Statistical Procedures for Reconciling Time Series of Large Systems of Accounts Subject to Low-Frequency Benchmarks

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Abstract
In this paper we study statistical procedures to reconcile large systems of annual time series subject to low-frequency benchmarks (e.g. available every five years). Our aim is to reconcile the preliminary levels of the series such that they (i) are consistent with the low-frequency benchmarks available, (ii) fulfill all the accounting relationships for any given year, and (iii) show movements that are as close as possible to the preliminary information. We propose to solve this kind of problems using a simultaneous least-squares procedure based on the proportional first difference (PFD) criterion, a movement preservation principle proposed by Denton (1971). However, we suggest that a pure proportional adjustment is adopted for series with breaks and high volatility that deteriorate the meaningfulness of growth rates. We apply this procedure for reconciling the 1998-2002 U.S. annual input-output accounts, GDP-by-industry accounts and expenditure-based GDP, subject to the 1997 and 2002 quinquennial benchmarks and all contemporaneous constraints of the input-output accounts for the in-between years.

Key Words: Benchmarking, Reconciliation, Temporal and Contemporaneous Constraints.

1. Introduction

Measurements of socio-economic phenomena are conducted at different frequencies, with different objectives. Monthly or quarterly information aims at providing a timely picture of the short-term movements. Annual data from sample surveys or register-based statistics rely on a large sample of units, and thus they provide a more accurate indication of medium- and long-term trends than infra-annual data. Finally, the Economic Census collects most comprehensive data on business activities and provides a detailed and accurate portrait of the Nation's economy once every five years. Higher frequency measurements are generally required to match corresponding lower frequency benchmarks.

At each frequency, socio-economic variables may be required to satisfy a number of aggregation and accounting relationships. A typical example is national accounts, where total aggregates of the economy must be consistent with the sum of detailed components (e.g. by industry) and identities are established between flows of production, expenditure, and income. Cross-sectional consistency between observed variables is not automatically met, and has to be restored.

In addition, when dealing with both high and low frequency series (e.g., quarterly and annual, respectively), in a system observed data need to be adjusted such that both temporal (across frequencies) and contemporaneous (within frequencies) constraints are satisfied. A reconciliation process aims at preserving as much as possible the content of the preliminary information available. Because the time-series dimension of socio-economic variables is relevant, it is often convenient that the movements (or the growth rates) of the preliminary information are preserved in the best possible way.

In this work we deal with the specific problem of reconciling annual (preliminary) estimates of U.S. national accounts aggregates subject to quinquennial benchmarks available

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from detailed Input-Output (IO) tables: given preliminary, not fully balanced IO accounts for five consecutive years (1998-2002), and given 2 fully balanced, benchmark IO accounts for years 1997 and 2002, we wish to obtain fully balanced, revised IO accounts for years 1998-2001 where the temporal profile of the preliminary aggregates is preserved as much as possible. We aim at adjusting the annual data such that they (i) are consistent with the quinquiennal benchmarks available, (ii) fulfill all the IO accounting relationships for any given year, and (iii) show movements that are as close as possible to the preliminary information. In addition, it is sensible to expect that the very different growth rates observed in many variables between 2001 preliminary and 2002 benchmark values as compared to those observed in the preliminary estimates are adjusted without drastically altering the temporal profiles of the variables originally estimated by the preliminary values.

The paper is structured as follows. In the next section we describe the national accounts problem faced in this paper. Section 3 briefly introduces benchmarking and reconciliation of economic time series. Then, in Section 4 we present and discuss the results achieved using a least squares reconciliation procedure based on alternative objective functions. Finally, some conclusions are drawn in the last section.

2. A Typical National Accounts Problem

The U.S. national accounts system measures gross domestic product (GDP) from IO, expenditure, and income accounts. For the system to be consistent, GDP measured as total value-added (VA) from the balancing items in the IO accounts must be consistent with GDP measured as total final expenditures from the national income and product accounts (NIPA). GDP measured from production and expenditure data should also be consistent with gross domestic income (GDI) measured as total VA from GDP-by-industry accounts.

Most time series data of the national accounts system are classified by attributes. This requires that the values of the component elementary series add up to marginal totals for each period. For example, the IO accounts, classified by N industries and M commodities, must satisfy N sets of industry and M sets of commodity cross-sectional aggregation constraints each period. Often individual series in the national account system must also add up to temporal benchmarks and, thus, must satisfy their respective temporal aggregation constraints. For example, each component series of quarterly GDP must add up to its annual aggregates, and each component series in the annual IO accounts should be consistent with its corresponding quinquennial benchmark.

Source data used to compile different sets of accounts in the U.S. national accounts system are obtained from different sources. Thus, inconsistencies often arise in source data items in the accounts due to differences in the definitions or classifications of some variables and due to various types of measurement errors in the source data. Consequently, initial data items of the accounts rarely satisfy all cross-sectional accounting constraints. Source data for the accounts also become available at different frequencies. Quinquennial benchmark data based on Economic Census contain more complete information and, thus, are more accurate, but they are not timely. High frequency source data, such as quarterly data for quarterly GDP estimates or annual survey data for the annual IO accounts are timely but often contain incomplete information. Hence, they are less accurate and often do not automatically satisfy all temporal aggregation constraints.

As for the U.S. benchmark and annual IO accounts (1997-2002), the compilation procedure is shown in figure 1 (see also Stewart *et al.*, 2007).

It should be noted that when a benchmark revision occurs, usually both levels and growth rates of the variables are affected. Figure 2 shows the percentage revisions produced by the 2002 benchmark estimates on the levels of GDP and some major final uses

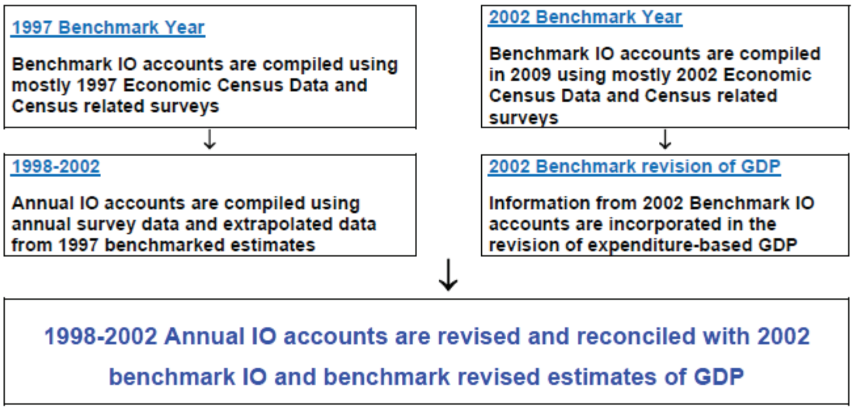

Figure 1: Compilation procedure of U.S. benchhmark and annual accounts (1997-2002)

aggregates: the impact of the 2002 benchmark ranges from -1.4% for Imports (the only one of the 7 considered aggregates with a downward revision), to 8.7% of Changes in business inventories, while the 2002 preliminary GDP level shows a 1.7% upward revision.

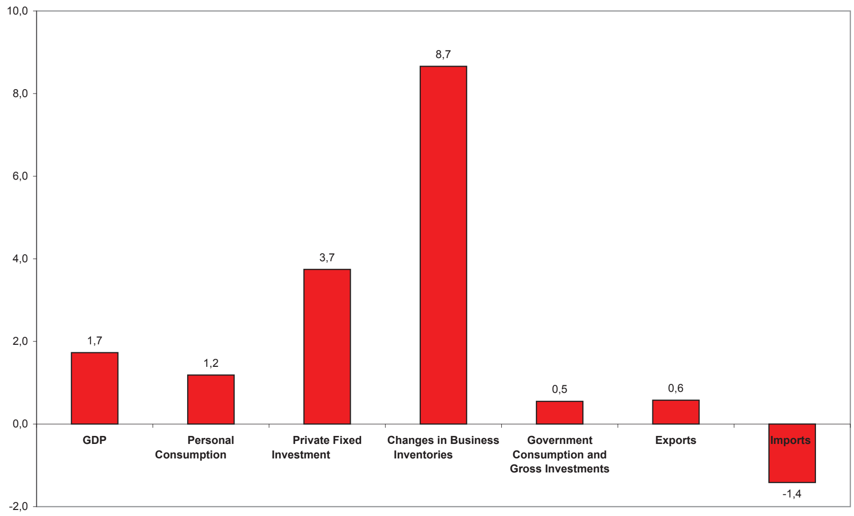

Figure 2: 2002 Benchmark revision to the levels of GDP and some major final uses aggregates (% of the preliminary 2002 value)

As for growth rates (fig. 3), the largest correction is for Private fixed investment (3.57%), while the 2002 preliminary GDP growth rate is clearly (1.79%) upward revised as well.

In this paper we consider the 1998–2002 U.S. annual IO accounts, GDP-by-industry accounts, and GDP from expenditures, subject to the 1997 and 2002 quinquennial benchmarks and all contemporaneous constraints of the system. Data from the 1998-2002 annual IO accounts, previously balanced and reconciled contemporaneously with the expenditure-based GDP, are the preliminary estimates in this application. The 1997 and 2002 benchmarks are GLS reconciled estimates based on the estimated reliabilities of all initial data items in the benchmark year IO accounts (Chen, 2012).

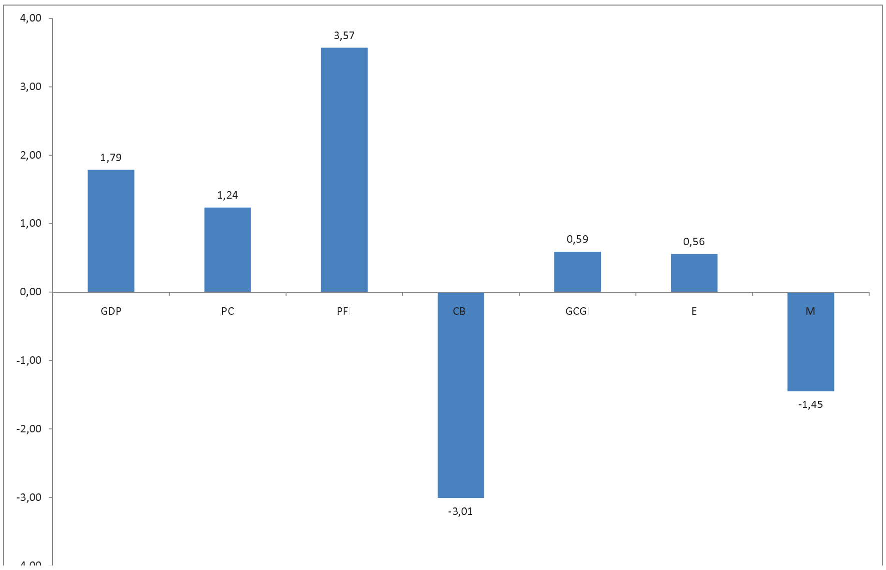

Figure 3: 2002 Benchmark revision to the growth rates of GDP and some major final uses aggregates

In our exercise, reconciliation is conducted at the level of detail of 65 industries, 69 commodities, 3 VA components and 13 final expenditure categories. The available preliminary data (within-benchmark-years and 2002 estimates) present *temporal* inconsistencies (preliminary and benchmark 2002 estimates are different, as previously shown), and *accounting* discrepancies (by industries, see figure 4, and by commodities, see figure 5).

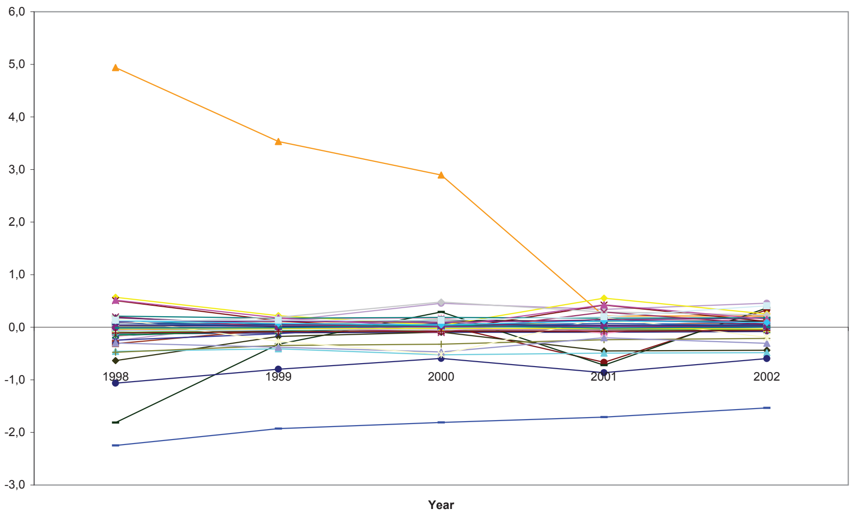

Figure 4: Discrepancies (%) by industry

To give an idea of the dimension of the problem, at the chosen level of detail, the system

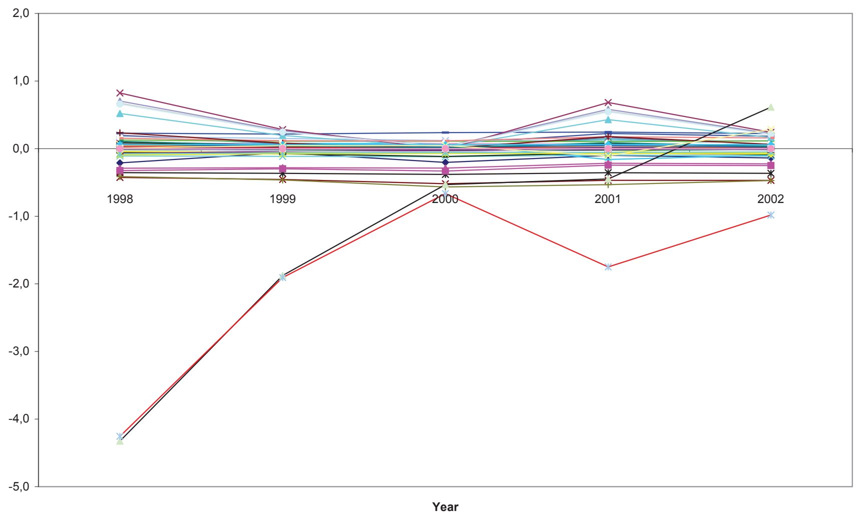

Figure 5: Discrepancies (%) by commodity

of IO accounts consists of a total of 10,062 series, 4,485 from the make table and 5,577 from the use table, which includes 4,485 intermediate inputs, 195 industry VA, and 897 final expenditure series. Of the 4,485 series from the make table, 694 are non-zero series, and of the 5,577 series from the use table, the non-zero series include 3,551 intermediate inputs, 193 VA and 300 final expenditures series. In what follows we show how to deal with all these issues in a consistent statistical framework.

3. Benchmarking and Reconciliation of Time Series

To restore temporal constraints in each component series, the modified Denton's proportional first difference (PFD) benchmarking method (Denton, 1971; Helfand *et al.*, 1977; Cholette, 1984) has been implemented at the U.S. Bureau of Economic Analysis (BEA) since 2006. To restore contemporaneous constraints in the annual accounts, the usual reconciliation procedures use accounting identities from different parts of the system to reduce accounting discrepancies as much as possible and to record the residual between GDP and GDI as aggregate statistical discrepancy. In a recent study, a generalized least-squares (GLS) procedure is used to reconcile GDP estimated from IO, expenditure, and income accounts for a benchmark year according to the estimated reliabilities of initial source data items (Chen, 2012).

Consistency in the time series of the national account system requires that temporal and contemporaneous constraints be satisfied simultaneously. In recent years, two alternative reconciliation procedures have been introduced to restore temporal and contemporaneous constraints in a system of series (Quenneville and Rancout, 2005; Di Fonzo and Marini, 2011). The two-step procedures consists of a univariate process to restore temporal constraints in each component series through benchmarking and a multivariate reconciliation process to restore contemporaneous constraints in the system in each period while preserving movements in each series. The two-step procedures are shown to be effective when low frequency benchmarks correspond to low frequency sums of the high frequency values (i.e. flow variables). However, each estimate in the quinquennial benchmark IO accounts pertains to the value of a variable at the end of the benchmark year, not the quinquennial sum of the values of the variable. In this case, the two-step procedure may not be able to preserve the temporal movements in each component series during the reconciliation process. What we need is a procedure which can simultaneously restore temporal and contemporaneous constraints in the annual IO accounts.

The reconciliation problem can be formalized in a compact matrix form as follows. The U.S. annual IO accounts consist of make and use tables. The 69×65 make table matrix contains the gross output of 69 commodities from 65 industries. The use table consists of a 69×65 matrix of intermediate inputs, a 3×65 matrix of industry VA from industry income, and a 69×13 matrix of final uses.

Let $\mathbf{X}_t$, $\mathbf{Z}_t$, $\mathbf{V}_t$, and $\mathbf{Y}_t$ denote the matrices of preliminary estimates of gross output, intermediate inputs, VA and final uses, respectively, in the annual IO accounts for $t = 1998, \ldots, 2002$. Let $\bar{\mathbf{X}}_{1997}$, $\bar{\mathbf{Z}}_{1997}$, $\bar{\mathbf{V}}_{1997}$ and $\bar{\mathbf{Y}}_{1997}$ denote the corresponding benchmark matrices for 1997 and let $\bar{\mathbf{X}}_{2002}$, $\bar{\mathbf{Z}}_{2002}$, $\bar{\mathbf{V}}_{2002}$ and $\bar{\mathbf{Y}}_{2002}$ denote benchmark matrices for 2002. The benchmark matrices have the same dimensions of the companion preliminary matrices. Our objective is to obtain matrices that satisfy the given benchmark level at $t = 1997$ and $t = 2002$, and preserve the movements in the preliminary matrices for $t = 1998, \ldots, 2002$.

The preliminary matrices can be conveniently rearranged into a one-dimensional vector of stacked time series. Let $\mathbf{x}_{i,j}$ denote the 6×1 column vector of the element (i, j) of the make table matrix $\mathbf{X}_t$, for $t = 1997, \ldots, 2002$[1]. We consider all (i, j) elements of the matrices even if they are zeros for all or for some years. The 4,485 time series in the make table can be stacked into a single $26,910 \times 1$ vector as

$$\mathbf{x} = \left[\begin{array}{ccccccc} \mathbf{x}'_{1,1} & \mathbf{x}'_{1,2} & \mathbf{x}'_{1,65} & \cdots & \mathbf{x}'_{69,63} & \mathbf{x}'_{69,64} & \mathbf{x}'_{69,65} \end{array} \right]'.$$

Vectors $\mathbf{z}$, $\mathbf{y}$, $\mathbf{v}$ can also be set up in the same fashion. Their row dimensions are 26,910, 5,382 and 1,170, respectively. The input vector of preliminary data of the problem is thus defined as

$$\mathbf{p} = \left[\begin{array}{cccc} \mathbf{x}' & \mathbf{z}' & \mathbf{y}' & \mathbf{v}' \end{array} \right]'$$

where $\mathbf{p}$ has dimension $60,372 \times 1$.

Let us now consider the constraints of the system. There are *exogenous* and *endogenous* constraints. The first type concerns the benchmark values for the years 1997 and 2002. Let $\mathbf{b}$ denote the vector of a two-element time series from the benchmarked matrices previously defined. That is,

$$\mathbf{b} = \left[\begin{array}{ccccccccc} \bar{x}^{1,1}_{1997} & \bar{x}^{1,1}_{2002} & \bar{x}^{1,2}_{1997} & \bar{x}^{1,2}_{2002} & \cdots & \bar{v}^{3,64}_{1997} & \bar{v}^{3,64}_{2002} & \bar{v}^{3,65}_{1997} & \bar{v}^{3,65}_{2002} \end{array} \right]',$$

with dimension $20,124 \times 1$. Let $\mathbf{H}_1$ denote the $20,124 \times 60,372$ mapping matrix for the exogenous constraints specified in $\mathbf{b}$ for the benchmark years 1997 and 2002. Given that, as we have previously said, preliminary and benchmark 2002 values are different, it is $\mathbf{H}_1\mathbf{p} \neq \mathbf{b}$.

The endogenous constraints are defined by the set of accounting relationships defined by the IO tables. There are 69 row constraints (commodities) and 65 column constraints (industries) per year. The aggregation constraint of total GDP equals total VA is redundant and can be disregarded, as it follows from adding up the first 134 constraints per year. In total, they add up to 804 constraints. Let $\mathbf{H}_2$ denote the $804 \times 60,372$ matrix mapping the

[1] In order to link the reconciled series to the 1997 benchmarks, we consider the benchmark matrices of 1997 as part of the group of preliminary matrices as well.

60,372 elements in the preliminary vector $\mathbf{p}$ to the 804 endogenous constraints. Clearly, it is $\mathbf{H}_2\mathbf{p} \neq \mathbf{0}_{804\times1}$.

In summary, we have

$$\begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix} \mathbf{p} \neq \begin{bmatrix} \mathbf{b} \\ \mathbf{0}_{804\times1} \end{bmatrix}, \qquad (1)$$

and we wish to derive the $60,372 \times 1$ vector of reconciled values $\mathbf{r}$

$$\begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix} \mathbf{r} = \begin{bmatrix} \mathbf{b} \\ \mathbf{0}_{804\times1} \end{bmatrix}, \qquad (2)$$

such that the temporal dynamics of $\mathbf{r}$ is 'close' to that of $\mathbf{p}$.

To reconcile a system of time series, we use adjustment procedures based on the constrained optimization of two different objective functions:

- Proportional adjustment (PROP):

$$\sum_{i=1}^{n} \sum_{t=1998}^{2002} \frac{(r_{t,i} - p_{t,i})^2}{|p_{t,i}|} \qquad (3)$$

- Proportional First Difference (PFD) adjustment, which is a multivariate extension of the univariate benchmarking solution proposed by Denton (1971) and modified by Cholette (1984):

$$\sum_{i=1}^{n} \sum_{t=1998}^{2002} \left(\frac{r_{t,i}}{p_{t,i}} - \frac{r_{t-1,i}}{p_{t-1,i}} \right)^2 \qquad (4)$$

where n is the number of non-null variables of the system.

In both cases, the system is adjusted simultaneously (i.e. all variables and all years at the same time). However, the adjustment principles operate very differently. The PROP criterion distributes the differences proportionally to the levels of the variables. On the other hand, the PFD criterion preserves the year-on-year movements of the variables. Because our target is to preserve the changes in the preliminary variables, we expect that the PFD method provide more satisfactory results for this exercise.

We also define a combined objective function (see Bikker *et al.*, 2013):

$$\sum_{i\in S^{PFD}} \sum_{t=1998}^{2002} \left(\frac{r_{t,i}}{p_{t,i}} - \frac{r_{t-1,i}}{p_{t-1,i}} \right)^2 + \sum_{i\in S^{PROP}} \sum_{t=1998}^{2002} \frac{(r_{t,i} - p_{t,i})^2}{|p_{t,i}|} \qquad (5)$$

where both the PFD criterion and the PROP criterion are utilized. The variables in the system are divided in two subsets (S^{PFD} and S^{PROP}, respectively): the PFD is used for those series showing meaningful and interpretable movements over time (namely movements that we would like to preserve), while for the rest of the series with breaks in the movements we switch to PROP[2] We call this procedure PFD-PROP.

[2]In this exercise S^{PROP} refers to changes in business inventories for 30 (non-zero) commodities, and to exports of commodity n. 66, which is null in 1997. Changes in inventories are very volatile and have many negative values, which makes the changes not very informative.

4. Results

In order to assess the global performance of the procedures, for each series we calculate the Mean Absolute Adjustment (MAA) and the Root Mean Squared Adjustment (RMSA) to the percentage levels:

$$MAA_i^L = 100 \times \frac{1}{5}\sum_{1998}^{2002}\left|\frac{\hat{r}_{t,i} - p_{t,i}}{p_{t,i}}\right|$$

$$RMSA_i^L = 100 \times \sqrt{\frac{1}{5}\sum_{1998}^{2002}\left(\frac{\hat{r}_{t,i} - p_{t,i}}{p_{t,i}}\right)^2}$$

and to the percentage growth rates:

$$MAA_i^R = 100 \times \frac{1}{5}\sum_{1998}^{2002}\left|\frac{\hat{r}_{t,i}}{\hat{r}_{t-1,i}} - \frac{p_{t,i}}{p_{t-1,i}}\right|$$

$$RMSA_i^R = 100 \times \sqrt{\frac{1}{5}\sum_{1998}^{2002}\left(\frac{\hat{r}_{t,i}}{\hat{r}_{t-1,i}} - \frac{p_{t,i}}{p_{t-1,i}}\right)^2}$$

for $i = 1, \ldots, n$, where n is the number of non-null series from the IO tables.

Table 1 shows the averages of indices MAA and $RMSA$ calculated for 43 main aggregates of national accounts (gross domestic product (GDP), gross output, intermediate inputs and VA of 12 major industries, and 6 final expenditure categories). These aggregates are calculated from the 10,062 reconciled series derived using the three procedures (PROP, PFD and PFD-PROP)[3].

Table 1: Summary measures of adjustment

	Levels		**Growth Rates**	
Criterion	MAA^L	$RMSA^L$	MAA^R	$RMSA^R$
PROP	1.380	4.146	1.607	4.501
PFD	4.174	8.288	2.352	6.631
PFD-PROP	3.530	6.073	1.604	2.273

As expected, PROP minimizes the adjustment in terms of levels (both MAA^L and $RMSA^L$ are minimum). Unexpectedly, PROP outperforms PFD in minimizing the adjustment in terms of growth rates. The PFD criterion is penalized by series in the system that show breaks in movements (e.g. new items from 1998) and present changes from positive to negative values (e.g. changes in business inventories). To overcome this difficulty, the PFD-PROP procedure adjusts the problematic series according to PROP while it maintains the PFD approach for the rest of the series. As it is noticed in Table 1, the PFD-PROP procedure achieves the minimum values for both MAA^R and $RMSA^R$.

[3] At first, we decided to verify the impact of the adjustment on the main aggregates of the national accounts. Further investigation on detailed components is currently being conducted.

Figure 6 displays the boxplots of $RMSA_i^L$ (top chart) and $RMSA_i^R$ (bottom chart) for the 43 aggregates, whereas those of indices MAA are shown in Figure 7. The visual inspection of the boxplots confirm that PFD-PROP produces the smallest adjustment of the growth rates, while PROP provides the best results in preserving the original levels. As for the growth rates, this conclusion is evident looking at the $RMSA$ statistics, whereas the absolute distance metric of MAA gives a less pronounced difference between the performance of PROP as compared to PFD-PROP.

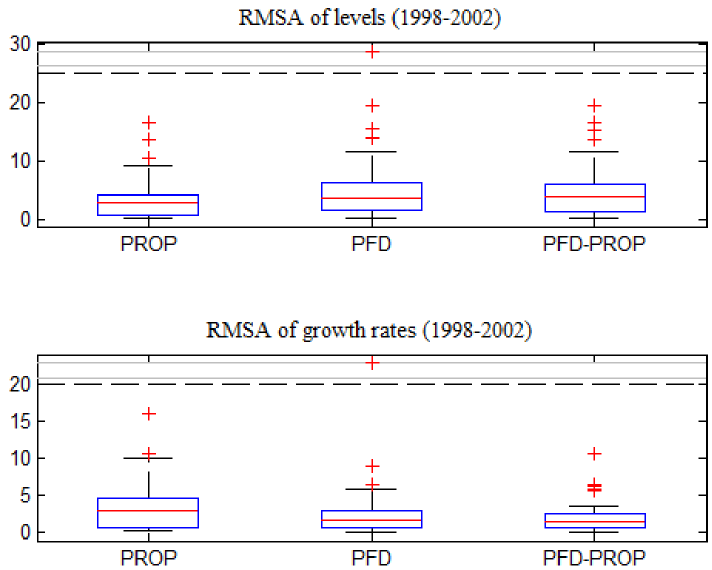

Figure 6: Boxplot of RMSA statistics

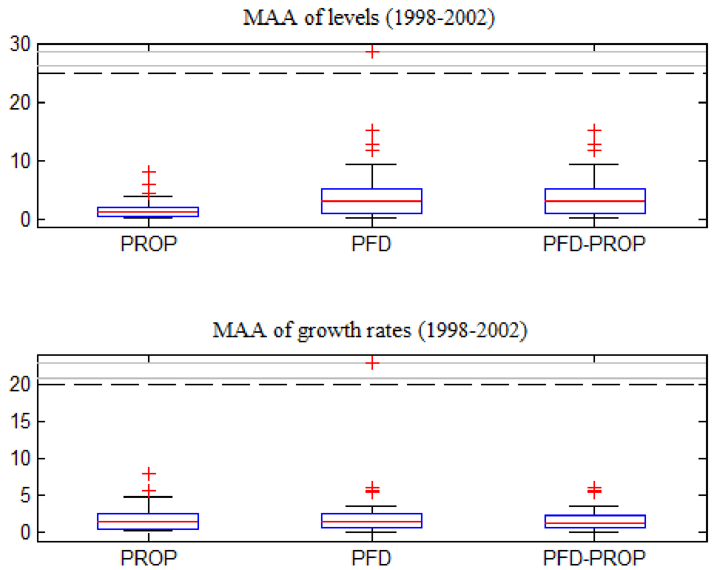

Figure 7: Boxplot of MAA statistics

To understand the different type of adjustment conducted by PROP and PFD-PROP, it is useful to look at the treatment of some representative series, like GDP (figure 8) and Output Agriculture (figure 9). In each figure, the left-hand charts refer to the levels, the right-hand charts to the growth rates, and the adjustments to both levels and growth rates are shown in the bottom charts.

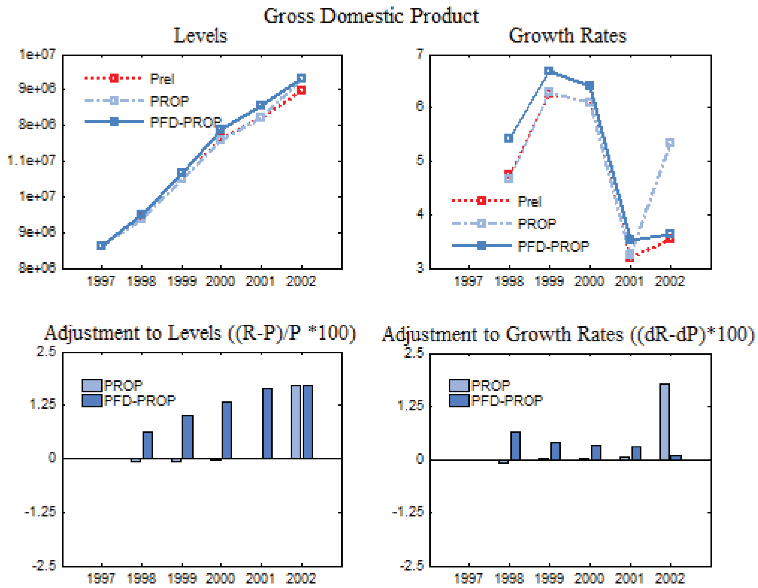

Figure 8: Gross Domestic Product: Adjustments to Levels and Growth Rates

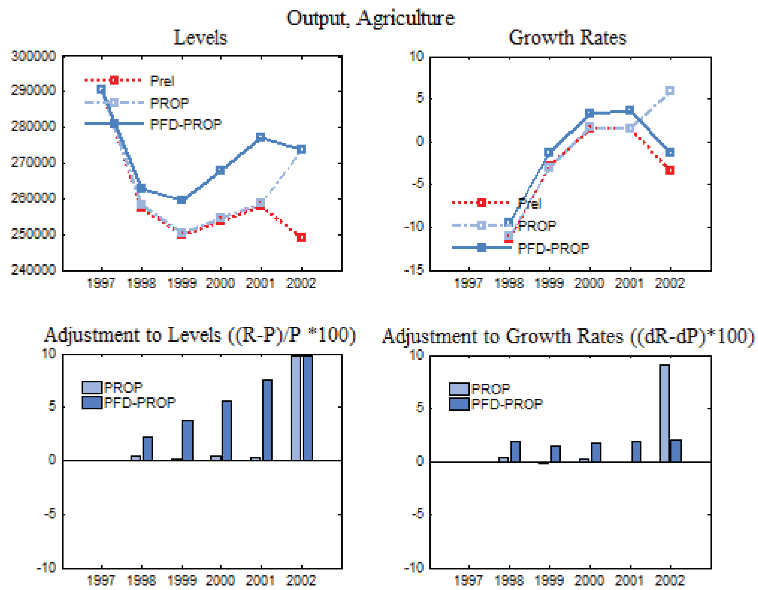

Figure 9: Output Agriculture: Adjustments to Levels and Growth Rates

It clearly appears that the adjustment done by PROP to GDP and Output Agriculture is all in the year 2002, differently from the reconciled estimates according to PFD-PROP, which produces (growing) adjustments to the levels for the entire period. This last feature permits to get ‘smoothed’ estimates of the growth rates, thus avoiding the abrupt ‘jumps’ produced by PROP, with a large positive correction of the preliminary 2002 growth rates. In the case of Output Agriculture this is worsened by the inversion of the direction of the change: while the preliminary series shows a decline of the 2002 level, and consequently a negative rate of change, the PROP-reconciled value shows a strong growth. As a result, the original movement from 2001 to 2002 is drastically changed. On the contrary, the PROP-PFD procedure distributes the difference between the preliminary 2002 value and the benchmark 2002 value over the 1998-2002 period. These patterns of adjustment that characterize the PROP-PFD and PROP procedures are evident in almost all the considered series.

However, it must be said that in some cases the reconciled figures produced by PROP and PFD-PROP are very close. In figure 10 the adjustments to levels and growth rates of Value Added Manufacture are shown. In this case we see that PROP and PFD-PROP produce similar corrections to the levels, thus giving rise to a very close dynamic profile of the reconciled series.

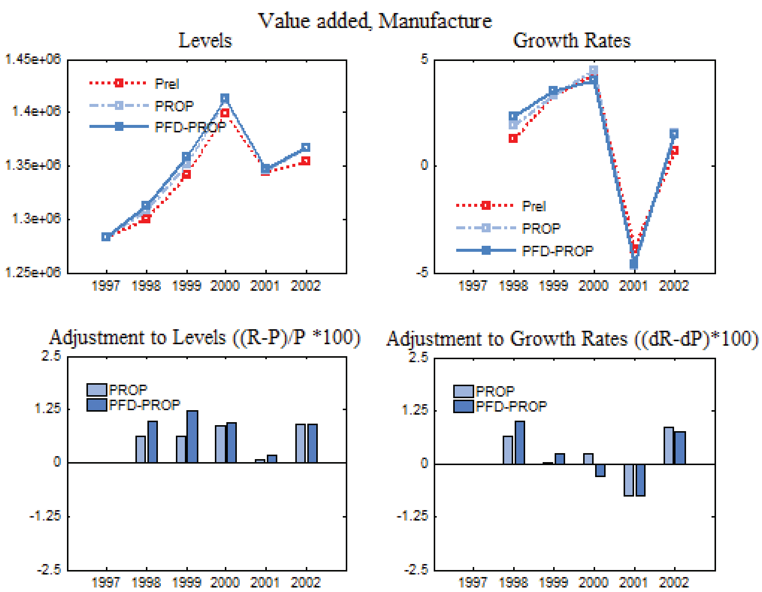

Figure 10: Value Added Manufacture: Adjustments to Levels and Growth Rates

5. Conclusions

In this paper we have shown how to reconcile annual preliminary series of national accounts with quinquennial benchmarks available from IO tables. Our objective was to minimize the impact of the adjustment on the movements in the preliminary series. In general, we have found that this objective is best achieved through a constrained optimization procedure based on a movement preservation principle, in our case the PFD criterion proposed by Denton (1971), modified by Cholette (1984). Looking at the temporal dynamics of the data, the PFD-based procedure is able to smooth the differences observed between the preliminary and the benchmark data of 2002, reducing the impact of the correction by distributing it over all the years.

However, we have noticed that a PFD adjustment provides unsatisfactory results for series that present breaks and changes from positive to negative values. Because these movements are more difficult to preserve, these series should be adjusted according to a pure proportional criterion. We have shown that a constrained optimization procedure that minimizes a combined PFD-PROP objective function improves the overall adjustment of the system, minimizing the impact on the year-on-year changes of the preliminary series.

REFERENCES

Bikker, R., Daalmans, J., Mushkudiani, N. (2013), "Benchmarking large accounting frameworks: a generalized multivariate model", *Economic Systems Research* (in press).

Chen, B. (2012), " A balanced system of U.S. industry accounts and distribution of the aggregate statistical discrepancy by industry", *Journal of Business and Economic Statistics*, 30, 202–221.

Cholette, P. (1984), "Adjusting sub-annual series to yearly benchmarks", *Survey Methodology*, 10, 35–49.

Dagum, E.B., Cholette, P.A. (2006), *Benchmarking, Temporal Distribution, and Reconciliation Methods for Time Series*. New York: Springer.

Denton, F. (1971), "Adjustment of monthly or quarterly series to annual totals: An approach based on quadratic minimization", *Journal of the American Statistical Association*, 66, 99–102.

Di Fonzo, T., Marini, M. (2011), "Simultaneous and Two-step Reconciliation of Systems of Time Series: Methodological and Practical Issues", *Journal of the Royal Statistical Society*, Ser. C, 60, 143–164.

Helfand, S.D., Monsour, N.J., Trager, M.L. (1977), "Historical revision of current business survey estimates", American Statistical Association, Proceedings of the Business and Economic Statistics Section, 246–250.

Quenneville, B., and Fortier, S. (2012), "Restoring accounting constraints in time series - Methods and software for a statistical agency", in Bell, W.R., Holan, S.H., and McElroy, T.S. (eds.), *Economic Time Series Modeling and Seasonality*, New York: Taylor & Francis, 231–253.

Quenneville, B., and Rancourt, E. (2005), "Simple methods to restore the additivity of a system of time series", paper presented at the workshop on 'Frontiers in benchmarking techniques and their application to official statistics', Luxembourg, 7-8 April 2005.

Stewart, R.L., Brede Stone, J., and Streitweiser, M.L. (2007), "U.S. Benchmark Input-Output Accounts, 2002", *Survey of Current Business*, 87, 19–48.