obtained for each set of random sample.

It can be observed from Table 1 and Table 2 that for fixed values of λ_2 as λ_1 increases, $\hat{A}_\infty$ and its corresponding 95% confidence interval decreases. In other words, as the unit in the standby will not fail and the failure rate of the online unit increases, in the long run, the probability of the system operating at a specified instance of time decreases. To observe the behaviour of the system reliability over a period of time, one more random sample of size n=1000 is observed from the bivariate exponential distribution by fixing the values of the various parameters as λ_1=5 , λ_2=2 and θ=0.5 respectively. The estimated values of the reliability based on moments $\hat{R}(t)$ given in (3.5) is evaluated for various choices of time periods t=0.05,0.1,…,1. The values of $\hat{R}(t)$ obtained for various choices of t and the line plot of (t, $\hat{R}(t)$) are given in Table 3 and Figure 1 respectively.

It is evident from Fig1 as t increases, the value of $\hat{R}(t)$ decreases agreeing with the theoretical results.

		λ_1					
		5	5.5	6	6.5	7	7.5
λ_2	2	0.394791	0.3656	0.340147	0.317804	0.298066	0.280524
	2.5	0.467705	0.43587	0.407684	0.38262	0.360232	0.340147
	3	0.530581	0.497463	0.467705	0.440907	0.416708	0.394791
	3.5	0.584585	0.551182	0.520751	0.493015	0.467705	0.444568
	4	0.630925	0.597934	0.567495	0.539437	0.513575	0.489724
	4.5	0.67073	0.638617	0.60864	0.580719	0.554741	0.530581

Table1: Estimated values of Steady state availability ($\hat{A}_\infty$)

		λ_1					
		5	5.5	6	6.5	7	7.5
λ_2	2	(0.376,0.403)	(0.345,0.376)	(0.318,0.352)	(0.295,0.331)	(0.275,0.312)	(0.258,0.295)
	2.5	(0.448,0.476)	(0.415,0.446)	(0.385,0.419)	(0.36,0.395)	(0.337,0.373)	(0.317,0.354)
	3	(0.511,0.538)	(0.476,0.507)	(0.445,0.479)	(0.418,0.453)	(0.393,0.429)	(0.371,0.408)
	3.5	(0.565,0.592)	(0.53,0.561)	(0.498,0.532)	(0.469,0.505)	(0.444,0.48)	(0.42,0.458)
	4	(0.611,0.639)	(0.576,0.608)	(0.545,0.578)	(0.516,0.551)	(0.489,0.526)	(0.465,0.503)
	4.5	(0.651,0.679)	(0.617,0.648)	(0.586,0.62)	(0.557,0.592)	(0.53,0.567)	(0.506,0.543)

Table 2: 95% Confidence interval for A_∞

t	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
$\hat{R}(t)$	0.975	0.917	0.844	0.766	0.688	0.614	0.546	0.484	0.428	0.377

t	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
$\hat{R}(t)$	0.333	0.293	0.258	0.227	0.2	0.176	0.155	0.136	0.12	0.105

Table 3: Reliability function

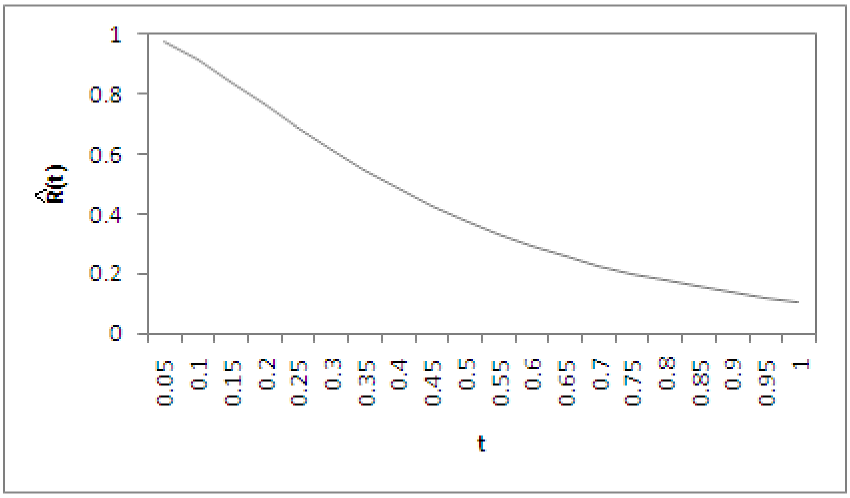

Fig1: The line plot of estimated values of the system reliability based on moments

Acknowledgment

The research of P.Chandrasekhar has been sponsored by the University Grants Commission, New Delhi, India under the Emeritus Fellowship.

References

1. Chandrasekhar, P. and Natarajan, R. (1994) – Confidence limits for steady state availability of a two unit standby system, Micro Electron. Reliab., Vol. 34, No.7, pp 1249-1251.
2. Chandrasekhar,P., Paul R. Savariappan and Feby George (2011) – Applications of bivariate and trivariate exponential distributions in Reliability Theory, JSM Proceedings, International Indian Statistical Association (IISA) Section, Miami Beach Florida: American Statistical Association, pp 3181 – 3191.
3. Gumbel,E.J. (1960) - Bivariate exponential distributions, Journal of the American Statistical Association, 55,292, pp 698-707.
4. John G. Rau(1970) – Optimization and Probability in Systems Engineering, Van Nostrand Reinhold, New York.

5. Marshall, A.W. and Olkin, I.(1967) - A Multivariate exponential distribution, J. Amer. Statist. Assoc., Vol.62, pp 30-44.
6. Osaki, S. and Nakagawa, T. (1976) – Bibliography for reliability and availability of stochastic systems, IEEE Transactions on Reliability, R-25, pp 284-287.
7. Paul Rajamanickam, S. and Chandrasekar, B. (1997) – Reliability measures for two-unit systems with a dependent structure for failure and repair times, Microelectron. Reliab., Vol. 37, No.5, pp. 829-833.
8. Paul Rajamanickam, S. and Chandrasekar, B. (1998) – Confidence limits for steady-state availability of a system with dependent structure for failure and repair times, Journal of Applied Statistical Science, Vol. 8, No.1, pp. 17-27.
9. Radhakrishna Rao, C. (1974) – Linear statistical inference and its applications, Wiley Eastern Pvt. Ltd., New Delhi.
10. Sinha, S.K. (1986) – Reliability and Life Testing, Wiley Eastern Ltd., New Delhi.