The Inequality Process is "Demonic": Conditional Entropy Maximization and Minimization and Wage Incomes

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Abstract

Maxwell's Demon intervenes into the Kinetic Theory of Gases (KTG), a particle system, removing particles with above mean kinetic energy, decreasing the entropy of the remaining particles. The Teacher intervenes into the Inequality Process (IP), a particle system of wealth distribution whose stationary distribution is approximated by a gamma pdf. An IP particle's probability of having its parameter retired by the Teacher is inversely proportional to its wealth. The Teacher replaces each retired particle parameter with that of a particle whose probability of selection is proportional to its wealth. The IP tends to transfer wealth to particles with small values of the particle parameter, by hypothesis of the IP's meta-theory and evidence representative of more productive workers. Teacher-driven evolution of the IP begins with large particle parameter values that the IP's meta-theory attributes to the earliest civilizations. Teacher-driven IP evolution reproduces features of the distribution of labor income over techno-cultural evolution, including the last half century in the U.S. Teacher-driven change in the IP's stationary distributions results in higher entropies of particle wealth. These increases in the entropy of particle wealth decelerate as the stationary distributions become more centralized. Simultaneously, the Teacher drives the increasing absolute value of the correlation between particle wealth and the particle parameter. This effect eventually decreases particle wealth entropy. This finding suggests that empirical analogues of Teacher and IP turn entropy increase in worker income and wealth into information gain over the course of technological evolution.

Key Words: Entropy, income distribution, inequality, Inequality Process, maxentropic, Maxwell's Demon, techno-cultural evolution, wealth.

1. Introduction

In 1867 James Clerk Maxwell imagined a nano-scale homunculus, the Demon, that, in terms of the Kinetic Theory of Gases (KTG), removes particles with above average kinetic energy from a population of particles (Leff and Rex, 1990). The Kinetic Theory of Gases (KTG) is a micro-model of thermally and physically isolated gas molecules in collision. Maxwell advanced the idea of the Demon perhaps to pique the common view of the inevitability of the macro-level 2nd Law of Thermodynamics. The 2nd Law asserts that in a physically and thermally isolated volume of gas, the entropy of molecular kinetic energy is non-decreasing. The Demon shows there might be, conceivably, a micro-level intervention into the KTG that, by removing particles with above mean kinetic energy, decreases the entropy of particle kinetic energy. Maxwell imagined the Demon operating a door in a vessel wall. When a particle with above average kinetic energy headed toward the door, the Demon allows only that particle to exit. By the mid-20th century, several physicists advanced the argument that any Demon-like intervention into a real gas would increase the entropy of the molecules remaining in the vessel (Leff and Rex, 1990). However, counter-factual assumptions can be inserted into a mathematical model. As formulated with Maxwell's assumptions, the Demon's intervention into the KTG implies a decrease in the entropy of the kinetic energy of the remaining particles. See Appendix A for proof.

1.1 Information Gain via Entropy Increase?

Leo Szilard redefined the Demon's thermodynamic entropy reduction as information gain (Leff and Rex, 1990). Since the KTG is a mathematical model, it is possible to set all particles' kinetic energy equal to mean kinetic energy, i.e., a spike distribution with zero entropy, before beginning the operation of the KTG. See Appendix B for the transition equations of the exchange of particle kinetic energy in the KTG. The operation of the KTG's transition equations disorders this initial spike distribution and the distribution of particle kinetic energy in the KTG converges to its stationary distribution, a negative exponential distribution. This is the distribution that results from maximizing the entropy statistic subject to the constraint of fixed mean kinetic energy. The information that all particles had equal kinetic energy is lost. The interpretation of information loss as entropy gain is well established in communications engineering (Shannon and Weaver, 1998[1949]).

But if, instead of removing particles with above mean kinetic energy, the Demon removed the same number of particles with below mean kinetic energy, it would have made a similar set of decisions and an identical number of door openings, although since the entropy statistic is nonlinear, the increase in entropy of the remaining particles would have not have had the same expected absolute value as the decrease of the entropy under the Demon's program. The present paper shows there can be information gain via entropy increase in the evolution of the distribution of personal wealth and income over the trajectory of techno-cultural evolution, negating the interpretation of entropy gain as necessarily information loss.

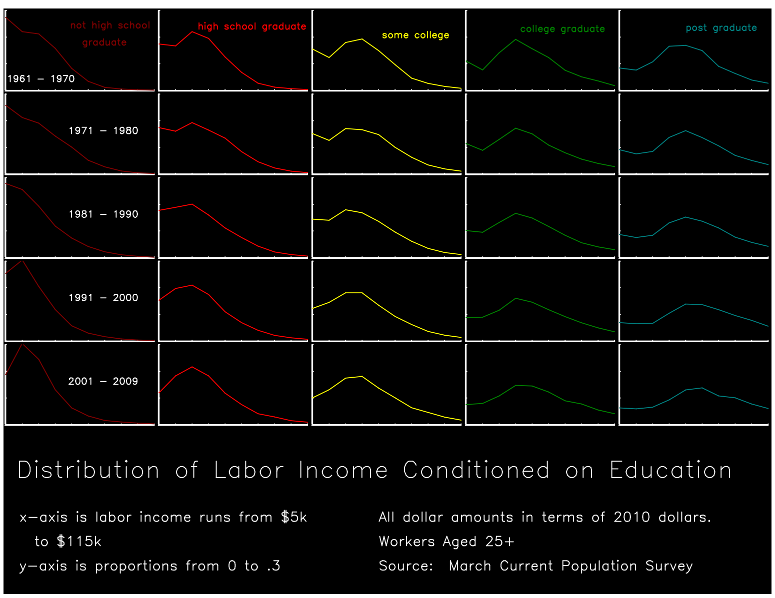


Figure 1

1.2 The Teacher's Intervention into the Inequality Process

The present paper introduces an intervention into a particle system that resembles that of the Demon. The particle system, the Inequality Process (IP), a model of personal income and wealth distribution (Angle, 1983-2012), is similar to that of the KTG. See Appendix B for the Inequality Process' (IP's) equations and the three substitutions into the KTG that transform it into the IP. The IP explains, inter alia, the distribution of the U.S. annual wage and salary income conditioned on education since 1961. See Figure 1. Figure 1 is estimated from Current Population Surveys (March 1962-2011) obtained from Unicon Research (2012) .

The intervention into the IP presented in this paper is the Teacher, so named to reflect its function and give a sense of its agency, as Maxwell gave the Demon. Like the Demon, the Teacher stochastically removes certain particles; unlike the Demon, the Teacher replaces each particle removed with a new particle. Like the KTG, the Inequality Process (IP) operates on an isolated population of particles, randomly paired for exchanges of a positive quantity. In the Inequality Process the label on the positive quantity exchanged by particles is 'wealth'. Unlike the empirical referent of the KTG, a monoatomic gas physically and thermally isolated in a vessel, the Inequality Process' (IP's) referent is a human population in which the distribution of personal wealth converges quickly enough to equilibrium that change in mean wealth and people entering or exiting the population are negligible effects. Unlike the Demon's, the Teacher's operations are, in the context of its empirical referent, possible, plausible, and endogenous.

The Inequality Process (IP), found and published as mathematical sociology, has been adopted as econophysics perhaps because of the Inequality Process' (IP's) similarity to the Kinetic Theory of Gases (KTG), discussed in Angle (1990). The IP dates from Angle (1983). The earliest interest in particle system models of wealth and income by econophysicists dates from 2000. Most citations to the Inequality Process are by physicists. See [http://scholar.google.com/citations?user=j2z9mg8AAAAJ&hl=en accessed on September 3, 2013]. In reviewing particle system models of income and wealth, Victor Yakovenko and J. Barkley Rosser write in "Colloquium: Statistical Mechanics of Money, Wealth, and Income" in ***Reviews of Modern Physics*** 81, 1705. [on-line at http://arxiv.org/abs/0905.1518] "Actually, this approach was pioneered by the sociologist John Angle (1986, 1992, 1993, 1996, 2002) already in the 1980s. However, his work was largely unknown until it was brought to the attention of econophysicists by the economist Thomas Lux (2005). Now, Angle's work is widely cited in econophysics literature (Angle, 2006).".

2. The Inequality Process' Verbal Meta-Theory and the Operationalization of Techno-Cultural Evolution

The Inequality Process (IP) is specified from the Surplus Theory of Social Stratification, an old theory of economic anthropology that explains why the first appearance of great inequality of wealth in the archeological record appears in the same layer as the first appearance of abundant stored food (Herskovits, 1940; Childe, 1944; Harris, 1959; Dalton, 1960, 1963). Explanations of the specification appear in Angle (1983, 1986, and 2006). See Appendix B for the equations of the Inequality Process (IP). That archeological layer corresponds to the transformation of a population that previously lived as hunter-gatherers, with few differences of wealth and no hereditary ruler, into the inegalitarian chiefdom, the society of the god-king. This simultaneity of events was universal: all times, all places, all cultures, all races. The Surplus Theory offers an elegantly simple explanation of it: a) there is widespread competition in all human groups, b) hunter-gatherers without abundant stored food live from hand to mouth, but c) when because of a richer ecological niche or the acquisition of agricultural technologies, the hunting and gathering population acquires an abundance of stored food, the competition that existed all along in the group concentrates control of that stored abundance in few hands and it becomes the first example of great wealth.

2.1 The Lenski Extension of the Surpus Theory: Human Capital Limits Competition For Wealth

While the Surplus Theory is an elegant verbal explanation of the universality of the transformation of the societal form anthropologists view as the most egalitarian, the hunter/gatherer, into the societal form they view as the most inegalitarian, the chiefdom, the Surplus Theory has no explanation for why further techno-cultural evolution beyond the chiefdom led to less concentration of wealth than in the chiefdom. Gerhard Lenski (1966) proposed a number of speculative amendments to the Surplus Theory to account for decreasing concentration of wealth as techno-cultural evolution moved beyond the chiefdom. The IP is specified from one of Lenski's speculations: that as technology advances, it requires more workers with more advanced skills. Worker skills are a valuable capital good that workers can withhold in bargaining for a larger share of the wealth they create. Consequently, a greater share of the wealth produced by advancing technology is retained by workers whose knowledge and skills embody that technology. Worker skill, human capital, becomes a larger fraction of aggregate societal wealth as populations attain a higher level of technology. In contemporary economies, human capital's share of national wealth is greater than that of tangible capital and natural resources combined (Hamilton and Liu, 2013; Jorgenson and Fraumeni, 1989). The data of the March Current Population Survey conducted by the U.S. Bureau of the Census asks questions about sources of personal money income (Current Population Surveys, March 1962-2011). The great majority of respondents report no money income from tangible property, e.g., dividends, interest, rent, or royalties. Consequently, in contemporary economies, personal income from work is the best measure of a personal stock of wealth.

The IP is abstracted from the Surplus Theory of Social Stratification as modified by Gerhard Lenski with the help of the principle of parsimony. In the specification of the IP the simplest model of competition consistent with the verbal meta-theory was sought. Thus, the model is a particle system. Its entities represent people but are so simple they qualify as particles. The IP's particles have only two characteristics, one transient, one semi-permanent. The transient characteristic is wealth; it changes with every competitive encounter with another particle. The semi-permanent characteristic is the fraction of wealth the particle gives up when it loses an encounter. It is semi-permanent in the way a worker's skill level is semi-permanent. This particle parameter, ω, operationalizes Lenski's extension of the Surplus Theory in a simple way. A smaller fraction lost operationalizes the more skilled worker, the worker more sheltered from loss due to competition. 'Skilled' means 'skilled at producing wealth'. A hunter/gatherer who produces almost no surplus wealth is represented in the IP by the fraction of wealth lost in an encounter of nearly 1.0. Competitive encounters are pairwise because a) pairwise is simplest, b) verbal theory offers no guidance on the organization of the extraction of surplus wealth from workers, and c) competition in groups, regardless of size or composition, that transfers wealth between people results in a net gain or loss for each person – just as in binary competition. Competition is zero sum because of its simplicity. Fixing mean particle wealth at 1.0 is a simplification, eliminating the need to model wealth production and consumption. The IP's properties are independent of the unconditional mean of wealth, i.e., the IP's properties are a consequence of the particle parameter ω, the intensity of competition and its distribution in the population of particles. Lenski treats per capita economic product as a function of technology, making no effort to create a theory of wealth production over the techno-cultural spectrum. The IP's operationalization of a society's level of technology is the harmonic mean of the values of the particle parameters. In the long run, the IP transfers wealth to particles that lose less when they lose, particles with a smaller ω parameter, the robust losers.

3. The Teacher's Intervention into the Inequality Process

There are three steps to the Teacher's intervention into the Inequality Process (IP). Their effect on the IP's stationary distribution and entropy of particle wealth is demonstrated via simulation. The Teacher's intervention begins the simulation of techno-cultural evolution with all particles assigned a parameter just under 1.0, indicative of a population unable to create surplus wealth. The Inequality Process (IP) is simulated with a large, finite population of 2,000 particles allowing 2,000 iterations to converge to its stationary distribution. The Unconditional mean of particle wealth is fixed at 1.0 in all simulations of the IP. The matrix language GAUSS is used for all calculations (Aptech, 2012).

3.1 Step 1 of the Teacher's Intervention into the IP: Remove a Particle's Parameter

The first step of the Teacher's intervention into the IP is the removal of the particle parameter of one percent of the population. The probability of the Teacher's choosing a particle for removal of its parameter is inversely proportional to that particle's wealth, i.e., poor particles are more likely to be tapped than wealthy:

$$p\,(removal\,of\;\; particle\;\; \psi) \;\;=\;\; \frac{1/x_{\psi}}{\sum_{i}^{N} 1/x_i}$$

where:

$x_{\psi} \;\;=\;\; particle\;\; \psi's\; wealth$

$N \;\;=\;\; number\; of\;\; particles\; in\;\; population$

3.2 Step 2 of the Teacher's Intervention into the IP: Replace a Particle's Parameter with That of Another Particle

Each particle whose parameter has been removed receives a new parameter. The new parameter is chosen and cloned from a particle with probability proportional to its wealth:

$$p\,(particle\;\; \psi's\;\; parameter\; being\;\; cloned\,) \;\;=\;\; \frac{x_{\psi}}{\sum_{i}^{N} x_i}$$

3.3 Step 3 of the Teacher's Intervention: Perturb All Particle Parameters Slightly

After replacing particles' parameters, the Teacher perturbs all particle parameters by adding a small random quantity, ε, to ω_{ψ}. E[ε] = 0.0. Here $\varepsilon = .003u_1 - .003u_2$, where u_1 and u_2 are independent uniform [0,1] continuous random varables. .9999 is a reflector for ω_{ψ}. .9999 is the initial value of ω_{ψ} for every particle. After the slight perturbation of all ω_{ψ}'s, the Inequality Process iterates 2,000 times, long enough to converge to a new stationary distribution. There are 7,000 simulations of the IP in tandem with the Teacher.

In the IP when particle ψ's parameter, ω_{ψ}, is large, particle ψ's wealth, x_{ψ}, is largely a matter of the length of particle ψ's winning streak, but with smaller ω_{ψ} wealth in the long run becomes more a matter of how small ω_{ψ} is. Smaller ω_{ψ} is the Inequality Process' (IP's) operationalization of greater worker skill. That operationalization follows from Lenski's extension of the Surplus Theory. This operationalization of worker skill explains the shapes of partial distributions of the distribution conditioned on ω_{ψ}. See Figures 1 and 2.

Techno-cultural advance depends on pre-requisite conditions and productive synergies of many people, conditions that for example came together in Europe in the 14th through the

17th centuries to spark the Renaissance, e.g., the invention of typeset printing. To mute the effect of population size on the rate of evolution and to make techno-cultural evolution more general than the rate of evolution of the most productive particles, two thirds of the harmonic mean of the ω_ψ's , $\tilde{\omega}_t$, is taken as the lower limit of ω_ψ's at any one time.

3.4 The Teacher's Resemblance to Natural Selection

The Teacher's intervention into the Inequality Process resembles Darwinian natural selection: a particle's $1/\omega_\psi$ is its fitness; fitness is stochastically related to a particle's niche (wealth); wealth stochastically determines its likelihood of passing its fitness on to offspring; particle fitnesses are continually perturbed. Given these Darwinian ingredients a larger population will evolve more quickly than a smaller population. Human culture can spread much more quickly than genes. Teachers, trainers, and coworkers can spread smaller ω_ψ's. The work skills of wealthier people are more likely to be spontaneously emulated than those of the poor, given the empirical relationship between work skills and wealth, and between ω_ψ and wealth in the Inequality Process (IP). The Teacher is more fine-grained in its operation than natural selection.

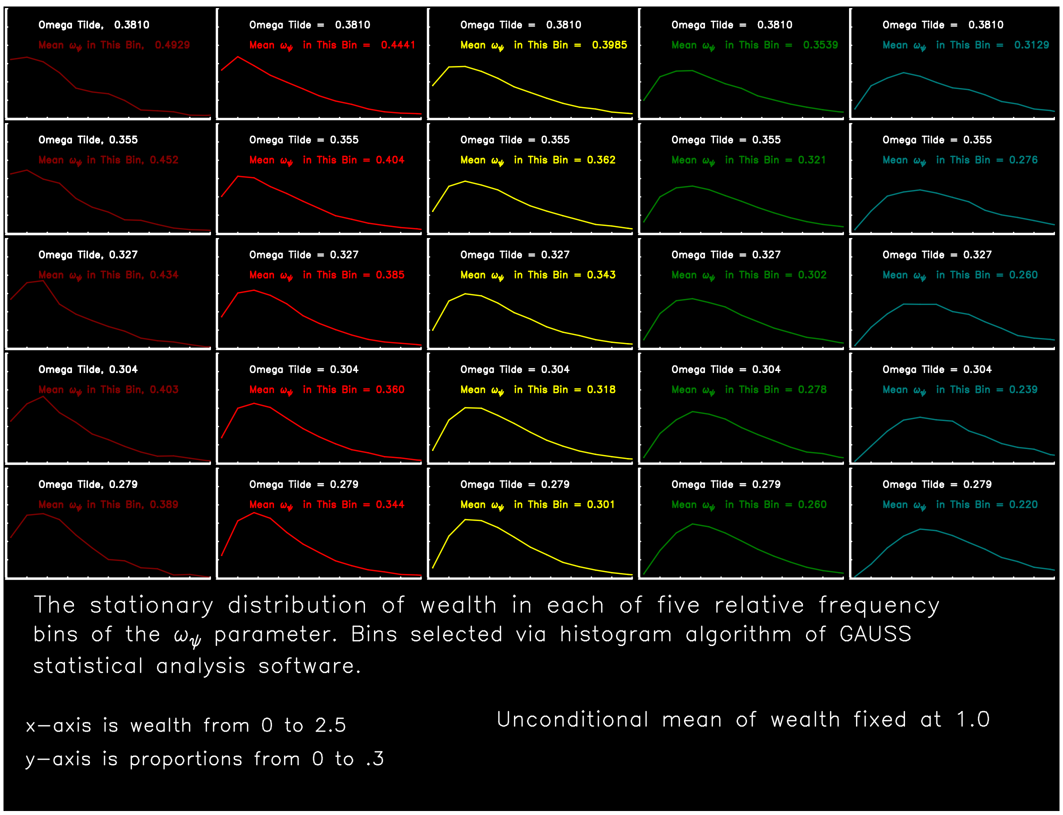


Figure 2

4. Findings

A comparison of Figure 2 to Figure 1 shows that the Teacher's intervention into the Inequality Process (IP) reproduces in some detail the time series of the partial distributions of the U.S. distribution of annual salary and wage income conditioned on the wokers' level of education. This result is surprising because the five groupings of particles by the size of their ω_ψ by the GAUSS histogram algorithm was not intended to mimic the grouping of workers by education category. However, both sets of decisions create roughly centralized distributions and ω_ψ ‘s estimated from U.S. income data scale by level of education. Another surprising finding is how much of the evolution of personal wealth distribution up to the present day is concentrated in the last half century in the U.S., about 13%. See Figure 3. Figure 3 shows that the Teacher’s intervention into the IP steadily drives down omega tilde, $\tilde{\omega}_t$, the harmonic mean of the ω_ψ‘s at a given time. The reciprocal of $\tilde{\omega}_t$ is the IP’s representation of a population’s level of technology. Figure 4 shows that the evolution of the Teacher’s intervention into the IP beginning with $\tilde{\omega}_t$ indicative of hunter/gatherers recently acquiring an abundance of stored food, i.e. $\tilde{\omega}_t = 1.0 - \varepsilon$, results in increasing but decelerating entropy of particle wealth. The entropy of particle wealth begins to decrease when $\tilde{\omega}_t$ reaches a value seen in the distribution of U.S. annual wage and salary income conditioned on education in the mid-20th century. See Figure 4. The reason entropy increases is that the Teacher’s intervention into the

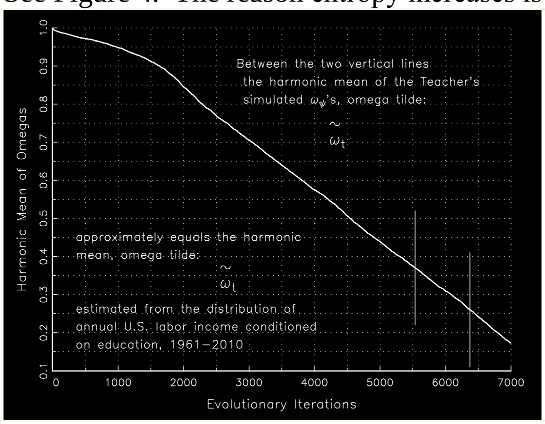


Figure 3

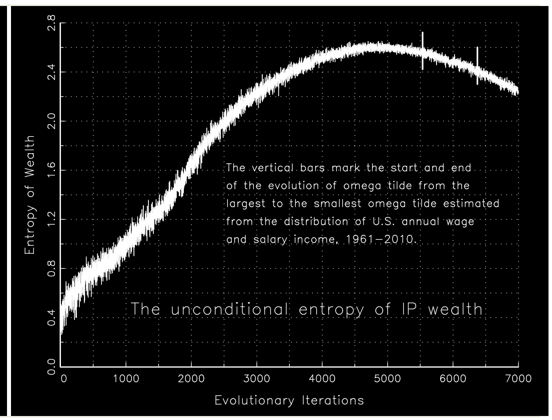
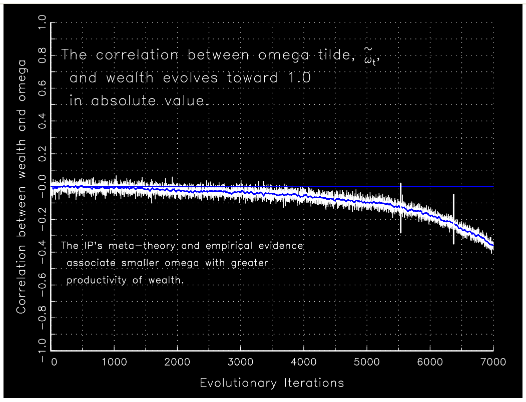


Figure 4

IP yields more centralized distributions of wealth. These have higher entropies. The distribution at the starting point of the evolution, $\tilde{\omega}_t = 1.0 - \varepsilon$, is a spike distribution just to the right of zero wealth with a very few particles far to the right with huge amounts of wealth. As these distributions become more like the distributions of Figure 2, the entropy of particle wealth decelerates. The reason entropy decelerates is the Teacher’s intervention into the IP increases the absolute value of the correlation between wealth and the particle parameters, the ω‘s. The increase in the correlation accelerates as $\tilde{\omega}_t$ becomes smaller, representing a society with more advanced technology. The acceleration is not uniform. See Figure 5. The empirical representation of this trend is an increasingly close association between worker productivity and wealth. This simulation does not model the effect of smaller $\tilde{\omega}_t$ on the unconditional mean of wealth.

The IP with the Teacher’s intervention replicates the time-series of inequality statistics of U.S. annual wage and salary income conditioned on education. One example is Figure 6. Figure 6 or a variant of Figure 6, the time-series of median annual wage and salary income by level of worker education in the U.S. is often pointed to in the labor economics literature and the U.S. media as evidence of out-of-control growth in inequality in the U.S. In Figure 6, the median income of the most educated appears to be racing up and away from the median incomes of the less well educated with the median incomes of successively less well educated groups of workers increasing less or not increasing at all in terms of constant dollars, i.e., purchasing power. Figure 7 displays median particle wealth of particles in five frequency bins defined by the size of their omegas, ranging from the smallest omegas (blue) to the largest omegas (red). A smaller omega is, in the IP's meta-theory, indicative of a more productive worker. As one would expect after seeing Figure 2, which nearly replicates the shapes of the distribution of annual wage and salary income conditioned on education in the U.S. 1961-2010 in Figure 1, Figure 7 nearly replicates Figure 6's wage and salary income medians conditioned on education, 1961-2010 within Figure 7's two vertical bars marking the range of $\tilde{\omega}_t$



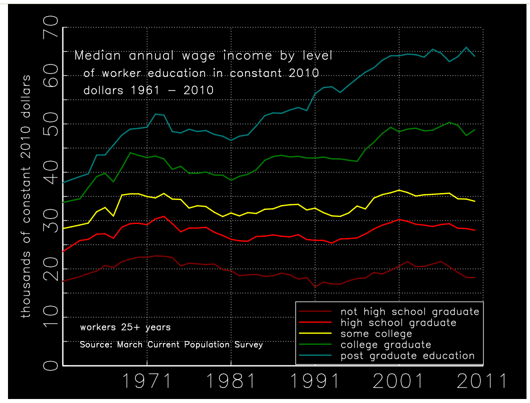


Figure 5 **Figure 6**

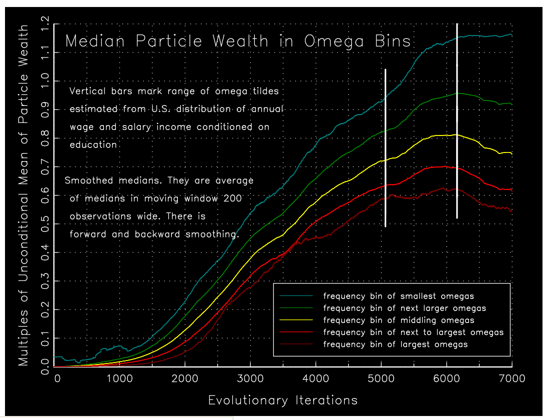


Figure 7

estimates from 1961 through 2010 in U.S. income data. Note in Figure 7 that the IP medians of particle wealth conditioned on ω diverge over the entire course of evolution due to the Teacher's intervention. The medians in Figure 7 increase over much of that evolution because as the distributions of wealth conditioned on ω are becoming more centralized and their means move closer to 1.0, the unconditional mean.

5. Conclusions

The Teacher intervenes into the Inequality Process (IP), a particle system similar to that of the Kinetic Theory of Gases (KTG). Appendix B.3 states the three substitutions into the KTG that yield the IP. Both the Teacher and Maxwell's Demon intervene into particle systems. The Demon removes certain particles. The Teacher removes the parameters of certain particles and replaces those parameters with parameters equal to those of certain other particles. The particles of the KTG represent molecules. They have one characteristic, a positive quantity called 'kinetic energy'. This quantity changes with each pairing with another particle. The particles of the IP represent people. They have two characteristics, one a positive quantity called 'wealth' that changes with each pairing of particles, and another, a parameter, that does not change until the Teacher intervenes.

This parameter is the fraction, ω, of each particle's wealth that it transfers to another particle if it loses an encounter. The operationalization of worker productivity in the IP's social science meta-theory is ω. The operationalization of a society's level of techno-cultural evolution in the IP is the harmonic mean of particle ω's, $\tilde{\omega}_t$.

In the IP winning or losing is a discrete 0,1, uniform random variable: a coin toss. In a loss, the losing particle gives up an ω fraction, $0.0 < \omega < 1.0$, of its wealth to the winner. In the KTG each particle gives up a fraction of its kinetic energy to the other. One particle gives up a [0.0,1.0] uniform random fraction of its kinetic energy; the other particle gives up the complement of that random fraction to the other. The Demon removes particles with above average kinetic energy. The Teacher draws a sample of particles whose parameter it will remove with probability proportional to the inverse of the particle wealth. The Teacher then draws another sample of particles with probability of each draw proportional to particle wealth. These particles' parameters are assigned randomly to particles whose parameters have been deleted. The Teacher then slightly randomizes the ω's of all particles.

Given the fact that the IP tends to transfer wealth to particles with smaller ω, the Teacher's intervention gradually decreases the harmonic mean of the ω's in the population of particles. This decrease accelerates. While the Teacher resembles aspects of Darwinian natural selection, the empirical representation of the Teacher does not necessarily require physical death. The Teacher's influence is more fine grained than natural selection. There are many cultural mechanisms changing a worker's productivity that can also be the empirical representation of the Teacher.

The start values of particle ω are just under 1.0 at the beginning of the simulation of the Teacher's intervention into the IP. According to the IP's meta-theory values of ω just under 1.0 represent people just beginning to acquire the capability of storing an ample supply of food. The presence of such wealth quickly leads to extreme concentration of it, the societal form known to anthropologists as the chiefdom. See the lower left corner of Figure 7. When $\tilde{\omega}_t$ is nearly 1.0, all wealth medians are near 0.0 even though mean wealth in all simulations is fixed at 1.0. See the tiny medians near the left edge of Figure 7. These medians are tiny because wealth is extremely concentrated as in the chiefdom.

The Teacher's intervention into the IP replicates change in the distribution of personal wealth over the course of techno-cultural evolution. The U.S. distribution of annual wage and salary income conditioned on education, 1961-2010, is approximated by the Teacher's intervention into the IP, the distribution of particle wealth conditioned on ω size. Surprisingly, the changes seen in the U.S. distribution account for about 13% of the simulated evolution of personal wealth distribution since the start of simulation with $\tilde{\omega}_t$ just under 1.0, the operationalization of the emergence of the chiefdom from hunter/gather society. These changes are due to rapidly rising levels of education.

The Teacher's intervention into the Inequality Process also reproduces the most frequently pointed to example of distressing growth in inequality in the U.S., Figure 6. Figure 7, between the markers for the largest $\tilde{\omega}_t$ and smallest $\tilde{\omega}_t$ estimated from U.S. annual wage and salary income conditioned on education in the last half century, faithfully reproduces the features of Figure 6. This paper locates the U.S. distribution of annual wage and salary income conditioned on education just past the maximum entropy of the Inequality Process driven by the Teacher's intervention (Figure 4). The growth in the entropy of wealth was driven by the Teacher's changing the distribution of wealth (starting with a spike over next to no wealth and near zero entropy). The growth in entropy decelerates as the correlation between a particle's ω and its wealth grows in absolute value (Figure 5). At the largest $\tilde{\omega}_t$ in the last half century in the U.S. (in the early 1960's), the absolute value of the correlation between ω and wealth in Teacher-driven evolution starts to accelerate upward.

Entropy maximization in the Inequality Process, driven by the Teacher, decelerates over the evolution of $\tilde{\omega}_t$ downward, yielding the distribution of wealth like that of the technologically most advanced societies (taking human capital as the primary indicator of wealth in such societies), and eventually at a point being reached in the last century in the U.S. begins to decrease. A process that, in the empirical analogue of the IP with the Teacher's intervention, can be interpreted as information gain (higher level of technology, greater worker productivity). In the IP it is information gain (the closer relationship between particle wealth and ω) so that ω is more accurately inferred from wealth, resulting from the Teacher's intervention that initially maximized the entropy of particle wealth.

Appendix A: The Demon Decreases Entropy

Maxwell found that the stationary distribution of the positive quantity exchanged by particles in the Kinetic Theory of Gases, a mathematical model of gas molecules, is a negative exponential pdf. It is a reasonably good approximation to the empirical distribution at temperatures and pressures in which people live. Boltzmann found the same solution by maximizing the entropy statistic, H:

$$H \equiv -\sum_{k=1}^{K} p_k \ln(p_k)$$

where there are K relative frequency bins of the distribution of molecular kinetic energy by size and p_k is the relative frequency in the k^{th} bin. H is subject to two constraints:

$$\sum_{k=1}^{K} p_k = 1.0$$

$$\sum_{k=1}^{K} p_k x_k = \textit{mean molecular kinetic energy in population of particles}$$

where x_k = mean kinetic energy in the k^{th} bin. Both Maxwell's and Boltzmann's solutions for the stationary distribution of molecular kinetic energy narrow the uniform widths of the frequency bins toward a limit of zero. At the limit the solutions yield the stationary distribution of molecular kinetic energy as a probability density function (pdf). This solution is the negative exponential pdf:

$$\begin{aligned} f(x) &= \lambda e^{-\lambda x} \\ \lambda &> 0 \\ x &\equiv \textit{molecular kinetic energy} \\ x &> 0 \end{aligned}$$

The entropy of the negative exponential pdf is found via integration of the expression:

$$H = -\int \ln(f(x)) \cdot f(x)\, dx$$

where $f(x) = \lambda e^{-\lambda x}$. The H of the negative exponential pdf is 1 – ln(λ). The mean, μ, of x is 1/λ. λ is less than or equal to μ; otherwise H would be negative. The usual assumptions of temperatures and pressures on the earth's surface made in conjunction with the KTG imply $\lambda < 1$ as well. A smaller λ both increases the mean of x, μ, and the H of a negative exponential pdf. And vice versa. Consequently, the Demon lowers the entropy of the kinetic energy of the remaining molecules.

Appendix B: The Equations of the Inequality Process (IP)

B.1 The IP's Transition Equations

The IP is defined by the equations for the transfer of wealth between particles in a competitive encounter, its transition equations:

$$x_{it} = x_{i(t-1)} + d_t \, \omega_{\theta j} \, x_{j(t-1)} - (1-d_t) \, \omega_{\psi i} \, x_{i(t-1)}$$
$$x_{jt} = x_{j(t-1)} - d_t \, \omega_{\theta j} \, x_{j(t-1)} + (1-d_t) \, \omega_{\psi i} \, x_{i(t-1)}$$

where:

$$x_{it} \equiv \textit{particle } i\textit{'s wealth at time-step } t$$
$$x_{j(t-1)} \equiv \textit{particle } j\textit{'s wealth at time-step } (t-1)$$
$$0 < \omega_{\theta j} < 1.0 \text{ fraction lost in loss by particle j}$$
$$0 < \omega_{\psi i} < 1.0 \text{ fraction lost in loss by particle i}$$
$$d_t = \textit{an i.i.d. } 0,1 \textit{ uniform discrete r.v. at time-step } t$$

Given the IP's assumptions, an isolated population of particles and random pairing of particles, the IP generates a stationary distribution of wealth in each ω_ψ equivalence class of particle that is approximately, but not exactly, a gamma probability density function (pdf). The IP's unconditional stationary distribution of wealth is thus approximately a mixture of gamma pdf's with different shape and scale parameters. Since the IP was first published in 1983, several related particle system models of personal income and wealth have been published (e.g., Chakraborti and Chakrabarti, 2000; Dragulescu and Yakovenko, 2000). The differences between these and the Inequality Process are discussed in Angle (2012).

B.2 The Macro Model of the Inequality Process' Stationary Distribution

The stationary distribution of the Inequality process (IP) can be approximated by a gamma pdf. The Macro Model of the Inequality Process (MMIP) is the approximating gamma pdf with shape and scale parameters expressed in terms of a particular value of the particle parameter, ω_ψ , and the harmonic mean of all the ω_ψ's, $\tilde{\omega}_t$.

$$f(x_\psi) \equiv \frac{\lambda_{\psi t}^{\alpha_\psi}}{\Gamma(\alpha_\psi)} x_\psi^{\alpha_\psi - 1} e^{-\lambda_{\psi t} x_\psi}$$
$$x_\psi \equiv \textit{wealth in the } \omega_\psi \textit{ equivalence class in multiples of } \mu_t$$
$$x_\psi > 0$$
$$\alpha_\psi \equiv \textit{shape parameter} \approx \frac{1-\omega_\psi}{\omega_\psi}$$
$$\lambda_{\psi t} \equiv \textit{scale parameter} \approx \frac{1-\omega_\psi}{\tilde{\omega}_t \, \mu_t}$$
$$\tilde{\omega}_t \equiv \textit{harmonic mean of } \omega_\psi\textit{'s}$$

Given the expression for the mean of a random variable in the two parameter gamma pdf, the MMIP's estimator of the mean of particle wealth, x_ψ , in the ω_ψ equivalence class is, $\mu_{\psi t}$, is:

$$\mu_{\psi t} = \frac{\alpha_\psi}{\lambda_{\psi t}} \approx \frac{(\tilde{\omega}_t \, \mu_t)}{\omega_\psi}$$

The unconditional mean of wage income is estimated from the fitted ω_ψ's, and the observed medians of wage income of workers at each level of education via the approximation to the gamma pdf's median (Salem and Mount, 1974):

$$x_{(50)\psi t} \approx \left(\frac{1-(4/3)\omega_\psi}{1-\omega_\psi} \right) \cdot \frac{(\tilde{\omega}_t \, \mu_t)}{\omega_\psi}$$

where,

$$x_{(50)\psi t} = \textit{median labor income of workers with a } \psi^{th} \textit{ level education education in year } t$$
$$\mu_t = \textit{unconditional mean of labor income in year } t$$
$$\omega_\psi = \textit{the proportion of wealth that a particle in the } \omega_\psi \textit{ equivalence class (representing workers with a } \psi^{th} \textit{ level education) gives up in a loss}$$
$$\tilde{\omega}_t = \textit{harmonic mean of } \omega_\psi\textit{'s in whole population (operationalizing level of education of population)}$$

The dynamics of the MMIP in each ω_ψ equivalence class are entirely exogenous. They are driven by the unconditional mean of wealth, μ_t, and the distribution of workers by level of education in the labor force as reflected in the harmonic mean of the ω_ψ's and are expressed solely in terms of the scale parameter, $\lambda_{\psi t}$. The shape of the stationary distribution of particles in the ω_ψ equivalence class does not change but $\lambda_{\psi t}$ does. The MMIP's model of the distribution of wealth is stretched to the right (over larger wealth (x) amounts), or compressed to the left (over smaller wealth amounts) according to whether the product $(\tilde{\omega}_t \ \mu_t)$ increases (stretches distribution to the right) or decreases (compresses distribution to the left.

When the MMIP is fitted to the distribution of annual wage and salary income conditioned on education (using education as the available indicator of worker skill) in the U.S. from 1961 on, the MMIP provides a good fit (Angle, 1993, 1996, 2002, 2006b, 2007a). ω_ψ varies inversely with worker education level as expected under the IP's meta-theory. The dynamics of the U.S. distribution of annual wage and salary income conditioned on education are in the scale of the distribution driven by two exogenous components, the unconditional mean of annual wage and salary income and the education level of the workers, measured by the harmonic mean of the ω_ψ's, $\tilde{\omega}_\psi$. As education levels of workers in the U.S. rose, the estimated fell, as implied by the IP's meta-theory. The two components of the product $(\tilde{\omega}_t \ \mu_t)$ drive the dynamics of the MMIP and the distribution of labor income in opposite directions.

Taking the partial derivative of the MMIP with respect to the driver of its dynamics, $(\tilde{\omega}_t \ \mu_t)$, gives an expression for the dynamics of the MMIP and the distribution of labor income:

$$\begin{aligned} \frac{\partial \ f_{\psi t}(x_0)}{\partial(\tilde{\omega}_t \mu_t)} &= f_{\psi t}(x_0) \ \lambda_{\psi t} \left(\frac{x_0 - \mu_{\psi t}}{\tilde{\omega}_t \mu_t} \right) \\ &= f_{\psi t}(x_0) \ \frac{(1-\omega_\psi)}{(\tilde{\omega}_t \ \mu_t)^2} \ (x_0 - \mu_{\psi t}) \end{aligned}$$

where x_0 is an arbitrary income or wealth amount. Note that the dynamics of the Macro Model are driven by its scale parameter, $\lambda_{\psi t}$. The Macro Model's shape parameter, α_ψ, is treated as constant over time because of the IP's meta-theory. The IP's operationalization of worker skill level is (1-ω_ψ) and wealth produced 1/ ω_ψ. There is no element of change in this proposition. So, the Macro Model's shape parameter, α_ψ, a sole function of ω_ψ, is constant. Other properties of the dynamics of the Macro Model, such as a great surge in the number of very large incomes when $(\tilde{\omega}_t \ \mu_t)$ increases, have been derived, tested, and confirmed with March CPS data (Angle, 2007a).

B.3 The Particle System Model of Kinetic Theory of Gases (KTG) Maps into the Inequality Process (IP)

Where the particles of the KTG have one trait, a positive quantity labeled 'kinetic energy', the Inequality Process' particles have two traits. One is, like kinetic energy in the KTG, a positive quantity, 'wealth' in the IP, that almost surely changes with every encounter with another particle. The other trait that IP particles have is a parameter value that determines how much wealth a particle loses to another particle in a loss.

The transition equations of the Kinetic Theory of Gases (KTG) are the equations for the exchange of kinetic energy between two particles, representing colliding gas molecules:

$$\begin{aligned} x_{it} &= \varepsilon_t \ x_{i(t-1)} + \varepsilon_t \ x_{j(t-1)} \\ x_{jt} &= (1-\varepsilon_t) \ x_{i(t-1)} + (1-\varepsilon_t) \ x_{j(t-1)} \end{aligned}$$

where:

$$\begin{aligned} x_{i(t-1)} &= \textit{particle i's kinetic energy at time-step } (t-1) \\ x_{jt} &= \textit{particle j's kinetic energy at time-step t} \\ \varepsilon_t &= \textit{an i.i.d.} [0,1] \textit{ uniform continuous r.v. at time-step t} \end{aligned}$$

Angle (1990) notes the close relationship between the KTG and the Inequality Process (IP) whose transition equations for the exchange of wealth between two particles, representing two people competing for each other's wealth:

$$x_{it} = x_{i(t-1)} + d_t \, \omega_{\theta j} \, x_{j(t-1)} - (1-d_t)\, \omega_{\psi i} \, x_{i(t-1)}$$
$$x_{jt} = x_{j(t-1)} - d_t \, \omega_{\theta j} \, x_{j(t-1)} + (1-d_t)\, \omega_{\psi i} \, x_{i(t-1)}$$

where:

$$\begin{aligned}
x_{it} &\equiv \textit{particle } i\text{'s wealth at time } t, \\
&\quad \textit{in multiples of } \mu_t \\
\mu_t &\equiv \textit{the unconditional mean at time } t \\
x_{j(t-1)} &\equiv \textit{the wealth of particle } j \textit{ at time } t-1, \\
&\quad \textit{in multiples of } \mu_t \\
0 &< \omega_{\theta j} < 1.0 \text{ fraction lost in loss by particle j} \\
0 &< \omega_{\psi i} < 1.0 \text{ fraction lost in loss by particle i} \\
d_t &= \textit{an i.i.d. } 0{,}1 \textit{ uniform discrete r.v. at time } t
\end{aligned}$$

Three transformations map the KTG's transition equations into the IP's:

1) ε_t -> d_t

i.e. from a continuous [0,1] uniform random variable to a discrete 0,1 uniform random variable;

2) $\omega = 1.0$ -> $0 < \omega < 1.0$

i.e., from 100% of a particle's positive quantity being at risk of loss to ω of it being at risk; and,

3) ω_ψ

i.e., in the KTG all particles have $\omega = 1.0$, but, in the IP, particle ψ has its own value, ω_ψ, $0 < \omega_\psi < 1.0$.

Appendix C: The Teacher Initially Increases Entropy in This Simulation

The Macro Model of the Inequality Process (MMIP) approximates the stationary distribution of the Inequality Process (Angle, 1993, 1996, 2002, 2006b, 2007a). The MMIP is:

$$\begin{aligned}
f(x_\psi) &\equiv \frac{\lambda_{\psi t}^{\alpha_\psi}}{\Gamma(\alpha_\psi)} x_\psi^{\alpha_\psi - 1} e^{-\lambda_{\psi t} x_\psi} \\
x_\psi &\equiv \textit{wealth in the } \omega_\psi \textit{ equivalence} \\
&\quad \textit{class in multiples of } \mu_t \\
x_\psi &> 0 \\
\alpha_\psi &\equiv \textit{shape parameter} \approx \frac{1-\omega_\psi}{\omega_\psi} \\
\lambda_{\psi t} &\equiv \textit{scale parameter} \approx \frac{1-\omega_\psi}{\tilde{\omega}_t \, \mu_t} \\
\tilde{\omega}_t &\equiv \textit{harmonic mean of } \omega_\psi\text{'s}
\end{aligned}$$

This formulation of the MMIP in terms of shape and scale parameters shows that it is a member of the two parameter family of gamma pdfs. The gamma pdf is a maxentropic distribution, the result of maximizing the entropy statistic:

$$H \equiv -\sum_{k=1}^{K} p_k \ln(p_k)$$

where k is the k^{th} relative frequency bin of the distribution of molecular kinetic energy by size and p_k is the relative frequency in the k^{th} bin, subject to the constraints:

$$\sum_{k=1}^{K} p_k = 1.0$$
$$\sum_{k=1}^{K} p_k\, x_k = \text{ mean molecular kinetic energy in population of particles}$$
$$\sum_{k=1}^{K} p_k \ln(x_k) = \text{ mean of } \ln(\text{molecular kinetic energy in population of particles})$$
where x_k = mean kinetic energy in the k^{th} frequency bin defined by an interval of values of kinetic energy.

The MMIP is a two parameter gamma pdf model of the stationary distribution of wealth in the ω_ψ equivalence classes of particles. Its shape parameter, alpha, α_ψ , and scale parameter, λ_ψ , are expressed in terms of ω_ψ and the harmonic mean of the ω_ψ's, $\tilde{\omega}_t$. The H_ψ of the MMIP is:

$$H_\psi = -\int_0^\infty \ln\left(\frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}\right)\left(\frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}\right) dx$$
$$= -\ln\left(\frac{\lambda^\alpha}{\Gamma(\alpha)}\right) - (\alpha-1)\ln(\text{geometric mean of } x) + \alpha$$

Salem and Mount (1974) give the natural logarithm of the geometric mean of a gamma distributed random variable, x, ln(geometric mean of x) = $\psi(\alpha) - \ln(\lambda)$, leaving the H of the MMIP as:

$$H = \ln(\Gamma(\alpha)) - (\alpha-1)\,\psi(\alpha) - \ln(\lambda) + \alpha$$

where $\psi(\alpha) = \frac{d}{d\alpha}\left(\ln(\Gamma(\alpha))\right) = \frac{\Gamma'(\alpha)}{\Gamma(\alpha)}$, the digamma function of α. Note that the ' ψ' of the digamma function is its traditional symbol and unrelated to the other use of ' ψ' in this paper. This paper's simulation of techno-cultural evolution begins with values of ω_ψ and the harmonic mean of the ω_ψ's, $\tilde{\omega}_t$, just below the maximum of 1.0, indicative of intense competition. Further evolution decreases these values. The MMIP's H increases initially via the Teacher's stochastic removal of particles with small wealth and indirectly the stochastic removal of particles with large ω_ψ's indicative of particles less productive of wealth, stochastically decreasing ω_ψ and increasing α. As ω_ψ decreases, α_ψ becomes larger, and the H of particle wealth with ω_ψ increases. Its increase decelerates and eventually turns negative.

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