Estimating the Renminbi Exchange Rate Basket – A Study on Numeraire

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Abstract
Numerous existing studies have investigated the undisclosed currency basket of the Renminbi (RMB) exchange rate, but the estimated basket depends on its numeraire currency. This paper studies the choice of numeraire from a statistical viewpoint. We show that a restricted least square estimator (RLSE) is numeraire-free, and more efficient than the least square estimator with any specific numeraire. We further clarify the impact of using a numeraire currency in relation to the proposed RLSE. The methodology will be demonstrated with analysis of up-to-date daily exchange rate data.

Key Words: exchange rate, numeraire, Renminbi, currency basket, restricted least square

1. Introduction

As the US dollar has been losing its ground as the key currency and fixed exchange rate regime has provoked currency crisis at times, basket peg system gets more popularity around the globe. While a few currencies such as Russian ruble (RUB) adopt a disclosed basket system which officially announces the currencies and their weights in the basket, many others such as Chinese yuan (CNY), Singapore dollar (SGD), Malaysian ringgit (MYR), Thai baht (THB) and Kuwaiti dinar (KWD) adopt an undisclosed basket system which keeps the details of the basket secret from outsiders. Under the rapidly spreading globalization, it is of great interest for enterprises, investors and foreign governments to reveal the undislosed currency baskets.

Currency values or exchange rates can only be observed relative to a *numeraire*, and to analyze connection between more than two currencies they are usually measured against the same numeraire. For instance, the value of CNY is evaluated against the US dollar (USD), Euro (EUR), Japanese Yen (JPY), the sterling pound (GBP), gold price or oil price. When you want to check if the exchange rate of CNY is mostly determined by USD, you may measure both CNY and USD against, say, GBP and investigate if CNY/GBP and USD/GBP have a high correlation. You may notice that such analytical result is affected by the choice of numeraire. Suppose that there is a minor currency AMC whose value is highly volatile and largely determined by its local climate. The correlation between CNY/AMC and USD/AMC should be very high, because both exchange rates are mostly affected by the local climate. However, most existing studies for currency basket fixed a numeraire before analysis often with an 'economic justification', and did not analyze the impact of numeraire choice. Frankel and Xie (2010) includes a survey on what numariares existing studies used: Frankel (1993) used purchasing power over a consumer basket of domestic goods; Frankel and Wei (1995, 2007) the special drawing right (SDR); Frankel and Wei

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(1994), Ohno (1999) and Eichengreen (2007) the Swiss Franc (CHF); Bnassy-Qur (1999) the USD; Frankel, Schmukler and Servén (2000) a GDP-weighted basket of five major currencies; Yamazaki (2006) the Canadian dollar (CAD).

This study focuses on analyzing effects of numeraire on the estimation of currency baskets both theoretically and empirically. As our best knowledge explicit studies on statistical aspect of numeraire choice are scarce, and it is worthwhile to investigate. A preliminary analysis is given in this section to provide an overview of currency basket estimation and demonstrate the numeraire issue. In the theoretical portion in Section 2, we emphasize numeraire-free estimate, efficiency of estimation and relationship between estimates by two different numeraires. In Section 3, the Renminbi exchange rate basket is analyzed for empirical study as it is one of the most crucial currency baskets to the global economy in this age. We conclude our study with a summary and future directions of currency basket estimation.

1.1 Background of Renminbi Exchange Rate

On July 21, 2005, the People's Bank of China (PBOC) announced to end pegging the Renminbi to the US dollar (1 USD = 8.27 CNY), and start floating the currency with an undisclosed basket of currencies of China's main trading partners (Figure 1). The PBOC published the list of main trading partners in three tiers by importance. The first tier consists of USD, EUR, JPY and Korean Won (KRW); the second tier consists of SGD, GBP, Malaysian ringgit (MYR), RUB, Australian dollar (AUD), THB and CAD; the third tier consists of Indian rupee (INR), Indonesian rupiah (IDR), Philippine peso (PHP), Saudi riyal (SAR), Turkish lira (TRY), UAE dirham (AED), Kazakhstani tenge (KZT), Taiwanese dollar (TWD), Vietnamese dong (VND), Angolan kwanza (AOA), South African rand (ZAR), Brazilian real (BRL), Chilean peso (CLP) and Mexican peso (MXN). This list is roughly consistent with China's trade amount with its trading partners (Table 1).

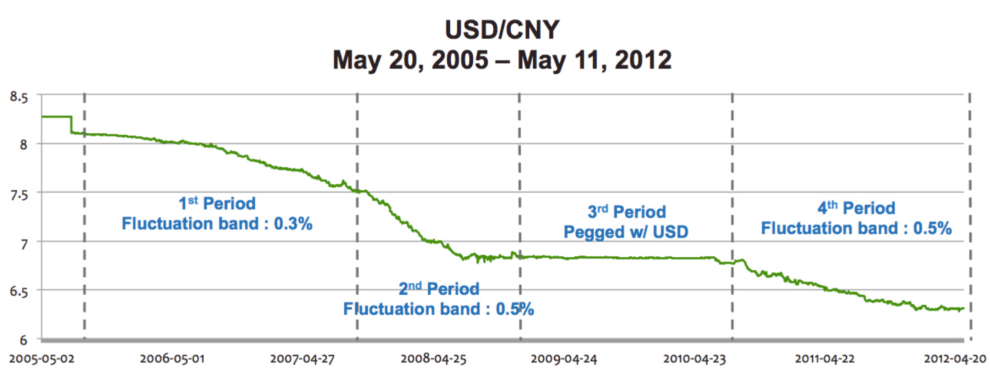

Figure 1: Historical RMB rate

If the government (or the central bank) strictly pegs its currency to the basket, outsiders can easily observe it in foreign exchange markets. However, the actual policy of the PBOC

Table 1: China's main trade partners (Imports plus exports in million Euro. Source: Direction of Trade Statistics, IMF, 2010.)

Rank	Country	Amount(*1)	Share(%)	Rank	Country	Amount(*1)	Share(%)
1	EU27	363224.3	17.0	14	Indonesia	32322.3	1.5
2	U.S.A.	291972.6	13.6	15	Canada	28026.9	1.3
3	Japan	224263	10.5	16	Vietnam	22761.3	1.1
4	Hong Kong	172371.4	8.1	17	Iran	22174.1	1.0
5	S. Korea	156414.4	7.3	18	Philippines	21016.6	1.0
6	Australia	65758.7	3.1	19	Chile	19501.2	0.9
7	Malaysia	56113.6	2.6	20	UAE	19392.6	0.9
8	Brazil	47474.5	2.2	21	Angola	18780.6	0.9
9	India	46697.7	2.2	22	Mexico	18713.1	0.9
10	Singapore	42999.7	2.0	23	South Africa	16820.3	0.8
11	Russia	41915.2	2.0	24	Kazakhstan	15359.8	0.7
12	Thailand	40061.4	1.9	25	Turkey	11452.4	0.5
13	Saudi Arabia	32741.8	1.5				

is more arbitrary and composition of the basket is not clear for outsiders. This is partly due to the central bank's effort to avoid speculation on the foreign exchange market. If an exchange rate is pegged to the level which is different from the market's equilibrium, there will be much speculation on the currency because investors have small risk to speculate. To discourage speculators, several countries adopts undisclosed basket for their currencies as mentioned above. On the other hand, the central bank may want to avoid too much volatility in the market since it harms the real economy. Under such a situation, the PBOC allows CNY to fluctuate within a specified percentage around the central parity against USD, EUR, JPY, HKD, GBP, AUD, CAD, MYR and RUB. The central parity is defined as the average currency rate of last one month. To be concrete, the specified percentage was $\pm 0.3\%$ for July 21, 2005 - May 20, 2007 (Period 1, hereafter), $\pm 0.5\%$ for May 21, 2007 - July 31, 2008 (Period 2), literally zero for Aug 1, 2008 - June 19, 2010 (Period 3), $\pm 0.5\%$ for June 19, 2010 - April 15, 2012 (Period 4), and $\pm 1.0\%$ after April 16, 2012. The scheme implies that the maximum daily change of the above rates is twice the floating band. The actual volatility of CNY against USD is largely affected by this floating band (Figure 2).

1.2 Preliminary Analysis

Note again that the value of currency is only observed as a ratio. Let Y_t be CNY and $X_{1,t}, \cdots, X_{p-1,t}$ be currencies in the Renminbi basket. The relationship among these currencies is analyzed with

$$\frac{Y_t}{N_t}, \frac{X_{1,t}}{N_t}, \cdots, \frac{X_{p-1,t}}{N_t}$$

where N_t is a numeraire. The numeraire may be another currency, a commodity price or price of anything. Then, the weight of the currencies in the basket is estimated by a linear model:

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_{p-1} x_{p-1,t} + \epsilon_t \quad (1)$$

where y_t ($x_{i,t}$) is defined as either Y_t/N_t ($X_{i,t}/N_t$) or the difference of logs (asset return) of Y_t/N_t ($X_{i,t}/N_t$). β_i represents the weight of the currency $X_{i,t}/N_t$. Assume ϵ_t's are

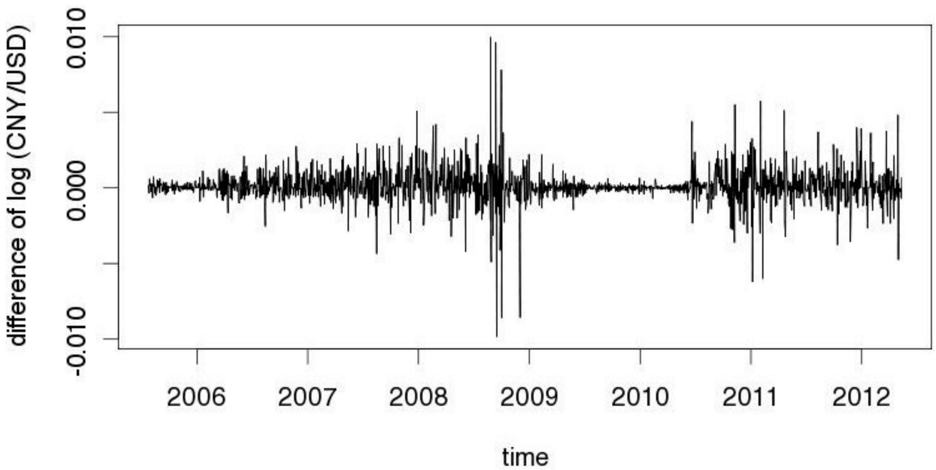

Figure 2: Daily change in difference of logs of CNY/USD (sample period: July 25, 2005-May 11, 2012)

independent and identically distributed (IID) errors with variance σ^2 in the preliminary analysis. Hereafter, we always use difference of logs for y_t and $x_{i,t}$. However, all theory and estimation in this study are applied for both definitions.

A sample result of the model (1) by three different numeraire are shown in Table 2. Difference in estimated results by numeraire is noticeable. While the USD dominates the basket with all three numeraires, its weight ranges from 94.86% to 103.4%. More importantly, the standard error for the estimates are totally different for USD and EUR with SDR as numeraire. The table implies that choice of numeraire is not a trivial issue.

Table 2: Estimated weights of currencies (Sample period: June 20, 2010 - April 16, 2012 (Period 4). (*1) New Zealand dollar. (*2) The SDR consists of: 0.66 USD (41.9%), 0.423 EUR (37.4%), 12.1 JPY (9.4%), and 0.111 GBP (11.3%).)

numeraire	NZD(*1)		GBP		SDR(*2)	
	estimate	S.E.	estimate	S.E.	estimate	S.E.
intercept	0.0002	0.0001	0.0002	0.0001	0.0002	0.0001
USD	0.9508	0.0142	0.9486	0.0154	1.0340	0.2037
EUR	0.0098	0.0111	0.0146	0.0121	-0.0068	0.0558
JPY	0.0035	0.0106	0.0043	0.0106	-0.0017	0.0180
KRW	0.0247	0.0112	0.0296	0.0105	0.0293	0.0104
Total	0.9887	-	0.9971	-	1.0547	-

2. Theoretical properties

Reconsider the equation (1) with a more general covariance matrix $\sigma^2 V$ for $\epsilon = (\epsilon_1, \cdots, \epsilon_n)$, where all diagonal elements of V are assumed to be one. Empirical results from CNY exchange rate basket indicates that ϵ_t has a significantly negative first-order autocorrelation of around -0.15.

2.1 Linear restriction

The estimated result of currency basket is subjective if the choice of the numeraire is arbitrary. Fortunately the following theorem provides a numeraire-free estimate for basket weights. For notational convenience, let $\Delta A := \log A_t - \log A_{t-1}$.

Theorem 1 *For any numeraire $\tilde{N}_t$, (1) is equivalent to:*

$$\tilde{y} = \beta_0 + \beta_1 \tilde{x}_1 + \cdots + \beta_{p-1}\tilde{x}_{p-1} + \left(1 - \sum_{i=1}^{p-1} \beta_i\right) \tilde{x}_p + \epsilon \tag{2}$$

where $\tilde{y} := \Delta \frac{Y}{\tilde{N}}$, $\tilde{x}_i := \Delta \frac{X_i}{\tilde{N}}$ $(i = 1, \cdots, p-1)$ and $\tilde{x}_p := \Delta \frac{N}{\tilde{N}}$.

Proof. Observe that

$$\log \frac{Y_t}{N_t} = \log Y_t - \log N_t + \log \tilde{N}_t - \log \tilde{N}_t = \log \frac{Y_t}{\tilde{N}_t} - \log \frac{N_t}{\tilde{N}_t}.$$

This yields

$$y = \Delta \frac{Y}{N} = \Delta \frac{Y}{\tilde{N}} - \Delta \frac{N}{\tilde{N}} = \tilde{y} - \tilde{x}_p,$$

and by the same reason it holds that $x_i = \tilde{x}_i - \tilde{x}_p$. Therefore, the equation (1) becomes

$$\begin{aligned} & \tilde{y} - \tilde{x}_p = \beta_0 + \beta_1(\tilde{x}_1 - \tilde{x}_p) + \cdots + \beta_{p-1}(\tilde{x}_{p-1} - \tilde{x}_p) + \epsilon \\ \Rightarrow \quad & \tilde{y} = \beta_0 + \beta_1 \tilde{x}_1 + \cdots + \beta_{p-1}\tilde{x}_{p-1} + \left(1 - \sum_{i=1}^{p-1} \beta_i\right) \tilde{x}_p + \epsilon. \quad \square \end{aligned}$$

The theorem above indicates that the old numeraire N_t is implicitly included in the basket. If we explicitly include N_t as a predictor as in (2), the total weight of currencies is 1. By letting $\beta_p := 1 - \sum_{i=1}^{p-1} \beta_i$, (2) can be seen as a linear regression with p covariates and one linear restriction $\sum_{i=1}^{p} \beta_i = 1$. The restricted generalized least square estimator (RGLSE) $\hat{\beta}_R$ of (2) can be obtained by estimating $\beta_1, \cdots, \beta_{p-1}$ and their standard errors with the original equation (1) by unrestricted GLSE, and calculating $\hat{\beta}_p$ and its standard error from these estimates. Although the estimation can be done in such a way, the following classic theorem on restricted least square further clarifies the properties of the estimator. Let $\hat{\beta} \in \mathbb{R}^p$ be the GLSE without the linear restriction on β_i's, then the following theorem holds. Note that σ^2 and V are assumed to be known.

Theorem 2 (Theil '71, Kreijger and Neudecker '77) *Among estimators $AY + s \in \mathbb{R}^{p+1}$ (A and s are constant matrix/vector),*

$$\hat{\beta}_R := \hat{\beta} + S^{-1}R'(RS^{-1}R')^{-1}\left(1 - R\hat{\beta}\right), \tag{3}$$

where $R = (0, 1, \cdots, 1)$ (a 1 by $(p+1)$ row vector) and $S = X'V^{-1}X$, is the unbiased estimator which minimizes the expected quadratic loss

$$E\left[\left(\hat{\beta}_R - \beta\right)' D \left(\hat{\beta}_R - \beta\right)\right]$$

for any non-negative definite matrix D. Moreover, the determinant of $\mathrm{Var}\left(\hat{\beta}_R\right)$ takes the minimum value.

Corollary 3 *Given σ^2 and V, $\hat{\beta}_R$ has a smaller variance (= is more efficient) than $\hat{\beta}$. To be more precise, for any non-negative definite matrix D,*

$$E\left[\left(\hat{\beta}_R - \beta\right)' D \left(\hat{\beta}_R - \beta\right)\right] \le E\left[\left(\hat{\beta} - \beta\right)' D \left(\hat{\beta} - \beta\right)\right].$$

The corollary guarantees that the RGLSE has smaller standard errors than any other unrestricted GLSEs. Hence, unless we know a specific choice of numeraire is correct, we should use the RGLSE. Even if we know a specific numeraire is correct, it is no loss of accuracy to include the numeraire as a currency in the basket and implement RGLSE. We now only have to choose an optimal set of predictors, not a numeraire. However, note that σ^2 and V are assumed to be known in the corollary, so the estimated variance of $\hat{\beta}_R$ can be larger than that of $\hat{\beta}$. Still estimated σ^2 for restricted model is the smallest among all RGLSE and GLSE when an optimal set of predictors are chosen. The minimality of V is mathematically less clear, but empirically estimated V is almost invariant for $\hat{\beta}_R$ and $\hat{\beta}$.

2.2 Stochastic Linear restriction

While the RGLSE with the optimal set of predictors is our suggested estimation method, we are also interested in interpreting the estimated basket by other numeraires. For instance, how should we read results in existing studies? Why was the estimated result so different when we chose SDR as the numeraire? To answer these questions, estimators by two different numeraires are compared. The comparison enables us to interpret estimates with a specific numeraire as compared to numeraire-free estimates by RGLSE. It will turn out that change in numeraire works as (i) a stochastic linear restriction on $\sum_{i=1}^{p} \beta_i$ and (ii) linear transformation of the response and predictors.

Suppose there are two numeraires N_t and $\tilde{N}_t$ and let $y := \Delta \frac{Y}{N}$, $\tilde{y} := \Delta \frac{Y}{\tilde{N}}$ and $n := \Delta \frac{\tilde{N}}{N}$, then $\tilde{y} = y - n$. Consider two linear models:

$$\begin{aligned} y &= \beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p + \epsilon, &(4)\\ \tilde{y} &= \tilde{\beta}_0 + \tilde{\beta}_1 \tilde{x}_1 + \cdots + \tilde{\beta}_p \tilde{x}_p + \tilde{\epsilon}. &(5)\end{aligned}$$

(5) is also written as

$$y - n = \tilde{\beta}_0 + \tilde{\beta}_1(x_1 - n) + \cdots + \tilde{\beta}_p(x_p - n) + \tilde{\epsilon} \tag{6}$$

Let $\hat{\beta}_{SR}$ be the GLSE (which is also Gaussian MLE) for $(\tilde{\beta}_0, \cdots, \tilde{\beta}_p)'$. Then, the theorem below holds.

Theorem 4 *$\hat{\beta}_{SR}$ is also obtained as Gaussian MLE of (4) with a stochastic linear restriction:*

$$\Sigma\beta_i - 1 \quad \sim \quad N\left(0, \frac{1}{n'V^{-1}n}\right) \tag{7}$$

if

$$V^{-1/2}n \perp sp\left\{V^{-1/2}\mathbf{1}, \mathbf{V^{-1/2}y}, \mathbf{V^{-1/2}x_1}, \cdots, \mathbf{V^{-1/2}x_p}\right\} \tag{8}$$

where $n = (n_1, \cdots, n_T)'$ ($y, x_1, \cdots, x_p$ are defined similarly), $sp\{\sim\}$ is the linear space spanned by $\sim$, and 1 is a vector of ones.

Remark. When V is an identity matrix, (8) means that n is orthogonal to $\{1, y, x_1, \cdots, x_p\}$. It roughly means that the change of numeraire is a sort of noise, since it is not correlated with the response and predictors.

Proof of Theorem 4. The GLS estimator $\hat{\beta}$ for (6) is determined by

$$\begin{aligned} \hat{\beta} &= \arg\min_{\beta} \tilde{\epsilon}' V^{-1} \tilde{\epsilon} \\ &= \arg\min_{\beta} \{z(\beta) + (\Sigma\beta_i - 1)n\}' V^{-1} \{z(\beta) + (\Sigma\beta_i - 1)n\} \end{aligned} \tag{9}$$

where

$$z(\beta) := y - \beta_0 - \Sigma\beta_i x_i$$

and all summations are for $i = 1$ to p.

If (8) is satisfied, then $n'V^{-1}z(\beta) = 0$ for any β and

$$(9) = \arg\min_{\beta} \left\{z(\beta)'V^{-1}z(\beta) + (\Sigma\beta_i - 1)^2 n'V^{-1}n\right\} \tag{10}$$

The first term on the right is the negative log-likelihood function of ϵ in the model (4) and the second term is seen as the log-likelihood function for:

$$\Sigma\beta_i - 1 \sim N\left(0, \frac{1}{n'V^{-1}n}\right). \quad \square$$

Existing theoretical result for stochastic linear restriction relates the estimate for (5) as opposed to the unrestricted estimate for (4):

Theorem 5 (Toutenburg '75) *The best linear unbiased estimator of the linear model (4) with the stochastic restriction (7) is:*

$$\hat{\beta}_{SR} = \hat{\beta} + S^{-1}R'\left(\frac{1}{n'V^{-1}n} + RS^{-1}R'\right)^{-1}(1 - R\hat{\beta}) \tag{11}$$

where $\hat{\beta}$ is the GLSE without the restriction, and

$$\mathrm{Var}(\hat{\beta}_{SR}) = \sigma^2 \left(S + (n'V^{-1}n)R'R\right)^{-1}$$

By comparing this theorem with Theorem 2, $\hat{\beta}_{SR} \to \hat{\beta}_R$ in probability as $n'V^{-1}n \to \infty$.

When the assumption (8) is not satisfied, the above theorems do not hold. Still it is possible to decompose n as

$$n = \alpha + \eta$$

where α is the projection of $V^{-1/2}n$ onto the space of $\{V^{-1/2}1$, $\{V^{-1/2}y$, $V^{-1/2}x_1, \cdots,$ $V^{-1/2}x_p\}$, multiplied by $V^{1/2}$ from the left. η is the remainder, and hence $V^{-1/2}\eta$ is orthogonal to $V^{-1/2}\vec{1}$, $V^{-1/2}y$, $V^{-1/2}x_1, \cdots, V^{-1/2}x_p$. Then, (6) becomes:

$$y - \alpha - \eta = \beta_0 + \beta_1(x_1 - \alpha - \eta) + \cdots + \beta_p(x_p - \alpha - \eta) + \tilde{\epsilon}$$

When we define $y^* := y - \alpha, x_1^* := x_1 - \alpha, \cdots, x_p^* := x_p - \alpha$, Theorem 4 holds for the linear model:

$$y^* = \beta_0 + \beta_1 x_1^* + \cdots + \beta_p x_p^* + \epsilon$$

with n replaced by η.

α works as a linear transformation of variables $1, y, x_1, \cdots, x_p$, and η works as n in the stochastic restriction (7). That implies that (i) a large $|\alpha|$ may make the estimate of $\tilde{\beta}$ distant from $\hat{\beta}$, but (ii) a large $|\eta|$ push the estimate of $\tilde{\beta}$ toward $\hat{\beta}_R$. In other words, if the change in numeraire has a large fluctuation irrelevant to currencies of our interest, the estimates are close to the RGLSE. Some studies such as Reinhart and Rogoff (2004) claims that a remote minor currency is good for a numeraire, but the above analysis clarifies that such estimate is always close to the restricted least square estimator.

3. Empirical results

3.1 Data Description

Daily exchange rate data published by the Federal Reserve Board (noon buying spot rates in New York for cable transfers) are used for analysis. The sample size is 458 for the Period 1; 305 for the Period 2; 456 for the Period 4. The first few days of the Period 1 (to be more specific, data before July 25, 2005) were excluded due to excessive volatility.

3.2 Variable selection results

We select the currency basket with RGLSE in three ways:

1. The best combination by AIC from all currencies in the tiers 1 and 2 except for RUB (a disclosed basket peg currency). Candidate currencies are USD, EUR, JPY, KRW; SGD, GBP, MYR, AUD, THB, CAD (Table 3).

2. The best combination by AIC from all currencies in the tiers 1 and 2 except for RUB, SGD, MYR and THB (disclosed and undisclosed basket peg currencies). USD, EUR, JPY, KRW; GBP, AUD, CAD (Table 4).

3. Fixed choice of currencies in the tier 1: USD, EUR, JPY, KRW (Table 5).

Highly significant large weights on MYR and SGD stand out in Table 3. The weights of 5.49% or 8.9% on MYR and 12.7% on SGD are way too high as compared to the trade share of these two countries for China (2.6% for Malaysia and 2.0% for Singapore). A natural interpretation is that Singapore and Malaysia link their currencies to a basket including RMB. In such a circumstance, the weights on these currencies are likely to be overestimated due to identification issues. This observation leads us to choose the best combination only from floating currencies. USD dominates the basket in Table 4, and EUR and KRW also have significant weights depending on the period. This is more reasonable outcome since all of these three currencies are in the tier 1. A possible reason for JPY not being in the basket is PBOC's buying of Japanese Government bond. If the PBOC's buying of JPY caused depreciation of JPY, the PBOC acceralated the fluctuation in the CNY/JPY exchange rate rather than pegged CNY partially against JPY. Since the tier 1 currencies always dominated the basket, we will use the fixed choice of these four currencies in the subsequent analysis (Table 5). The main focus in the following is the effect of numeraire choice, and not variable selection.

3.3 Linear restriction and efficiency

Corollary 3 claims the RGLSE always has a smaller standard error for all estimates, given σ^2 and V. Table 6-8 demonstrates this fact. Estimates are compared for four different numeraires SDR, CAD, Norwegian Krone (NOK), NZD and the RGLSE. It turned out that RGLSE makes all standard errors the smallest except for the USD in Period 2. The exception was caused by the fact that NZD had a predictive power for CNY in this period and consequently lowered the estimated $\hat{\sigma}^2$. If more thorough variable selection was done, NZD should have been selected in the basket and RGLSE should have performed the best. In all other estimates, RGLSE gets up to 50% reduction in standard error as compared to CAD, NOK and NZD numeraire, and even more reduction as compared to SDR. It is found again that estimates are quite different with the SDR numeraire. This issue is clarified in the next section.

3.4 Stochastic Linear restriction

As seen in Section 2.2, numeraire change n can be decomposed into α and η, and the magnitude of these quantities characterizes the numeraire change. With the RGLSE as the base line, Table 9 shows the magnitude of n, α and η. We can find that the magnitude of the noise term η is much smaller for SDR. This is largely caused by the fact that the most fluctuation of SDR is explained by USD, EUR and JPY which are in the basket. The noise term of SDR comes from GBP, which has only 11.3% share in SDR.

To demonstrate the effect of $|\eta|$ on estimates, Table 10 simulates the basket weights when the noise term η is artificially inflated for SDR. As the inflation factor k increases, all estimates approach the RGLSE. This result illustrates that discrepancies of estimates by different numeraires largely stem from too small noise in the numeraires.

Table 3: Selection from all 9 currencies (Ljung-Box tests, skewness, (excess) kurtosis and Anderson-Darling tests were done for the transformed residuals $\hat{V}^{-1/2}\hat{\epsilon}$. The same note applies for Tables 4-5 also.)

	Period 1		Period 2		Period 4	
	estimate	S.E.	estimate	S.E.	estimate	S.E.
(Intercept)	0.0001	0.0000	0.0003	0.0001	0.0002	0.0001
USD	0.9180	0.0145	0.8734	0.0252	0.9110	0.0134
KRW	0.0271	0.0085	-	-	-	-
MYR	0.0549	0.0154	-	-	0.0890	0.0134
SGD	-	-	0.1266	0.0252	-	-
Sample Size	458	(p-value)	305	(p-value)	456	(p-value)
$\hat{\sigma}$	0.000639	-	0.001269	-	0.001253	-
ρ	-0.166	-	-0.138	-	-0.121	-
LB(10), RES	22.27	0.0138	6.93	0.7324	7.11	0.7153
LB(10), SQRES	71.07	0.0000	9.20	0.5129	9.22	0.5112
skewness	0.31	-	0.03	-	0.03	-
kurtosis	2.22	-	1.37	-	1.38	-
AD	6.97	0.0000	2.02	0.0000	2.02	0.0000

Table 4: Selection from USD, EUR, JPY, KRW, GBP, AUD, CAD

	Period 1		Period 2		Period 4	
	estimate	S.E.	estimate	S.E.	estimate	S.E.
(Intercept)	0.0001	0.0000	0.0003	0.0001	0.0002	0.0001
USD	0.9615	0.0079	0.9294	0.0141	0.9605	0.0093
EUR	-	-	0.0706	0.0141	-	-
JPY	-	-	-	-	-	-
KRW	0.0385	0.0079	-	-	0.0395	0.0093
Sample Size	458	(p-value)	305	(p-value)	456	(p-value)
$\hat{\sigma}$	0.000647	-	0.001278	-	0.001290	-
ρ	-0.166	-	-0.168	-	-0.135	-
LB(10), RES	19.39	0.0356	6.56	0.7658	12.86	0.2317
LB(10), SQRES	66.69	0.0000	6.51	0.7704	11.40	0.3269
skewness	0.32	-	0.04	-	0.07	-
kurtosis	2.18	-	1.01	-	4.12	-
AD	6.35	0.0000	1.35	0.0016	7.73	2.20E-16

Table 5: Fixed combination: USD, EUR, JPY, KRW

	Period 1		Period 2		Period 4	
	estimate	S.E.	estimate	S.E.	estimate	S.E.
(Intercept)	0.0001	0.0000	0.0003	0.0001	0.0002	0.0001
USD	0.9596	0.0090	0.9261	0.0216	0.9411	0.0139
EUR	0.0005	0.0080	0.0695	0.0156	0.0196	0.0097
JPY	0.0029	0.0073	0.0015	0.0116	0.0082	0.0105
KRW	0.0370	0.0086	0.0029	0.0152	0.0310	0.0103
Sample Size	458	(p-value)	305	(p-value)	456	(p-value)
$\hat{\sigma}$	0.000649	-	0.001282	-	0.001288	-
ρ	-0.169	-	-0.168	-	-0.156	-
LB(10), RES	19.49	0.0344	6.55	0.7671	12.66	0.2432
LB(10), SQRES	67.79	0.0000	6.51	0.7707	13.04	0.2214
skewness	0.32	-	0.04	-	0.13	-
kurtosis (*1)	2.18	-	1.01	-	3.98	-
AD	6.30	0.0000	1.37	0.0015	7.57	0.0000

Table 6: Comparison of standard error of $\hat{\beta}$ by numeraire: Period 1

numeraire	SDR		CAD		NOK		NZD		(RGLSE)	
	Est.	S.E.	Est.	S.E.	Est.	S.E.	Est.	S.E.	Est.	S.E.
USD	0.8863	0.1376	0.9558	0.0102	0.9609	0.0091	0.9606	0.0093	0.9596	**0.0090**
EUR	0.0214	0.0400	-0.0015	0.0084	-0.0103	0.0120	0.0022	0.0087	0.0005	**0.0080**
JPY	0.0082	0.0124	0.0032	**0.0073**	0.0036	**0.0073**	0.0025	0.0074	0.0029	**0.0073**
KRW	0.0372	**0.0086**	0.0368	**0.0086**	0.0359	0.0087	0.0372	**0.0086**	0.0370	**0.0086**
sum	0.9532	-	0.9943	-	0.9901	-	1.0024	-	1.0000	-
$1000\hat{\sigma}$	0.649	-	0.649	-	**0.648**	-	0.650	-	0.649	-

Table 7: Comparison of standard error of $\hat{\beta}$ by numeraire: Period 2

numeraire	SDR		CAD		NOK		NZD		(RGLSE)	
	Est.	S.E.	Est.	S.E.	Est.	S.E.	Est.	S.E.	Est.	S.E.
USD	0.9499	0.2586	0.9319	0.0230	0.9285	0.0219	0.9250	**0.0215**	0.9261	0.0216
EUR	0.0625	0.0770	0.0761	0.0176	0.0541	0.0306	0.0439	0.0193	0.0695	**0.0156**
JPY	0.0000	0.0195	-0.0018	0.0124	0.0036	0.0119	0.0135	0.0128	0.0015	**0.0116**
KRW	0.0029	0.0152	0.0039	0.0152	0.0024	0.0152	-0.0052	0.0155	0.0029	**0.0152**
sum	1.0153	-	1.0101	-	0.9887	-	0.9772	-	1.0000	-
$1000\hat{\sigma}$	1.284	-	1.284	-	1.283	-	**1.273**	-	1.282	-

Table 8: Comparison of standard error of $\hat{\beta}$ by numeraire: Period 4

numeraire	SDR		CAD		NOK		NZD		(RGLSE)	
	Est.	S.E.	Est.	S.E.	Est.	S.E.	Est.	S.E.	Est.	S.E.
USD	1.0265	0.2015	0.9402	0.0152	0.9423	0.0141	0.9424	0.0140	0.9411	**0.0139**
EUR	-0.0034	0.0550	0.0189	0.0107	0.0132	0.0158	0.0149	0.0108	0.0196	**0.0097**
JPY	0.0022	0.0178	0.0086	0.0107	0.0082	**0.0105**	0.0074	**0.0105**	0.0082	**0.0105**
KRW	0.0301	0.0105	0.0303	0.0111	0.0295	0.0107	0.0261	0.0113	0.0310	**0.0103**
sum	1.0554	-	0.9979	-	0.9932	-	0.9909	-	1.0000	-
$1000\hat{\sigma}$	1.289	-	1.290	-	1.289	-	1.288	-	1.288	-

Table 9: Size of n, α and η: Period 4

	SDR	CAD	NOK	NZD	USD
$\lvert n \rvert$	0.00325	0.00615	0.00872	0.00901	-
$\lvert \alpha \rvert$	**0.00322**	**0.00385**	**0.00742**	**0.00608**	**-**
$\lvert \eta \rvert$	**0.00046**	**0.00479**	**0.00459**	**0.00665**	**-**
intercept	0.00016	0.00017	0.00017	0.00016	0.00017
USD	1.02650	0.94015	0.94227	0.94245	0.94113
EUR	-0.00337	0.01890	0.01324	0.01487	0.01963
JPY	0.00216	0.00856	0.00819	0.00742	0.00825
KRW	0.03010	0.03033	0.02954	0.02613	0.03099
sum	1.05539	0.99794	0.99324	0.99087	1.00000

Table 10: Effects of η: Period 4 (numeraire is $\alpha + k\eta$, and is SDR when $k = 1$.)

-	k=1	k = 1.5	k=2	k = 5	k = 10	RGLSE
$\lvert \alpha \rvert$	0.00322	0.00322	0.00322	0.00322	0.00322	-
$\lvert \eta \rvert$	**0.00046**	**0.00069**	**0.00093**	**0.00232**	**0.00463**	**-**
intercept	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
USD	1.0265	0.9797	0.9632	0.9450	0.9422	0.9411
EUR	-0.0034	0.0092	0.0137	0.0186	0.0193	0.0196
JPY	0.0022	0.0055	0.0067	0.0080	0.0082	0.0082
KRW	0.0301	0.0306	0.0307	0.0309	0.0310	0.0310
Total	1.0554	1.0250	1.0143	1.0025	1.0007	1.0000

4. Summary and future directions

4.1 Summary

We have analyzed the effects of numeraire choice on currency basket estimation theoretically and empirically.

The theoretical study shows that (i) the restricted least square estimator is numeraire-free and more efficient than unrestricted least square with any numeraire, given σ^2 and V, and (ii) change of numeraire n is decomposed into a linear transformation of response and predictors α, and a noise term η which works as a stochastic restriction on the basket weights.

The empirical analysis reveals that while USD is still dominant (90-96%) in the RMB currency basket, there is evidence that the other currencies such as EUR, KRW, SGD, MYR are in the basket. However, two or more currencies managed by currency basket system induce some identification problems. Lastly, estimation with the SDR as numeraire is much different from other numeraires, and it is largely due to its small noise term η.

4.2 Future Topics

There are some general open issues for analysis of currency basket estimation. Firstly, many people believe that the weight of each currency should be positive. Such constraint can be imposed by some statistical techniques, but the optimal strategy is not clear. Secondly, simultaneous analysis of multiple currency baskets is interesting, and can clarify the competitive devaluation problem in foreign exchange markets.

In addition to these general issues, there are specific problems for the RMB basket. The fluctuation band set by the PBOC is likely to affect the error distribution of the linear model, and hence it is worthwhile to develop a model to account for the band. High frequency data may illuminate the impact of the band also. As another topic, gradual appreciation of the CNY can be incorporated in the model. Funke and Gronwald (2008) fitted a curve for the evolution of CNY and predicted the RMB exchange rate surprisingly well.

Finally, we emphasize that numeraire choice affects wide range of economic analysis. For an example, analysis based on real price and nominal price can be seen as a numeraire choice problem.

References

[1] Bénassy-Quéré, A., Coeuré, B., Mignon, V., 2004. On the Identification of de facto Currency Pegs. *Journal of the Japanese and International Economies* 20(1): 112-127.

[2] Eichengreen, B., 2007. China's Exchange Rate Regimu: The Long and Short of It. In *China's Financial Transition at a Crossroads*, Charles W. Calomiris ed., Columbia University Press.

[3] Fidrmuc, J., 2010. Time-Varying Exchange Rate Basket in China from 2005-2009. Comparative Economic Studies, 52, 515-529.

[4] Frankel, J., 1993. Is Japan Creating a Yen Bloc in East Asia and the Pacific? In *Regionalism and Rivalry: Japan and the US in Pacific Asia*, edited by J. Frankel and M. Kahler. Chicago: University of Chicago Press: 15085.

[5] Frankel, J., Schmukler, S., and Servén, L., 2000. Verifiability and the Vanishing Intermediate Exchange Rate Regime. In *Brookings Trade Forum 2000*, edited by Susan Collins and Dani Rodrik. Washington, DC: Brookings Institution: 59- 123.

[6] Frankel, J., and Wei, S.-J., 1994. Yen Bloc or Dollar Bloc? Exchange Rate Policies of the East Asian Economies. In *Macroeconomic Linkages* edited by Takatoshi Ito and Anne Krueger, 295329. Chicago: University of Chicago Press.

[7] Frankel, J., and Wei, S.-J., 1995. Emerging Currency Blocs. *The International Monetary System: Its Institutions and its Future* , Hans Genberg, ed., Springer, Berlin, 111-143.

[8] Frankel, J., Wei, S.-J., 2007. Assessing China's Exchange Rate Regime. *Economic Policy* 51, (July): 575614.

[9] Frankel, J.A., Wei, S-J., 2007. Assessing China's Exchange Rate Regime. *NBER Working Paper*, 13100.

[10] Frankel, J.A., Xie, D., 2010. Estimation of De Facto Flexibility Parameter and Basket Weights in Evolving Exchange Rate Regimes. *American Economic Review*, 100(2), 568-572.

[11] Funke, M., Gronwald M., 2008. The Undisclosed Renminbi Basket: Are the Markets Telling Us Something about Where the Renminbi-US Dollar Exchange Rate is Going? *World Economy*, 31, 1581-1598.

[12] Kreijger R.G., Neudecker, H., 1977. Exact Linear Restrictions on Parameters in the Generalized Linear Model with a Singular Covariance Matrix. J. Amer. Stat. Assoc., 72, 430-432.

[13] Ohno, K., 1999. Exchange Rate Management in Developing Asia. *Asian Development Bank Institute Working Paper*, No.1. (January).

[14] Rao C.R., Toutenburg, H., 1999. *Linear Models - Least Squares and Alternatives.* Springer.

[15] Reinhart, Carmen, Kenneth, Rogoff, 2004. The Modern History of Exchange Rate Arrangements: A Reinterpretation. *Quarterly Journal of Economics*, 119(1), 1-48.

[16] Theil, H., 1971. *Principles of Econometrics*, Wiley, New York.

[17] Toutenburg, H., 1975. The use of mixed prior information in regression analysis. *Biometrische Zeitschrift*, 17, 365-372.

[18] Yamazaki, K., 2006, Inside the Currency Basket. Columbia University and Mitsubishi UFJ Trust and Banking Corp., December.