Complementary Properties of an F-Test and a Spectral Diagnostic for Detecting Seasonality in Unadjusted and Seasonally Adjusted Time Series

David F. Findley* Demetra P. Lytras†

Abstract
We consider the comparative strengths of spectral diagnostics and the F^M test statistic of Lytras, Feldpausch and Bell (2007) for detecting significant seasonality in two contexts: (i) Deciding whether a time series meets the minimum requirement to be a candidate for seasonal adjustment, the focus of the comparisons of Lytras et al. (ii) Deciding whether a seasonally adjusted time series still has seasonality that is detectable in its more recent data. Our results show that the best ways to apply the diagnostics differ according to the context. The scope of the results is increased by the analysis of simulation results for stationary seasonal autoregressive models and a theoretical result showing their relevance.

Key Words: Seasonal adjustment, Residual seasonality, Seasonal autoregressive models, Stationary seasonality, X-13ARIMA-SEATS

1. Diagnostics for Detecting Seasonality

Lytras, Feldpausch and Bell (2009), hereafter LFB, presented a new F-statistic, designated F^M, to test for stable seasonality in monthly time series, which has been incorporated into X-13ARIMA-SEATS (U.S. Census Bureau, 2012) and into versions of TRAMO-SEATS (Gómez and Maravall, 1996) and TSW (Caporello and Maravall, 2004) to be released soon. (Personal communication from A. Maravall.) Its formula is given below in Subsection 3. They compared its performance, on simulated series from non-seasonal ARIMA models and from Airline models, to the performance of the autoregressive spectrum diagnostic of X-12-ARIMA, reprised below in Section 2, and to the performance of historic X-11 F-statistics and other X-l1-ARIMA diagnostics inherited by X-12-ARIMA and X-13ARIMA-SEATS (a combination we abbreviate as X-12/13). Having "significant" stable seasonality has historically been a necessary condition for seasonally adjusting a series in software incorporating the X-11 method. (It does not establish that an adjustment of acceptable quality can be found.)

LFB did not consider the detection of residual seasonality in seasonally adjusted series. This is the most fundamental deficiency a seasonal adjustment can have. We investigate the efficacy of F^M, AR spectra, and the periodogram for this task in Section 4. There we analyze seasonal adjustments of 28 U.S. Census Bureau Service Sector Statistics obtained with deliberately incorrect X-11 seasonal filter options. A least one diagnostic indicates residual seasonality in 22 of these series. When a better seasonal adjustment is obtained with X-12/13A-S's automatic moving seasonality ratio seasonal filter selection procedure inherited from X-11-ARIMA (Dagum , 1980), none of these diagnostics indicates residual seasonality. Our analysis shows that a useful conclusion of LFB regarding the length of the data interval used to calculate the diagnostics for detecting seasonality in the unadjusted

*Center for Statistical Methods Research, U.S. Census Bureau, Washington DC, 20233
†Time Series Methods Staff, U.S. Census Bureau, Washington DC, 20233

series is reversed in the new context: for detecting residual seasonality, the short default 8-year span of X-12/13A-S should be used for F^M and for the spectrum diagnostics, not a much longer span.

In Section 5, we examine the performance of the diagnostics with simulated series having positive stationary autocorrelation of varying amounts at seasonal lags $12, 24$, and beyond, but no stable seasonality. A formal definition of seasonality is lacking. The operational heuristic concept is that seasonality consists of movements in the series away from its underlying level or trend that recur with the same direction and a similar magnitude from one year to the next, for several years at least. Thus, substantial positive "correlation" of some kind at seasonal lags $12, 24$ and 36 is expected. The simplest mathematical model is that of perfect correlation/repetition, i.e. fixed or stable seasonality, but this is rarely realistic. Equally simple and more versatile are stationary first-order seasonal autoregressive models with a positive coefficient $0 < \Phi < 1$. For such a model, autocorrelations at seasonal lags are powers of this coefficient, $\rho_{12k} = \Phi^k$ for $k = 1, 2, \ldots$. TRAMO-SEATS and TSW usually use this model for the seasonal component when an automatically identified seasonal ARIMA model does not include a seasonal difference and Φ is not below a specified threshold.

2. Spectral Diagnostics

Here we review the autoregressive spectral density and periodogram diagnostics as implemented in X-12/13 for monthly data, which could be the appropriately transformed original series or seasonally adjusted series.

2.1 The AR(p) Spectral Density

Here we briefly review the spectrum diagnostics of X-13ARIMA-SEATS and its X-12-ARIMA predecessors (hereafter X-12/13) for monthly data x_t. Let $\hat{\phi}(B) = 1 - \sum_{j=1}^{p} \hat{\phi}_j B^j$ denote the estimated $AR(p)$ polynomial of a p-th order autoregressive model $\phi(B)(x_t - \mu) = a_t$ for data $x_t, t = d+1, \ldots, n$ resulting from differencing of order $d \geq 0$. The estimated model's spectral density is

$$\hat{g}(\lambda) = \frac{\hat{\sigma}_a^2}{\left|1 - \sum_{j=1}^{p} \hat{\phi}_j e^{i2\pi j\lambda}\right|^2}, -1/2 \leq \lambda \leq 1/2, \tag{1}$$

where $\hat{\sigma}_a^2 = (n-d-p)^{-1} \sum_{t \geq d+p+1}^{n} \left(x_t - \sum_{j=1}^{p} \hat{\phi}_j x_{t-j}\right)^2$.

Without the assumption being made that x_t is AR(p), AR spectral densities are a standard diagnostic for periodic components, see Priestley (1980). (Note that a perfectly repeating seasonal component satisfies the autoregression $z_t = z_{t-12} + 0$.) For monthly data, for the BAYSEA seasonal adjustment program (Akaike and Ishiguro, 1982), introduced as a diagnostic the decibel-scaled version of $\hat{g}(\lambda)$ with $p = 30$, calculated at 61 discrete frequencies,

$$arspec(\lambda) = 10 \log_{10}(\lambda) \tag{2}$$

calculated at 61 frequencies, $\lambda_k = k/120, k = 0, 1, \ldots, 60$.Thus $\lambda_k, k = 10, 20, \ldots, 50, 60$ are the seasonal frequencies 1/12, 2/12,..., 1/2 cycles per month or 1, 2,..., 6 cycles per year. There are no statistical tests for a significant seasonal peak. Instead, the"visual significance" or v.s. criterion of Soukup and Findley (1999) is used in X-12/13, which was developed empirically to detect trading day effects. With $R = \max_k arspec(\lambda_k) - \min_k arspec(\lambda_k)$, a peak at any of λ_k, $k = 10, 20, 40, 50$ (which we designate by S1, S2,

... , S5) than the median of the *arspec* values and also more than $(5/32)R$ greater than the neighboring values *arspec*(λ_{k-1}) and *arspec*(λ_{k+1}). A v.s. peak at 5 cycles per year (λ_{50}) is sometimes ignored, especially if it is the only v.s. peak, because such peaks often seem to have little connection to seasonality, especially in a seasonally adjusted series. (The frequency could be an "alias" of a non-seasonal frequency.) To detect trading day effects, the equispaced λ_k closest to the main and secondary trading day frequencies, .348 and .432 respectively, are replaced by the latter frequencies.

In X-12/13, to detect seasonality, either in the original series or its log transform (after adjustment for regression effects) first differencing, i.e. $(1-B)$, is usually applied and, in the software's default setting, the AR model is estimated from the last 8 years of data. The same approach is used to residual seasonality in the adjusted series or its log transform. The results of LFB discussed in Section 3, indicates that the longest available data span is usually advantageous for confirming seasonality in the unadjusted data. We present results in Section 4 showing that the 8 year span is better for detecting residual seasonality in a seasonally adjusted series, also for F^M.

2.2 The Periodogram

With

$$I_n(\lambda) = \frac{1}{n-d}\left|\sum_{t=d+1}^{n} x_t e^{-i2\pi t\lambda}\right|^2, \tag{3}$$

the periodogram diagnostic *pdg*(λ) is defined as

$$pdg(\lambda) = 10\log_{10} I_n(\lambda) \tag{4}$$

at the 61 values used for *arspec* $(\lambda.)$

Under assumptions too restrictive for our application, hypothesis test statistics for detecting periodic components in correlated series are available for this diagnostic see Priestley (1980). Therefore, the same v.s. criterion is used by the software, with the 8 year span length as default.

2.3 An Example

Here we look at several diagnostic graphs for series 45291, Sales of Warehouse Clubs and Superstores, after preadjustment for effects estimated by the regression component of a regARIMA model and removed before seasonal factor estimation (the B 1 series in the X-11 nomenclature). This is the series graphed in Figure 1. Figures 2 and 3 show the v.s. seasonal spectral peaks of *arspec* and *pdg* confirming seasonality in this series, after log transformation and first differencing. Figures 4 and 5 show v.s. peaks indicating residual seasonality in the last 8 years of the similarly transformed seasonal adjusted series obtain with the x11 spec command `seasonalma=stable`. This causes each calendar month's seasonal factors to remain constant over these years. There are no such v.s. peaks in these spectra (not shown), when the seasonal adjustment filter is chosen automatically with `seasonalma=msr`.

Figure 6 shows an overlay of the seasonal factors of 45291 by calendar month, centered around their calendar month averages. For about half of the months, these averages are substantially different from the averages of the factors over the last eight years. This reveals why removing fixed seasonality like that produced by seasonalma = stable, whose factors (not shown) are quite close the averages in the Figure, leaves residual seasonality in the later years.

Figure 7 shows that the constant seasonal factors for 45291 obtained by the `seasonalma = stable` setting in the `x11 spec` of X-13A-S are very close to the stable seasonal factors estimated by `variable = seasonal`. This reveals that the F^M test discussed next is effectively testing if the average seasonality in each calendar month is zero. In conjunction with Figure 6, this shows why F^M must be calculated from the last eight years to detect the residual seasonality revealed by *arspec* and *pdg*.

3. The F^M Test Statistic for Stable Seasonality

The F^M statistic of LFB tests the joint significance of the coefficients of the monthly indicators for January, ... , November in contrast with December, so that they sum to 0 over the year. That is, they are the $M_{j,t}$ defined for months $j = 1, \ldots, 11$ as follows. When $t = h + 12\,(m - 1)$, i.e. month h of year m, then

$$M_{j,t} = \begin{cases} 1 & h = j \\ -1 & h = 12 \\ 0 & h \neq j, 12 \end{cases} . \tag{5}$$

Denote by $\hat{\chi}^2 = \hat{\beta}' \left[\widehat{var}\left(\hat{\beta}\right)^{-1}\right] \hat{\beta}$ the estimated chi-square statistic of the maximum likelihood estimates of the regression coefficient of an estimated regARIMA model whose with k regressors, with coefficient vector $\beta = (\beta_1, \ldots, \beta_k)'$, include the eleven $M_{j,t}$. LFB shows via simulations that

$$F^M = \frac{\hat{\chi}^2}{11} \times \frac{n - d - k}{n - d}, \tag{6}$$

approximately follows an $F_{11,n-d-k}$ distribution when there is no stable seasonality.

They conclude from their analyses that F^M is better than other X-11-ARIMA F-statistics and that the significance text with F^M from 20 year span much more reliable that the "visual significance" (v.s.) criterion of the AR(30) spectral density of the last 8 years of the series. They that find v.s. improves substantially with a 20 year span, but even then F^M remains substantially better.

However, our study indicates that when looking for residual seasonality, in X-12/13A-S, the F^M-statistic should be obtained from only the last 8 years of the seasonally adjusted and zero-weighted extreme value adjusted series (X-11 Table E 2) via commands of the form

```
series {file ="series.E2" modelspan = (2000.jan,) ....   }
regression {variables = (seasonal)
automdl {}
```

In our study, the input series for testing for the presence of stable seasonality in the original series was the log transform of the regression-adjusted series (X-11 Table B 1).

4. Detection of Residual Seasonality

To study residual seasonality properties of the diagnostics, deliberately inadequate seasonal adjustments (SA's) of 28 U.S. Service Sector series were obtained by removing the estimates of a stable (unchanging) seasonal pattern using the x11 option `seasonalma=stable`.

As the table below shows,and Subsection 2.3 illustrated, this option left residual seasonality, confirmed by at least two diagnostics, in the last eight years of all but 6 series, based on diagnostics calculated for the last 8 years.(For the full seasonally adjusted series, the p-value of F^M is .98 for two series and 1.0 on the rest, because the X–11 filters remove stable seasonality, exactly for additive adjustments, see Bell (2010) and effectively for multiplicative adjustment as this result shows. spectrum finds residual seasonality in 17, with 2+ peaks in 10.)

For the 28 SA's obtained with the better, automatic option `seasonalma=msr`, none has a "visually significant" seasonal spectral peak or an F^M with a p-value at or below .05. Results from last-10-year spans are also presented to address the question of whether a longer span improves overall diagnostic performance. Overall, it degrades performance.

In the table below ar = *arspec*; *pdg* = Periodogram; F = F^M; (p-value) shows $p > .05$ when ar or *pdg* indicate seasonality; and ? signifies a seasonality indication unconfirmed by another diagnostic.

5. Performance with Stationary Seasonal AR(1) Series

All of the diagnostic considered above are motivated by their properties with data having a perfectly repeating (i.e. periodic) component, whereas most economic time series that are seasonally adjusted monthly have estimated seasonal factors that evolve over time, rather than being periodic. So what are the diagnostics responding to? It is clear that the AR(30) spectrum can have peaks at frequencies with stationary AR data that has strong correlations at lags 2,3,4,5, 6 and 12, but how strong does the correlation need to be. What about F^M? The size studies of LFB only considered series that did not have correlations beyond lag 13. The power simulations studies of LFB only considered series whose seasonality. When, if ever, is it appropriate to seasonally adjust series lacking a stable seasonal component? To what extent do the spectral and F-tests (mis)identify such series? We investigated these issues using simulated series x_t of lengths from 84 to 288 satisfying

$$x_t = \Phi x_{t-12} + e_t, \tag{7}$$

with normal i.i.d. e_t. The lag j autocorrelations of such x_t are $\rho_j = \Phi^k$ for $|j| = 12k, \quad k = 0, 1, \ldots$ and $\rho_j = 0$ otherwise We considered $\Phi = 0.1, _{\ldots}, 0.4, \ldots 0.8, 0.9$. For example,

$$(\rho_{12}, \quad \rho_{24}, \quad \rho_{36}) = \begin{cases} (0.4, \quad 0.16, \quad 0.064), & \Phi = 0.4 \\ (0.9, \quad 0.81, \quad 0.648), & \Phi = 0.9 \end{cases} . \tag{8}$$

Table 1. Residual Seasonality Diagnostics for the Final 8- and 10-year Spans

Stable SA	ar-8yr	*pdg*-8yr	F-8yr	ar-10yr	*pdg*-10yr	F-10yr
44000	s2,s4	s2,s4	.00	s1,s4	s1,s2	.00
44100	s5	s2,s5	(.14)			
44130			(.15)		s4?	(.67)
441x0	s5	s2,s5	(.20)			
44200			(.11)	s1?		(.25)
44300	s2,s4	s2,s4	.00	s2,s4		.03
44312	s2		.05	s1,s3		(.82)
44400	s2	s1	.04	s2	s1	(.59)
44410			(.34)		s1?	(.23)
44500	s4?		(.13)	s4?		(.68)
44510	s1	s1	(.13)	s1?		(.72)
44530					s2?	(.30)
44600	s2	s5	(.31)			(.63)
44611			.00?	s2?		(.55)
44700		s1?	.00			
44800	s1,s3	s1,s2	.00	s1,s3		.00
44811	s2	s1,s2	.00	s1,s2	s1,s2	.02
44812	s1,s3	s1	.00	s1,s3		.03
44820		s2	.02	s4	s2	(.29)
45100			.00?			
45200	s1,s2,s3	s1,s2	.00	s1,s2,s3	s1	(.45)
45210	s1		.00	s2	s1	.04
45291	s1,s2,s5	s2,s5	.00	s1,s2,s3	s2	(.11)
45299	s1	s1	(.18)	s1?		(.46)
45300		s1?	(.13)	s2	s1	(.27)
45400	s2	s1,s2	.00	s1,s2	s1,s2	.01
45410	s1,s2	s1,s2	.00	s1,s2	s1,s2	.01
72200	s1,s2	-	.02	s1,s2	s1	.00
Totals	18/1?	16/2?	15/2?	15/4?	11/2?	9/0?

We present graphs and simulation results for 1000 series from each of $\Phi = 0.4$, 0.9 with series lengths $n = 84$, 288. Figures 8 and 9 show SEATS and X-11 seasonal factors and their seasonal adjustments for a realization with $\Phi = 0.4$ for which the diagnostics indicated seasonality. Figures 10 and 11 do the same for $\Phi = 0.9$. For the graphs, the $n = 288$ realizations are assigned to January, 1990 – December 2013. In the Tables 2 and 3, the second column (1+ peaks) and third column (2+ peaks) indicate the proportion of realizations with *arspec* has one or more, respectively, 2 or more v.s. peaks. The $F^M_{.05}$ column shows the proportion for which the value of F^M is significant at this level. The final column, 2+ peaks; $F^M_{.05}$, indicates the proportion with both this property and two or more v.s. peaks from *arspec*. The term Type I error is correct for $F^M_{.05}$ in the literal sense that none of the series has a fixed seasonal component.

Table 2. Simulated Proportions of v.s. Peaks and Type I Errors of $F^M_{.05}$ **for** $\Phi = 0.4$

n	Model for F^M	1+ peaks	2+ peaks	$F^M_{.05}$	2+ peaks; $F^M_{.05}$
84	Automdl + S	0.529	0.130	0.642	0.116
84	$(1,0,0)_{12}$ + S	0.536	0.128	0.388	0.083
84	$(0,0,1)_{12}$ + S	0.537	0.128	0.384	0.085
84	$(0,0,0)_{12}$ + S	0.537	0.129	0.624	0.116
288	Automdl + S	0.520	0.125	0.382	0.056
288	$(1,0,0)_{12}$ + S	0.519	0.126	0.141	0.021
288	$(0,0,1)_{12}$ + S	0.519	0.126	0.297	0.048
288	$(0,0,0)_{12}$ + S	0.519	0.126	0.661	0.084

(9)

Table 3. Simulated Type I Errors of *arspec* and $F^M_{.05}$ **for** $\Phi = 0.9$

n	Model	1+ peaks	2+ peaks	$F^M_{.05}$	2+ peaks; $F^M_{.05}$
84	Automdl + S	0.982	0.865	1.000	0.865
84	$(1,0,0)_{12}$ + S	0.981	0.860	0.930	0.811
84	$(0,0,1)_{12}$ + S	0.983	0.856	1.000	0.856
84	$(0,0,0)_{12}$ + S	0.984	0.864	1.000	0.864
288	Automdl + S	0.987	0.890	0.999	0.889
288	$(1,0,0)_{12}$ + S	0.987	0.889	0.669	0.599
288	$(0,0,1)_{12}$ + S	0.988	0.886	0.999	0.885
288	$(0,0,0)_{12}$ + S	0.989	0.900	1.000	0.900

With the correct model, $F^M_{.05}$ more often correctly identifies that there is no fixed seasonal component. We are now starting to look at other diagnostics including stability diagnostics to learn what values of Φ tend to produce adjustments that many would find unacceptable.

6. Stationary Seasonal AR Factors from Incorrect ARMA Models.

At its simplest, the ARIMA model-based method implemented in TRAMO-SEATS and X-13ARIMA-SEATS uses and ARIMA model for a series X_t to derive ARIMA or ARMA models for the components of signal plus noise decomposition,

$$X_t = S_t + N_t. \tag{10}$$

With correct differencing operator $\delta(B) = \delta_S(B)\,\delta_N(B)$ and using $\tilde{\vartheta}(B)$, $\tilde{\phi}_N(B)$, and $\tilde{\phi}_S(B)$ to denote the correct total MA and correct signal and noise AR polynomials, with others polynomials being possibly incorrect, the model for the stationarized signal estimator $\delta_S(B)\,\hat{S}_t$ from bi-infinite data is

$$\left\{\vartheta(B)\,\tilde{\phi}_N(B)\right\}\left[\tilde{\phi}_S(B)\,\vartheta(F)\right]\delta_S(B)\,\hat{S}_t = \tag{11}$$

$$\frac{\sigma_b^2}{\sigma_a^2}\left\{\tilde{\vartheta}(B)\,\phi_N(B)\right\}\delta_N(F)\,a_t. \tag{12}$$

If N_t is the seasonal component and S_t is the seasonally adjusted series, then a non-constant $\tilde{\phi}_N(B)$ will be seasonal (and a non-constant $\tilde{\phi}_S(B)$ will be non-seasonal). So, with an incorrect model, if $\tilde{\phi}_N(B) \neq \phi_N(B)$, for example if $\phi_N(B) = 1$, then $\delta_S(B)\,\hat{S}_t = (1-B)^{d+D}\,\hat{S}_t$ can be expected to have autocorrelation at the main seasonal lag 12 that is induced by $\phi_N(B)/\tilde{\phi}_N(B)$. Differences, if there are any, in the seasonal factors of of $\tilde{\vartheta}(B)$ and $\vartheta(B)$ are another source of seasonal lag autocorrelation, through $\tilde{\vartheta}(B)/\vartheta(B)$.

References

[1] Akaike, H. and M. Ishiguro (1980), BAYSEA, *A Bayesian Seasonal Adjustment Program*, Computer Science Monographs No. 13, Tokyo: The Institute for Statistical Mathematics.

[2] Bell, W. R. (1984), "Signal Extraction for Nonstationary Time Series". *Annals of Statistics* 12, 646-664.

[3] Bell, W. R. (2011),"Unit Root Properties of Seasonal Adjustment and Related Filters (revised 8/30/2011)," Research Report RRS2010-08, Center for Statistical Research and Methods, U.S. Census Bureau, available at http://www.census.gov/srd/papers/pdf/rrs2010-08.pdf.

[4] Caporello, G. and A. Maravall (2004) *Program TSW: Revised Manual*, Documentos Ocasionales 0408, Bank of Spain.

[5] Burman, J. P. (1980) "Seasonal Adjustment by Signal Extraction," *Journal of the Royal Statistical Society A*,143, 321–337.

[6] Dagum, E. B. (1980). *The X-11-ARIMA Seasonal Adjustment Method*, Statistics Canada.

[7] Findley, D. F. (2012). "Uncorrelatedness and Other Correlation Options for Differenced Seasonal Decomposition Components of ARIMA Model Decompositions,". Center for Statisical Research and Methodology Research Report Series, Statistics #2012-06, Washington, D.C. U.S. Census Bureau, available at http://www.census.gov/ts/papers/rrs2012-06.pdf

[8] Gómez, V. and A. Maravall (1996), *Programs TRAMO and SEATS : Instructions for the User* (beta version:June 1997), Banco de España, Servicio de Estudios, DT 9628. (Updates and additional documentation at http://www.bde.es/webbde/es/secciones/servicio/software/econom.html.

[9] Hillmer, S. C. and G. C. Tiao (1982), "An ARIMA-Model-Based Approach to Seasonal Adjustment," *Journal of the American Statistical Association*, 77 63–70.

[10] Lytras, D.P., R.M. Feldpausch, W.R. Bell (2007) "Determining Seasonality: A Comparison of Diagnostics from X-12-ARIMA," Proceedings of the Third International Conference on Establishment Surveys (Montreal, June 2007). Available at http://www.census.gov/ts/papers/ices2007dpl.pdf

[11] Priestley, M. (1981), *Spectral Analysis and Time Series*, London: Academic Press.

[12] Soukup, R. J. and Findley, D. F. (1999). On the Spectrum Diagnostics Used by X-12-ARIMA to Indicate the Presence of Trading Day Effects After Modeling or Adjustment. *1999 Proceedings of the American Statistical Association*, Business and Economic Statistics Section, pp. 144-149. Alexandria, VA: American Statistical Association. http://www.census.gov/ts/papers/rr9903s.pdf

[13] U.S. Census Bureau (2012), *X-13-ARIMA-SEATS Reference Manual*, *Ver. 1.0*. Available at http://www.census.gov/ts/x13as/docX13ASHTML.pdf

7. Figures

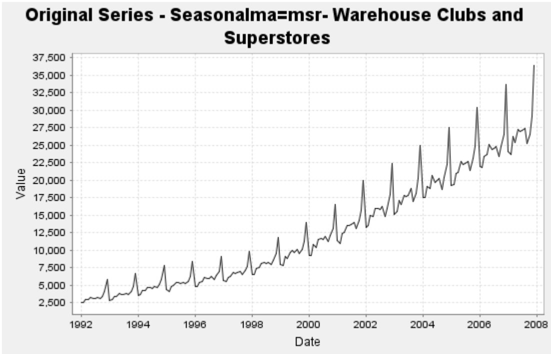

Figure 1: Series 45291 from January, 1992 through December, 2007 is strongly seasonal, with seasonal amplitudes increasing in proportion to the level.

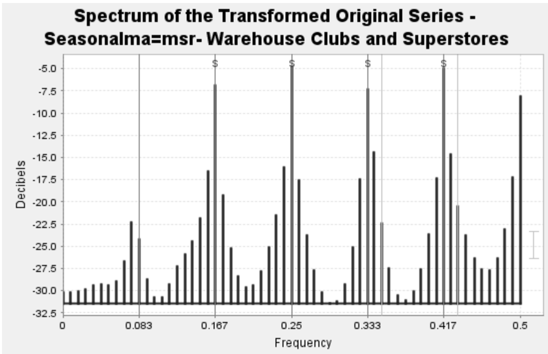

Figure 2: *Arspec* detects v.s. peaks at the seasonal frequencies s2, ... , s5 in the first difference of the logs of the original series.

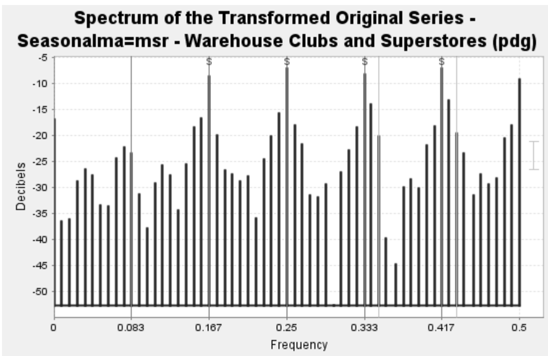

Figure 3: *pdg* also detects v.s. peaks at the seasonal frequencies S2, ... , S5 in the first difference of the logs of the original series.

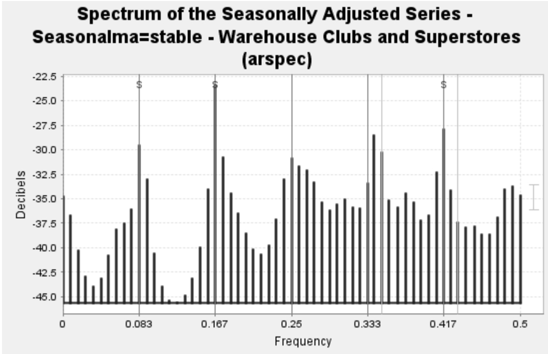

Figure 4: In the adjusted series from stable seasonal factors, *arspec* for the last 8 years detects residual seasonality from v.s. peaks at S1 and S2, and a v.s. trading day peak at the second Trading Day frequency.

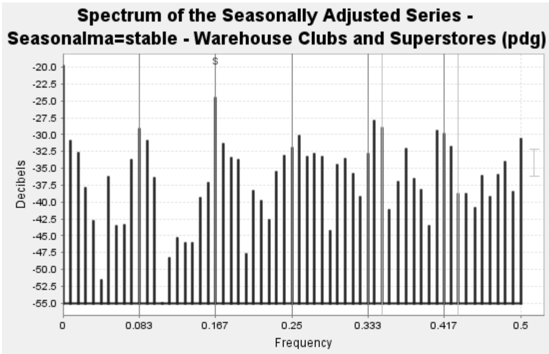

Figure 5: In the adjusted series from stable seasonal factors, *pdg* for the last 8 years detects residual seasonality from a v.s. peak at S2.

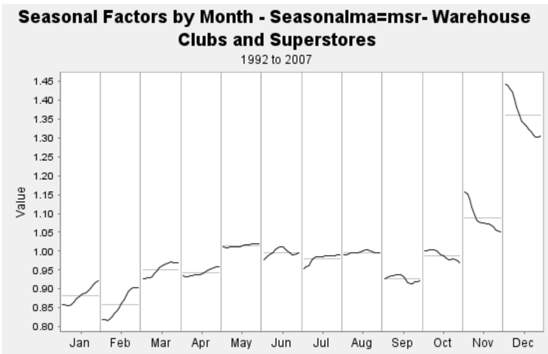

Figure 6: Note the many calendar months for which the seasonal factors of the last eight years are consistently above or consistently below their average over the last sixteen years(horizontal lines). The X-11 stable seasonal adjustment of the full span essentially divides the calendar months by latter averages, and thus leaves residual seasonality in these months.

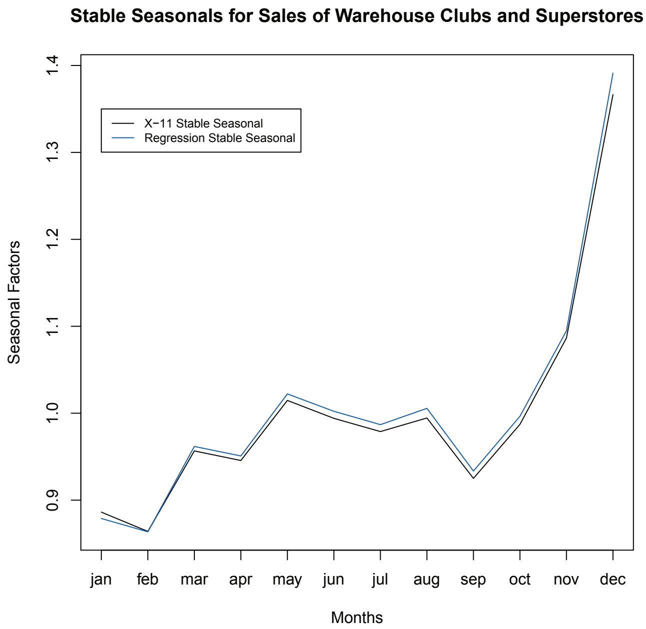

Figure 7: Continuous line graph connecting the constant seasonal factors of 45291for the 12 calendar months from seasonalma=stable (blue) and those implied by the fixed seasonal regressors used for the F^M test. The factors are quite close.

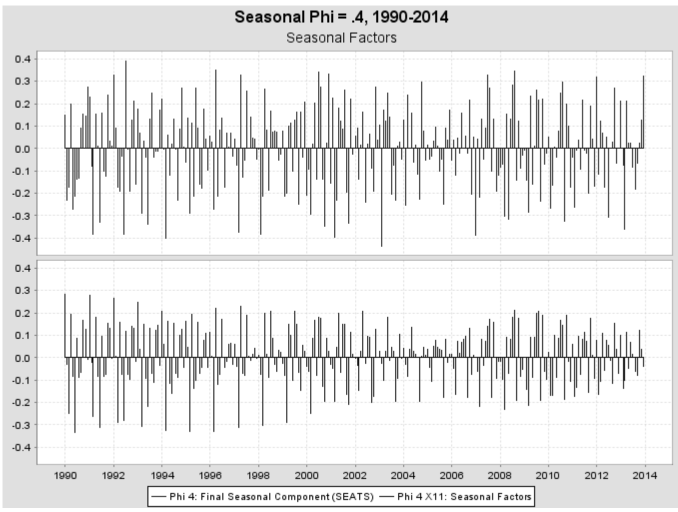

Figure 8: With $\Phi = 0.4$ data, the SEATS seasonal factors are very erratic. For example, the months with the largest and smallest seasonal factors are rarely the same from one year to the next. The more limited set of X-11 seasonal filters results in less erratic factors, but for a given calendar month, they also change direction one or more times over the data span.

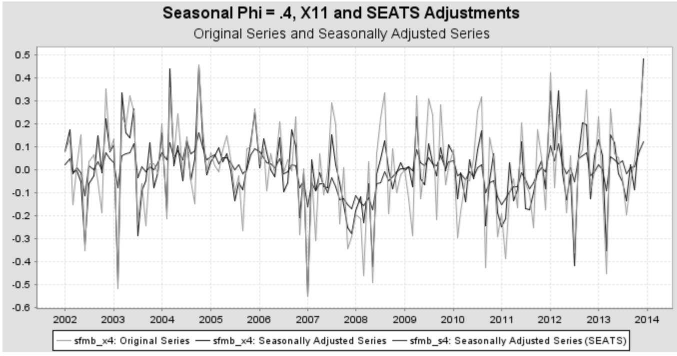

Figure 9: SEATS seasonal adjustment via very erratic factors does much more smoothing than the X-11 filters, but what has it removed from the series?

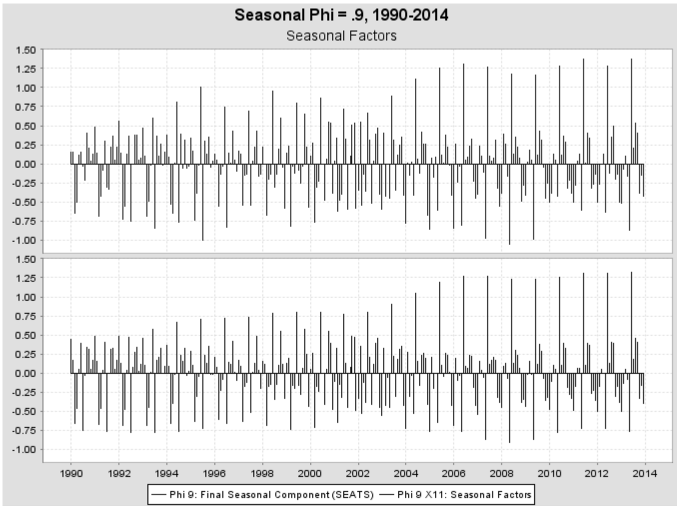

Figure 10: With $\Phi = 0.9$ data, both methods seasonal factors are much more consistent from year to year.

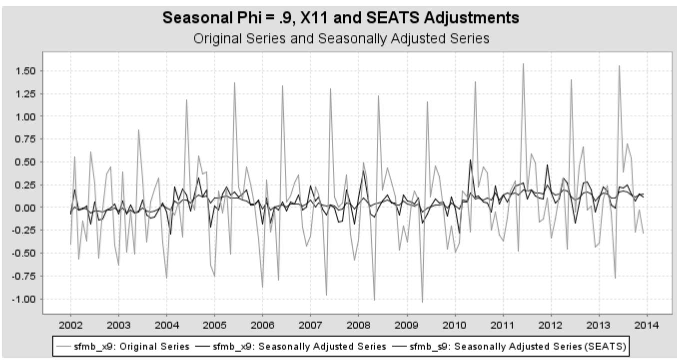

Figure 11: With $\Phi = 0.9$ data, the series looks clearly seasonal, and both seasonal adjustments seem credible. For example, for the X-11 filter adjustment, the Q and M statistics have values less than one. But other diagnostics need to be considered also, a future research project.