Insider Trading and Strategies: A Simulation Study

Feng Liu, Ruiwen Zhang

Abstract
Market microstructure studies market friction and asymmetric information impact on security prices. Kyle (1985)'s strategic trade model is one of the most cited models in the field of microstructure. Kyle develops the optimal trading behavior for the informed trader and shows that the agent will trade on his information only gradually, rather than exploit it to the maximum extent possible. Due to the iterative nature of original solutions, it requires numeric approximation to get optimum for a given set of inputs. This become computationally challenging if the inputs are assumed random or number of periods tends to be large. We have yet to see Kyle's simulation or statistical inference using exact solutions on large scale, most papers relate to modified Kyle's price impact coefficient in explaining market behavior or on asset pricing. This paper focus on simulation results using our novel derived unique solution, and its capability to fit large data application. We also show patterns for insider's optimal strategies under equilibrium.

Key Words: Strategic trade model, Kyle model, Microstructure, Asymmetric Information

1. Introduction

Financial markets deviate from the perfect-market ideal in which there are no impediments to trade. Microstructure deals primarily with trading mechanism and market frictions, such as transaction costs as reflected in the bid-ask spread and commissions and asymmetric information. The market for information deals with the supply and demand for information. The informational friction will arise if one investor is better informed than another. The informed trader with positive news will bid up asset price with advantages to the uninformed trader who buys them. Similarly, when selling the stocks, the informed will receive a better pricing than the uninformed. The existence of informed trader redistribute income between the uninformed and the informed and such friction has an impact on asset price.

Kyle (1985)'s strategic trading model is one of the celebrated microstructure models. There are large number of literature interpreting or extending Kyle's model. Brennan and Subrahmanyam (1996) show that required returns are related to Kyle's price coefficient. Holden and Subrahmanyam (1992) study the competition among multiple insiders each endowed with perfect private information. And Foster and Viswanathan (1996) consider the competition with heterogeneous private signals. Huddart, Steven, Hughes and Levine (2001) study the insider's announcement of his trading volume right after submission. Cochrane (2005), Vayanos and Wang (2009) have surveyed on liquidity and asset pricing. Those research often enable us to interpret the economic aspects of certain model parameters proposed in the model. We have yet to see Kyle's simulation or statistical inference using original Kyle's equilibrium solution on large data.

This paper examines Kyle's model and its equilibrium solutions. We provide simulation studies on Kyle's model using our derived method. Our unique solution is computationally efficient and direct. We also show its application on statistical inference which could be a daunting task before. We especially focus on simulation of insider's strategies which sheds light on insider trading for large number of periods N, or N tends to infinity.

The rest of the paper is organized as follows: Section 2 presents Kyle's model and its equilibrium solution. Section 3 discuss the original solution and numeric approximation.

We present our simulation study using our new method in Section 4. Finally, section 5 makes concluding comments.

2. Kyle's equilibrium solution

Kyle proves the existence and uniqueness of a linear equilibrium solution in which the parameters are derived via a set of recursive formulas. One asset that pays off, $v \sim N(p_0, \Sigma_0)$, Σ_0 is value uncertainty. One informed trader and the uninformed investors are placing orders. Trading by the uninformed traders Δu_n is exogenous and normally distributed $N(0, \sigma_u^2 \Delta t_n)$. The informed trader knows the distribution of the uninformed order flow (but not its value) and takes account of his order flow on the market clearing price. The informed trader observes v and submitted order flow Δx_n. The competitive risk-neutral market-maker determines the auction price to reflect the information contained in the aggregated order flow $\Delta y_n = \Delta x_n + \Delta u_n$.

Definition 1 *A sequential auction equilibrium is defined as a pair (X, P) such that the following conditions hold:*

$(C1)$ *(profit maximization)* *For $n = 1, ..., N$ and all $X^{'} = (\Delta x_1^{'}, ..., \Delta x_N^{'})$ with $\Delta x_i^{'} = \Delta x_i$, $i = 1, ..., n-1$, we have*

$$E[\pi_n(X, P)|\mathcal{F}_{n-1}^I] \geq E[\pi_n(X^{'}, P)|\mathcal{F}_{n-1}^I]. \quad (1)$$

$(C2)$ *(market efficiency)* *For $n = 1, ..., N$ we have*

$$p_n = E(v|\mathcal{F}_{n-1}^U, \Delta y_n). \quad (2)$$

Definition 2 *A sequential auction equilibrium (X, P) is called a linear equilibrium if the component functions of X and P are linear, and a recursive linear equilibrium if there exist parameters $\lambda_1, ..., \lambda_N$ such that*

$$p_n = p_{n-1} + \lambda_n \Delta y_n, \quad n = 1, ..., N. \quad (3)$$

Theorem 1 *There exists a unique linear equilibrium (X, P), represented as a recursive linear equilibrium, characterized by (for $n = 1, ..., N$)*

$$\begin{aligned} \Delta x_n &= \beta_n \, (v - p_{n-1}) \, \Delta t_n, & (4) \\ p_n &= p_{n-1} + \lambda_n \Delta y_n, & (5) \\ \Sigma_n &= Var(v|\mathcal{F}_n^U), & (6) \\ E[\pi_n|\mathcal{F}_{n-1}^I] &= \alpha_{n-1} \, (v - p_{n-1})^2 + \delta_{n-1}; & (7) \end{aligned}$$

Given Σ_0 and σ_u^2, the parameters $\beta_n, \lambda_n, \Sigma_n, \alpha_n, \delta_n$ are the unique solutions to equations

$$\begin{aligned} \alpha_{n-1} &= [4\lambda_n(1 - \alpha_n \lambda_n)]^{-1}, & (8) \\ \delta_{n-1} &= \delta_n + \alpha_n \, \lambda_n^2 \, \sigma_u^2 \, \Delta t_n, & (9) \\ \beta_n \, \Delta t_n &= (1 - 2\alpha_n \lambda_n) \, [2\lambda_n(1 - \alpha_n \lambda_n)]^{-1}, & (10) \\ \lambda_n &= \beta_n \, \Sigma_n \, \sigma_u^{-2}, & (11) \\ \Sigma_n &= (1 - \beta_n \lambda_n \, \Delta t_n) \, \Sigma_{n-1}, & (12) \end{aligned}$$

subject to $\alpha_N = \delta_N = 0$ and the second order condition

$$\lambda_n(1 - \alpha_n \lambda_n) > 0. \quad (13)$$

3. Numeric approximation

As is suggested on page 1325 in Kyle (1985), combining (10) and (11) yields

$$(1-\lambda_n^2\sigma_u^2\Delta t_n/\Sigma_n)(1-\alpha_n\lambda_n)=\frac{1}{2}, \tag{14}$$

this is a cubic equation in λ_n given nonnegative values of α_n, Σ_n and σ_u^2. (14) has three real roots. The middle root is the only one that satisfies the second order condition. Overall, the sequences $\{\lambda_n\},\{\beta_n\},\{\alpha_n\},\{\delta_n\}$ and $\{\Sigma_n\}$ can be determined by iterating $n=N,N-1,...,1$ backwards given a pair (Σ_0,σ_u^2) and the boundary condition $\alpha_N=\delta_N=0$. Since Σ_N is unknown, Kyle proves that only one terminal value Σ_N exists such that the correct initial value Σ_0 is obtained at the last step of the backward iteration. To search for Σ_N, one needs to start with an arbitrary initial value and run a search until it converges. A numeric approximation is given below.

Given Σ_0, σ_u^2 and the boundary condition $\alpha_N=\delta_N=0$, an iterative algorithm:

S1: Make an initial guess Σ_N;

S2: Get $\lambda_N=\frac{\sqrt{\Sigma_N}}{\sqrt{2\Delta t_n\sigma_u^2}}$ using $\alpha_N=0$ from (14);

S3: Get β_n and Σ_{n-1} from (11) and (12) for $n=N$;

S4: Get α_{n-1} from (8);

S5: Get λ_{n-1} by solving the cubic equation (14);

S6: Repeat S3 if $n>0$ and replace n by n-1;

S7: Repeat S1 with a different initial value Σ_N if $|\Sigma_0-\Sigma_0|>\epsilon$ where ϵ is a prescribed error bound.

This backward induction search algorithm contains an outside loop and an inside loop: the outside loop, as shown in S1 — S7, determines Σ_N up to an acceptable error, while the inside loop S3 — S6 mainly solves a cubic equation for each n. Solving cubic equation and taking middle root could be numeric approximation also. For a given input (Σ_0,σ_u^2), the computational complexity for a desirable target result is $O(N^2)$ or $O(N^3)$.

This approach might be feasible for simulation given the inputs are known and number of periods N is relatively small. While for large amount of data or empirical study, model inputs have to be treated as random and inferred from the data. Conceivably, the required computational complexity will increase dramatically. This issue exists in all Kyle-type model framework due to the backward induction process.

4. Simulation study using new method

We re-derive the equilibrium solution, and propose a new method. This paper is not focused on theoretical aspect of the algorithm, instead, we present our simulation study using the new method. We could simulate very large number of periods efficiently, for example, $N=10,000$. For computational complexity, we reduce it to O(N) or O(1). The ”trial-and-error” phase with different values for Σ_N is eliminated. We validate the new method with the previous search algorithm, it gives the same result.

We assume $P_0=4.0$, $v=4.5$, $\Sigma_0=1$, and $\sigma_u=0.5$. The following figures show model parameters, order flows and one-instance of price evolution with changing period numbers $N=10,50,100,1000$.

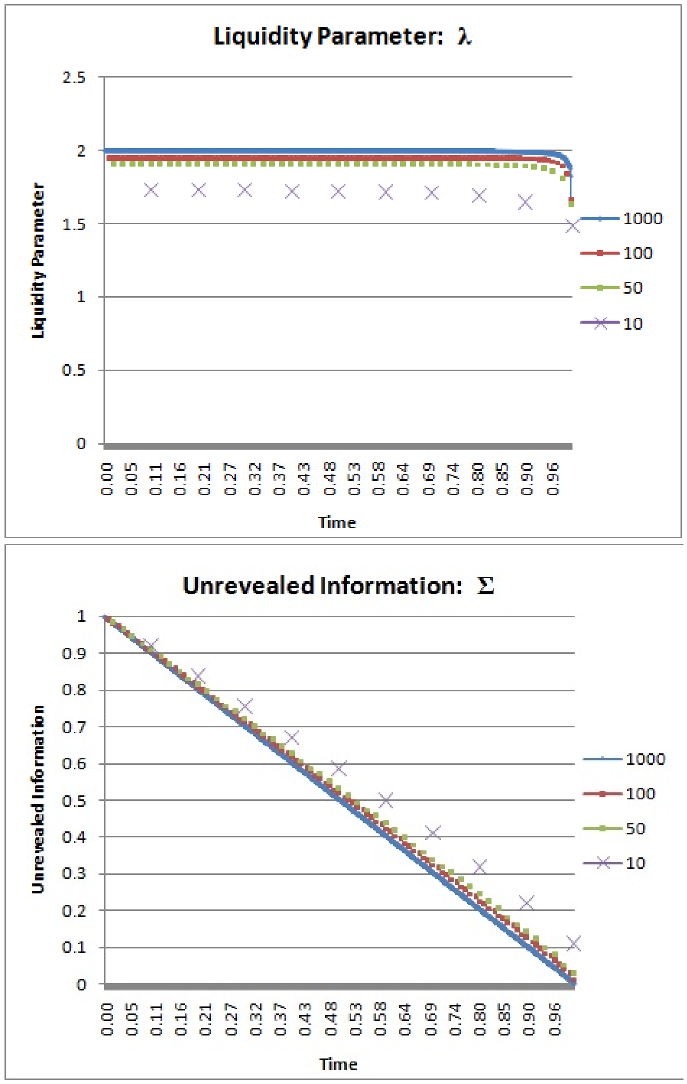

Figure 1: Liquidity Parameter and Unrevealed Information with changing period numbers $N = 10, 50, 100, 1000$

The order flows and price evolution are illuminating. The informed trader is actively disguise his informed orders among the uninformed orders. As trading unfolds, informed orders become more aggressive. The volatility of trade prices is determined by the noise traders and not by the insider. There is a sense in which the "trading volume" of the insider is small. Despite his small trading volume, however, the insider ultimately determines what price is established at the end of trading. He does this because his trades, unlike the trades of noise traders, are positively correlated from period to period. The trade price moves toward the true value while trade price may not converge to the true value at the end of trading. In our case, $p_{100} = 4.57$ is close to true value 4.5.

5. Conclusion

Our study examines the Kyle's equilibrium solution through our new method. The simulation shows it can solve large number of inputs, or large number of periods efficiently. It reduce computational complexity by two order of magnitude. This methodology could be applied to statistical inference on Kyle's model or Kyle-type framework.

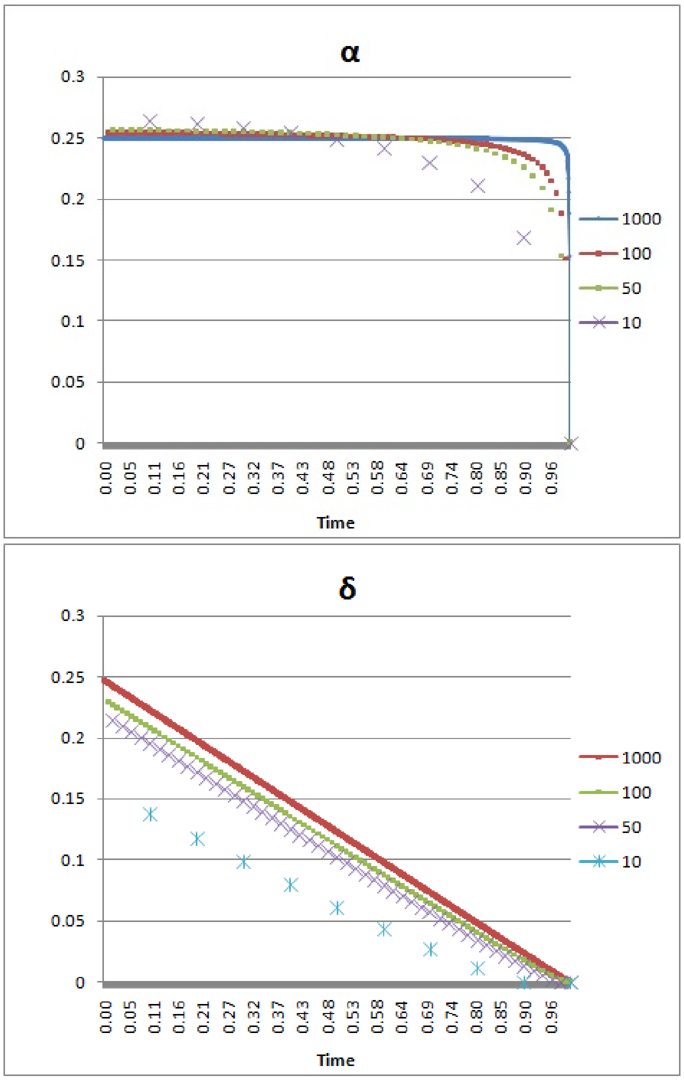

Figure 2: Profit parameters with changing period numbers $N = 10, 50, 100, 1000$

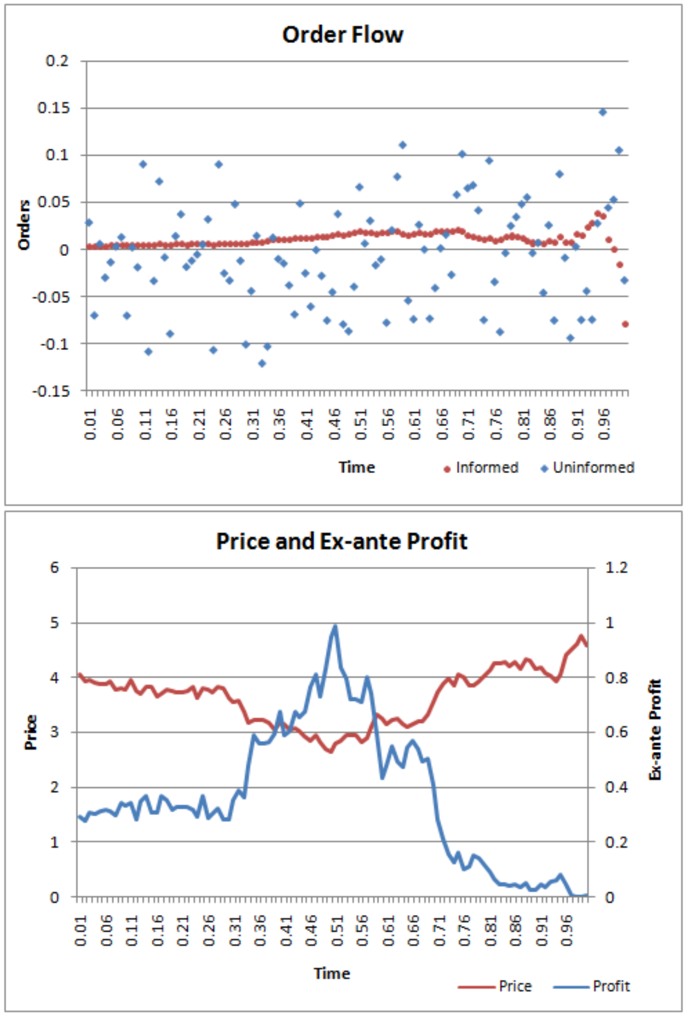

Figure 3: Orders, Price and Ex-ante Profit with $N = 100$

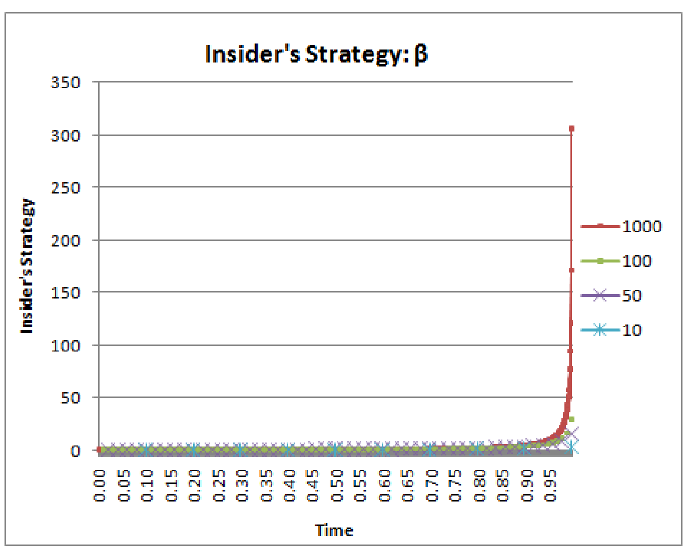

Figure 4: Insider's strategy with changing period numbers $N = 10, 50, 100, 1000$

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