Restoring Accounting Constraints in the System of U.S. Industry Accounts

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Abstract

This study illustrates a two-step method for restoring temporal and contemporaneous constraints in the U.S annual input-output (IO) accounts between the Quinquennial benchmarks. Step 1 is a univariate benchmarking process to restore temporal constraints in each component series, and step 2 is a multivariate reconciliation process to restore contemporaneous constraints while maintaining the movements preserved via benchmarking. Two alternative procedures for reconciliation are compared. The two-step method is shown to be simple to implement and computational effective. It allows reconciliation to be conducted independent of the benchmarking prediction errors and allows reconciliation of a large system of accounts to be conducted period-by-period, thus greatly reducing the computational requirement.

Key Words: Benchmarking, Large Accounts Reconciliation, Temporal and Contemporaneous Constraints

1. Introduction

A majority of time series data produced by the system of U.S. national and industry accounts are part of a system of series classified by attributes. This requires that the values of the component elementary series add up to marginal totals for each period. For example, the U.S. input-output (IO) accounts are classified by N industries and M commodities. Each period, the system must satisfy N sets of industry cross-sectional aggregation constraints over commodities in each industry and M sets of commodity cross-sectional aggregation constraints over industries in each commodity. Moreover, for the system of national accounts to be consistent, GDP measured using production data from the IO accounts must be consistent with GDP measured using expenditure data from the national income and product accounts (NIPA). GDP measured via production and expenditure approaches should be consistent with GDP measured via income approach.

Often individual series in the system of accounts must also add up to temporal benchmarks and, thus, must satisfy their respective temporal aggregation constraints. For example, each component series of quarterly GDP estimates interpolated using annual data and quarterly indicators must add up to its annual aggregates. Similarly, the component series of annual IO accounts should be benchmarked to the Quinquennial benchmark IO accounts compiled primarily using Economic Census data or Economic Census related surveys.

Source data used to construct the system of national accounts are obtained from a variety of sources. Thus, inconsistencies often arise in initial data items in different accounts due to differences in the definitions or classifications of some variables and due to various types of measurement errors in the source data. Consequently, initial source data items of NIPA and industry accounts rarely satisfy all accounting constraints. The usual reconciliation procedures use accounting identities from different parts of the system to reduce accounting discrepancies as much as possible and to record the residual between the major aggregates. For example, national accounts estimates include a statistical discrepancy, which measures the difference between GDP compiled using expenditure and income data. In two recent studies, a GLS method was implemented to balance the Quinquennial benchmark IO accounts and to reconcile GDP via production, expenditure and income approaches (Chen; 2010, 2012). The GLS procedure was able to restore all contemporaneous constrains in the system of accounts according to the reliabilities of all initial estimates and to distribute the aggregate statistical discrepancy between final expenditures and industry VA estimates.

Source data become available at different frequencies. Low frequency data contain more complete information and, thus, are more accurate, but they are not timely. High frequency source data are timely but often contain incomplete information. Therefore, they are less accurate and often do not satisfy all temporal aggregation constraints. Since 2007, the modified Denton proportional first difference method has been adopted in NIPA for interpolation of quarterly or monthly GDP component series to restore annual aggregation constraints.

However, to ensure consistency in the system of accounts over time, it requires that temporal and contemporaneous constraints be satisfied simultaneously through benchmarking and reconciliation. In particular, methods for restoring temporal constraints in the IO accounts over multiple years and methods for restoring contemporaneous constrains in the accounts annually are yet to be fully integrated. Currently, compilation of the IO accounts in each Quinquennial benchmark cycle consists of the following sequential steps. *Step 1*, for a Quinquennial benchmark year, benchmark IO accounts are compiled using data primarily from Economic Census and Economic Census related surveys. *Step 2*, for the five years following the Quinquennial benchmark year, annual IO accounts are compiled using both annual survey data when available and data extrapolated from estimates in the previous benchmark IO accounts. Annual IO accounts are balanced and reconciled contemporaneously with the expenditure-based GDP. *Step 3*, five years after the previous Quinquennial benchmark, new benchmark IO accounts are compiled for the new Quinquennial benchmark year. *Step 4*, information from the new benchmark IO accounts is then used in the comprehensive revision of expenditure-based GDP. *Step 5*, upon comprehensive revision of GDP, annual IO accounts between the two Quinquennial benchmark years are revised and reconciled with the benchmark revised GDP. This sequential process repeats for the next benchmark cycle. Traditionally, the revised annual IO accounts were not benchmarked to the benchmark IO accounts. During the most recent benchmark revision, the revised annual IO accounts were benchmarked to the new benchmark IO accounts using a prorating procedure. However, they were not linked to the previous Quinquennial benchmark IO accounts. Thus, there is a need for a statistical method that can fully integrate benchmarking and reconciliation processes to restore all temporal and contemporaneous constraints in the time series of the system if accounts.

The objective of this study is to illustrate a two-step benchmarking-reconciliation method for jointly restoring temporal and contemporaneous constraints in the time series of the system of IO accounts. The two-step benchmarking-reconciliation method was first developed by Quenneville and Rancourt (2005) and an alternative procedure was later proposed by Di Fonzo and Marini (2010). The major difference between the two alternatives lies in the specifications of the weights used in the least squares reconciliation process, representing different adjustment rules in reconciliation, which are discussed in greater details in Section 2.4.

In particular, the two-step method is implemented to restore temporal and contemporaneous constraints in the U.S. IO accounts from 1997 to 2002. 1997 and 2002 are Quinquennial benchmark years. At the outset, a GLS procedure (Chen; 2010, 2012) is applied to reconcile the 1997 and 2002 accounts according to the estimated reliabilities of all initial source data items. The accounts reconciled include the IO accounts, GDP-by-industry accounts, and expenditure-based GDP. The GLS reconciled estimates from the 1997 and 2002 benchmark accounts are then used as the Quinquennial benchmarks for restoring temporal constraints in the system of accounts. Series to be benchmarked and reconciled in this application are from annual IO accounts from 1998 to 2002 prior to the 2002 comprehensive revision. To be consistent in the terms used in this paper, component series to be benchmarked are considered the original estimates of the annual IO accounts, and benchmarked estimates are then considered initial estimates in the reconciliation process. The two-step benchmarking and reconciliation is conducted at the level of detail of 65 industries, 69 commodities, 3 value-added components and 13 final expenditure categories.

The plan for the paper is as follows. Section 2 describes the methods for independent and joint benchmarking and reconciliation. Section 3 discusses the application and presents the results from two alternative two-step benchmarking and reconciliation procedures. Section 4 discusses further research and concludes the paper.

2. Methods for Restoring Accounting Constraints

This section briefly describes methods for independent benchmarking and reconciliation problems. It will also describe methods for jointly restoring both temporal and contemporaneous constraints in the system of accounts. Subsection 2.1 describes a generalized least squares (GLS) method for restoring contemporaneous constraints in the system of accounts in each period, and subsection 2.2 describes the modified Denton's proportional first difference method for restoring temporal aggregation constraints. Subsection 2.3 and 2.4 describes the method for jointly restoring all accounting constraints in the system of the accounts.

2.1 Multivariate Reconciliation to Restore Contemporaneous Constraints

A consistent system of national accounts requires that in each period the underlying elementary components in the accounts must satisfy all cross-sectional aggregation constraints and different sets of accounts in the system must also be reconciled. This subsection describes a GLS approach for balancing and reconciling the system of accounts. Let α_t denote the Nx1 vector of true, non-stochastic, and unknown value of variables in a linear system of national accounts for period t. The system consists of M + 1 accounts and α_t is said to be reconciled when it satisfies the linear accounting system

(1) $H\alpha_t = \beta_t$.

System (1) imposes M (< N) independent linear constraints on the N variables in α_t, for a given MxN matrix H and a given Mx1 vector β_t. Independence of the constraints means that H has full row rank M, the elements of H are either 0 or ±1, and in the overall accounting there is usually one more constraint not included in (1) so as to preserve H's full row rank. The M+1th account is redundant because it follows from adding up the first M accounts.

Let α_t^0 denote an initial, unreconciled, estimate of α_t, produced by a statistical agency, so that error $e_{0t} = H\alpha_t^0 - \beta_t \neq 0$. Following Byron (1996), suppose that α_t^0 is considered a stochastic and unbiased estimate of true α_t, with positive definite covariance matrix Ω_t. The GLS method computes an adjusted and reconciled estimate denoted by α_t^* which is as close as possible to α_t^0. Let $r_t = \alpha_t^* - \alpha_t^0$ denote the adjustment from reconciliation. If $E(\alpha_t^*) = \alpha_t$, i.e., α_t^* is unbiased, $E(\alpha_t^0 - \alpha_t^*)(\alpha_t^0 - \alpha_t^*)' = \Omega_t$, and α_t^* and α_t^0 are independent, then, given α_t^0, β_t, H, and Ω_t, the least square problem minimizes

(2) $$S(\alpha_t^*) = (\alpha_t^* - \alpha_t^0)'\Omega_t^{-1}(\alpha_t^* - \alpha_t^0)$$

with respect to α_t^*, subject to $H\alpha_t^* = \beta_t$. If indeed H has full row rank and Ω_t is positive definite, then, the problem has the unique solution

(3) $$\alpha_t^* = \alpha_t^0 - \Omega_t H'(H\Omega_t H')^{-1}(H\alpha_t^0 - \beta_t).$$

Ideally, survey data would provide enough information to estimate all elements in Ω_t. However, often survey data underlying the initial estimates in the accounts provided only information to estimate variances in Ω_t. In such cases Ω_t is restricted to be diagonal, with positive diagonal elements set to estimated variances of elements in α_t^0, which implies that relative variances of the elements determine the adjustments in the reconciliation process. For a detailed discussion on the construction of Ω_t for national accounts data in the GLS reconciliation, see Chen (2012). If, however, information is not available to estimate variances in Ω_t, it could be specified as a weighting matrix with positive diagonal elements (Dargum and Cholette, 2006; Quenneville and Rancourt, 2005; Quenneville and Fortier, 2009; Di Fonzo and Marini, 2010).

2.2 Univariate Benchmarking to Restore Temporal Constraints

For a system of time series, each component series in the system must satisfy its temporal aggregation constraints. This subsection describes the modified Denton proportional first difference method for benchmarking the series to their temporal aggregates. Let $\alpha_i = (\alpha_{i,1} \dots \alpha_{i,T})'$ for i = 1, …, N be the Tx1 vector of true, non-stochastic and unknown high frequency values of the i^{th} series for t = 1, … T. Let α_i^0 and $\hat{\alpha}_i$ be the corresponding original and benchmarked estimates of α_i. Let $a_i = (a_{i,1} \dots a_{i,L})$ be the Lx1 low frequency temporal aggregates. Let j_i denote the temporal aggregation order for series α_i. Then, for given α_i^0, a_i, and j_i, the modified Denton Proportional 1st Difference (MPFD) benchmarking model is

(4) $$\text{Min } S(\hat{\alpha}_i) = \sum_{t=2}^{T}\left(\frac{\hat{\alpha}_{it}-\alpha_{it}^0}{\alpha_{it}^0} - \frac{\hat{\alpha}_{it-1}-\alpha_{it-1}^0}{\alpha_{it-1}^0}\right)^2$$

(5) $$\text{s.t. } \sum_{t=t_1}^{t_L} j_i\hat{\alpha}_{it} = a_{i\ell},\ \ell=1,\dots,L,\ 1 \le t_1 \le t_L \le T.$$

Let J denote the LxT temporal sum operator matrix, V denote the NTxNT weighting matrix, $\alpha^0 = (\alpha_1^{0\prime} \dots \alpha_N^{0\prime})'$ be the NTx1 vector of original series, and $a = (a_1' \dots a_N')'$ be NLx1 vector of temporal benchmarks. Then, for the system of N series and given α^0, a, J and V, the MPFD benchmarking problem in matrix form is

(6) Min $S(\hat{\alpha}) = (\hat{\alpha} - \alpha^0)' V^{-1} (\hat{\alpha} - \alpha^0)$

(7) s.t. $(I_N \otimes J)\, \hat{\alpha} = a$.

Let $\hat{J}$ denote $(I_N \otimes J)$. Then the known unique solution to the MPFD benchmarking problem is

(8) $\hat{\alpha} = \alpha^0 + V\hat{J}'(\hat{J}V\hat{J}')^{-1}(a - \hat{J}\alpha^0)$,

where $V = (I_N \otimes D^{1\prime}D^1)$, and the (T-1)xT matrix D^1 is the 1st-order differencing operator matrix. For a detailed discussion of the MPFD method, see Dagum and Cholette (2006).

2.3 Simultaneous Benchmarking-Reconciliation Method

For a consistent time series of the system of accounts, both temporal and contemporaneous aggregation constraints must be satisfied. This subsection describes a method that simultaneously restores both temporal and contemporaneous constraints of the system. Let $\beta = (\beta_1', \ldots, \beta_T')'$ be the MTx1 vector of given constants and let Σ denote the NTxNT weighting matrix for t = 1, …, T. Then, for given α^0, a, β, H, J and Σ and ℓ = 1, … L, the joint benchmarking-reconciliation problem is to minimize the weighted sum of squared adjustments from benchmarking and reconciliation such that all temporal and contemporaneous constraints of the system are satisfied simultaneously, i.e.

(9) Min $S(\alpha^*) = (\alpha^* - \alpha^0)'\Sigma^{-1}(\alpha^* - \alpha^0)$

(10) s.t. $\begin{pmatrix} H \otimes I_T \\ I_N \otimes J \end{pmatrix} \alpha^* = \begin{pmatrix} \beta \\ a \end{pmatrix}$.

Different specifications of Σ matrix distinguish different optimization methods. For the MPFD method, $\Sigma = (\alpha^0)^{-1}(I_N \otimes D^{1\prime}D^1)(\alpha^0)^{-1}$, and D^1 is the (T-1)xT 1st-difference matrix.

The first order condition of the benchmarking-reconciliation problem can be written compactly as

(11) $Ax = \theta$,

where coefficient matrix $A = \begin{pmatrix} \Sigma & G' \\ G & 0 \end{pmatrix}$ is (NT+MT+LN)x(NT+MT+LN), $x = \begin{pmatrix} \alpha^* \\ \lambda \end{pmatrix}$ is (NT+MT+LN)x1, λ is the (MT+LN)x1 vector of Lagrange multipliers, and $\theta = \begin{pmatrix} 0 \\ a \end{pmatrix}$.

The computational challenges of using the simultaneous approach are that the dimensions of A matrix can be quite large if the system is large, and the inversion of a large and sparse A matrix could be computationally difficult. See Di Fonzo and Marini (2010) for a detailed discussion of matrix A.

2.4 A Two-Step Approach for Restoring All Accounting Constraints

To reduce the computational burden of restoring all accounting constraints in a large system of accounts, a two-step benchmarking-reconciliation method was first introduced by Quenneville and Rancourt (QR) (2005). Step one is a univariate process to benchmark each component series to its temporal aggregates. Step two is a multivariate process to restore all contemporaneous constraints in the system using a weighted least squares approach (WLS) with a diagonal covariance matrix. The major advantage of the QR two-step approach is its simplicity. The univariate benchmarking process can be conducted using a variety of existing optimization or regression-based methods (Dagum and Cholette, 2006; Quenneville and Fortier, 2010). The MPFD method, as one of the benchmarking alternatives, is widely adopted. SAS and FORTRAN programs for benchmarking are also available from various sources. The QR two-step method allows reconciliation to be independent of benchmarking prediction errors. Thus, reconciliation of a time series of the accounts can be conducted period-by-period, which breaks the problem of reconciling a large system of accounts into smaller ones and greatly reduces the computational requirement.

The authors argued that this sequential approach could preserve the movements in the component series in the reconciliation process without using covariance terms from benchmarking process, because, according to a mathematical result from Hyndman et al (2007), GLS estimates of a linear regression model obtained using the Moor-Penrose inverse can be independent of the covariance matrix and, thus, can be obtained from OLS if, in this case, the discrepancies are small enough. Since temporal discrepancies are zero after benchmarking, the only discrepancies are those related to the contemporaneous constraints in the system. Thus, reconciliation can be independent of benchmarking prediction errors and temporal constraints are preserved during reconciliation.

Formally, the QR's WLS reconciliation procedure, $S^1(\alpha^*; \omega_1)$, is

$$(12) \qquad S^1(\alpha^*; \omega_1) = \sum_{i=1}^{N} \sum_{t=(\ell-1)s+1}^{\ell s} \frac{(\alpha^*_{i,t} - \hat{\alpha}_{i,t})^2}{|\hat{\alpha}_{i,t}|},$$

for ℓ = 1, …, L and s denotes the temporal sum operator. Under the least squares procedure, the weights specified in (12) implies that variance of the i^{th} benchmarked estimate is $\omega_1 = \sigma^2_{i,t} = |\hat{\alpha}_{i,t}|$ and $CV(\hat{\alpha}_{i,t}) = 1/\sqrt{|\hat{\alpha}_{i,t}|}$.

Di Fonzo and Marini (2010) pointed out that weights used in the QR's WLS procedure imply different reliabilities for all variables in the reconciliation, admitting heteroscedasticity in the variances, and reliabilities are determined by the sizes of the variables. Consequently, large variables are considered relatively more reliable and, thus, are adjusted relatively less than small variables. They also pointed out the difficulty of applying More-Penrose inverse when the coefficient matrix A is large and sparse. Instead, they proposed an alternative least squares procedure, $S^2(\alpha^*; \omega_2)$, for reconciliation,

$$(13) \qquad S^2(\alpha^*; \omega_2) = \sum_{i=1}^{N} \sum_{t=(\ell-1)s+1}^{\ell s} \left(\frac{\alpha^*_{i,t} - \hat{\alpha}_{i,t}}{\hat{\alpha}_{i,t}}\right)^2,$$

which implies that variance of the i^{th} element is $\omega_2 = \sigma^2_{i,t} = \hat{\alpha}^2_{i,t}$ and the degree of reliabilities is constant and identical for all variables, i.e., $CV(\hat{\alpha}_{i,t}) = 1$ for all i. Instead of More-Penrose inverse, Gaussian elimination method was used to invert the large and sparse coefficient matrix A. Using this procedure, adjustments from reconciliation are based on the relative variances of the component variables, a concept consistent with the principal underlining the GLS approach.

3. An application of the Two-Step Approach

This section discusses an application of the two-step benchmarking-reconciliation method using 1997 to 2002 data from the U.S. IO accounts, GDP-by-industry accounts, and final expenditures from the NIPA. At the outset, a GLS procedure is used to reconcile the 1997 and 2002 Quinquennial benchmark accounts according to the estimated reliabilities of all initial estimates. The GLS reconciled estimates are then used as the Quinquennial benchmarks in benchmarking each component series in the annual IO accounts from 1998 to 2002. The GLS reconciled estimates reflect the reliabilities of the underlying source data in the 1997 and 2002 benchmark IO accounts. Because information is unavailable on the reliabilities of the source data for the 1998-2002 annual IO accounts, benchmarking the component series to the GLS reconciled benchmark estimates allows information on the changes in the reliabilities between 1997 and 2002 to be incorporated in the benchmarking process.

Data in the 1998-2002 annual IO accounts were previously balanced and reconciled with the expenditure-based GDP, but prior to the 2002 benchmark revision. Thus, they are considered the original estimates in this two-step benchmarking-reconciliation process. Benchmarking and reconciliation are conducted at the level of detail of 65 industries, 69 commodities, 3 VA components and 13 final expenditure categories. At this level of detail, the system of IO accounts consists of a total of 10062 series, 4485 from the make table and 5577 from the use table, which include 4485 intermediate inputs, 195 VA and 897 final expenditures series. Of the 4488 series from the make table, 694 are non-zero series, and of the 5577 series from the use table, the non-zero series include 3551 intermediate inputs, 193 VA and 300 final expenditures.

Both reconciliation procedures discussed in subsection 2.4 are implemented in this application for a comparison. Subsection 3.1 and 3.2 present, respectively, the results from univariate benchmarking and multivariate reconciliation. Subsection 3.3 compares the two alternative reconciliation procedures using simple summary indices.

3.1 Results from Univariate Benchmarking

Six benchmarking methods are evaluated in this study. They are the additive and multiplicative regression-based methods developed by Dagum and Cholette (2006), the modified Denton additive and proportional first difference methods (Denton, 1971; Helfand et al. 1977; Cholette, 1977), the growth preservation method (Causey and Trager, 1981) and the numerical Pro-rating procedure. After comparing the results, the MPFD method is selected for benchmarking in this application, because the results from MPFD are very close to those from the best alternative of the additive regression-based method, and it is simple to implement and has already been adopted in the national accounts for routine interpolation.

Percentage corrections, percentage differences between the original and benchmarked estimates, are used to measure the adjustments from benchmarking, and the adjustments are evaluated at the aggregate and disaggregated industry or commodity levels. Table 1-a display the percentage corrections in each variable in the annual IO accounts at the aggregate level from 1998 to 2002. Total intermediate inputs needed the largest percentage corrections in each year followed by total final uses. This is probably because a significant portion of the source data used to compile intermediate inputs in the annual IO accounts are extrapolated from the previous Quinquennial benchmark estimates. Data used to compile final uses are from a variety of sources, which explains the corrections needed to bring original estimates to the levels determined by the Quinquennial benchmarks. In comparison, percentage corrections in total gross outputs and total VA are much smaller, because source data for gross output in the annual IO accounts are compiled from annual surveys conducted by the Census Bureau, and source data of VA in the GDP-by-industry accounts are largely compiled from regulatory data and from business tax returns from the Internal Revenue Services (IRS). Moreover, Figure 1 shows that larger percentage corrections in total intermediate inputs and total final uses are accompanied by larger sample dispersions.

Table 1-a: Percentage Corrections from Benchmarking at Aggregate, 1998~2002

Year	Gross Output	Value Added	Final Uses	Intermediate Inputs
1998	0.43	0.48	0.97	1.19
1999	0.86	0.91	1.35	2.20
2000	1.18	1.20	1.01	3.11
2001	1.59	1.49	2.89	3.37
2002	1.82	1.78	2.45	3.07
Mean	1.17	1.17	1.73	2.59
Stdv.	0.56	0.50	0.88	0.90

[Figure 1 is here]

Benchmarking results at disaggregated level provide information on the adjustments needed to restore temporal constraints in each industry and expenditure category. For example, Table 1-b shows industry averages and variantions of percentage corrections are in general larger for intermediate inputs and final uses, except that variations in the percentage corrections were larger in VA than in final uses and gross output in 1999 and 2002.

Table 1-b: Mean and Standard Deviation (in parentheses) of Percentage Corrections of 65 Industries and 13 Expenditure Categories from Benchmarking, 1998~2002

Year	Gross Output	Intermediate Inputs	Value Added	Final Uses
1998	0.29	1.39	0.55	4.12
	(2.25)	(5.21)	(3.06)	(13.72)
1999	0.57	2.51	1.07	1.68
	(4.49)	(10.47)	(6.00)	(3.58)
2000	0.50	3.66	1.20	-6.46
	(6.84)	(15.57)	(8.66)	(26.46)
2001	1.10	4.83	1.85	-9.08
	(8.90)	(20.98)	(11.18)	(36.80)
2002	1.12	5.37	2.61	2.07
	(11.07)	(26.21)	(14.47)	(3.30)

3.2 Results from Multivariate Reconciliation

Both reconciliation models, $S^1(\alpha^*; \omega_1)$ by Quenneville and Rancourt (2005) and $S^2(\alpha^*; \omega_2)$ by Di Fonzo and Marini (2010), are used to reconcile the annual IO accounts from 1998 to 2001. Benchmarked estimates are used as initial estimates. Both alternatives use a least squares approach but differ in the specifications of the weights used to determine the adjustments. Each alternative has shown better results in terms of smaller root mean squared adjustments in separate examples. We implemented both procedures to compare the results in this case. Software program of General Algebraic Modeling System (GAMS) with CPLEX solver is used to compute the reconciliation models. Both procedures successfully reconciled the system of the accounts for each sample year, and the results are evaluated and compared at the aggregate and disaggregated industry levels.

Percentage adjustments, the percentage difference between reconciled and benchmarked estimates, are used to evaluate the results from reconciliation. Table 2-a compare the results from the two reconciliation models at the aggregate level. A positive (negative) percentage adjustment indicates that the benchmarked estimates are adjusted upwards (downwards) in order to restore the contemporaneous constraints in the accounts. Table 2-a offers three observations. First of all, percentage adjustments from both reconciliation models are very small, with sample averages of at most .3% for each variable. This is a desirable result, empirically validating the two-step approach for joint benchmarking and reconciliation. Although reconciliation process inevitably adjusts the already benchmarked estimates to restore contemporaneous constraints, the smaller the adjustments from reconciliation the better the period-to-period movements preserved through benchmarking are maintained. Secondly, at the aggregate, both sets of results show relatively larger adjustments in VA and final uses than in gross outputs and intermediate inputs. Thirdly, there is no clear ranking in the sizes of the adjustments from the two models. Figure 2 shows that adjustments from the two models trend to slightly opposite directions in total gross output, but follow similar patterns in the other variables.

Table 2-a: Percentage Adjustments from Reconciliation at Aggregate Level Using Benchmarked Estimates as Preliminary Estimates, 1998~2001

$S^2(\alpha^*; \omega_2 = \hat{\alpha}_{i,t}^2)$

Year	Gross Output	Value-Added	Final Uses	Intermediate Inputs
1998	-0.01	0.26	0.48	-0.02
1999	0.16	0.27	0.38	0.00
2000	0.09	0.00	0.61	-0.21
2001	0.05	0.09	-0.28	-0.94
Avg.	0.07	0.16	0.30	-0.29
Stdv.	0.07	0.13	0.40	0.44

$S^1(\alpha^*; \omega_1 = |\hat{\alpha}_{i,t}|)$

Year	Gross Output	Value-Added	Final Uses	Intermediate Inputs
1998	0.03	0.22	0.44	0.12
1999	0.07	0.25	0.36	-0.18
2000	0.15	0.10	0.71	-0.20
2001	0.24	0.06	-0.31	-0.48
Avg.	0.12	0.16	0.30	-0.19
Stdv.	0.09	0.09	0.43	0.24

[Figure 2 is here]

At the disaggregated level, reconciled estimates allow us to see the effects of reconciliation on initial estimates by industry and by expenditure category. Table 2-b compares the means and dispersions of percentage adjustments by variable based on the two sets of results. The first observation of Table 2-b is that using either model the average percentage adjustments are significantly larger in VA and final uses than in gross output and intermediate inputs. The second observation is that there is no clear ranking in terms of percentage adjustments from the two models. Moreover, because data are generally noisier at disaggregated levels, dispersions in the adjustments by industry and by expenditure category are larger than those at the aggregate level.

Table 2-b: Means and Standard Deviations (in parentheses) of Percentage Adjustments of 65 Industries and 13 Expenditure Categories from Reconciliation, 1998~2001

		$S^1(\alpha^*;\omega_1=\lvert\hat{\alpha}_{i,t}\rvert)$		
Year	Gross Output	Inter-mediate Inputs	Value Added	Final Uses
1998	-0.09	0.22	0.49	-0.61
	(0.96)	(1.94)	(1.68)	(0.97)
1999	0.01	0.02	0.59	-0.79
	(1.29)	(2.05)	(1.97)	(2.44)
2000	0.10	0.01	0.27	7.04
	(1.79)	(2.24)	(2.12)	(24.26)
2001	0.10	-0.34	0.16	-2.37
	(1.29)	(1.75)	(1.82)	(7.92)
		$S^2(\alpha^*;\ \omega_2=\hat{\alpha}^2_{i,t})$		
Year	Gross Output	Intermediate Inputs	Value Added	Final Uses
1998	-0.13	-0.03	0.62	-0.87
	(0.73)	(2.48)	(1.68)	(1.48)
1999	0.16	0.00	0.90	0.11
	(2.62)	(0.00)	(5.07)	(1.76)
2000	0.10	-0.02	0.30	0.04
	(1.89)	(2.58)	(2.12)	(5.92)
2001	-0.04	-0.69	0.20	-4.74
	(1.07)	(2.15)	(1.81)	(13.99)

Figure 3 depicts the dynamics of the mean percentage adjustments of 65 industries and 13 expenditure categories from the two sets of results. While the mean percentage adjustments in gross output, intermediate inputs and VA follow very similar paths based on the two models, the mean percentage adjustments in final expenditures are noticeably different. The larger average percentage adjustments in 2000 based on $S^1(\alpha^*;\omega_1)$ and the large negative adjustments in 2001 based on $S^2(\alpha^*;\omega_2)$ are caused by big adjustments in the initial estimates of inventory change. The larger dispersion in the adjustments at expenditure category level mirrors the larger dispersion at the aggregate.

[Figure 3 is here]

An important aspect of accounts reconciliation is to remove the aggregate statistical discrepancy between GDP and industry VA. The aggregate discrepancy in this application is computed using the benchmarked estimates. With all variables allowed to be adjusted, the aggregate discrepancy is distributed between final expenditures and industry VA estimates. Table 3-a compares the two distributions at the aggregate based on the two sets of results. Column 2 shows the aggregate discrepancy in millions of dollars for each sample year. A negative value in column 2 implies that GDP is less than total industry VA, as were the cases for 1998, 1999 and 2000, whereas a positive value implies that GDP is greater than total industry VA, as were the cases for 1997, 2001 and 2002. Columns 3 and 4 show the distribution between total final uses and total VA based on model $S^2(\alpha^*; \omega_2)$, and columns 5 and 6 show the distribution based on model $S^1(\alpha^*; \omega_1)$. A positive value in columns 3-6 indicates that initial estimates are adjusted upward in order to remove the statistical discrepancy, whereas a negative value signals a downward adjustment. Because final expenditures were considered final and were not adjusted for the 1997 data, the aggregate statistical discrepancy in 1997 was fully distributed to the industry VA in the GDP-by-industry accounts.

Table 3-a: Estimated Aggregate Statistical Discrepancy Distributed to GDP Industry VA, 1998~2002 (In million $)

		$S^2(\alpha^*; \omega_2 = \hat{\alpha}_{i,t}^2)$		$S^1(\alpha^*;\omega_1 = \lvert\hat{\alpha}_{i,t}\rvert)$	
Year	SD	SD(GDP)	SD(VA)	SD(GDP)	SD(VA)
1997	46541.0	0.0	46541.0	0.0	46541.0
1998	-18857.9	41498.8	22640.9	38340.8	19482.9
1999	-10253.8	35262.9	25009.1	33725.6	23471.8
2000	-60028.0	60031.5	3.5	69951.9	9923.9
2001	38783.3	-29224.7	9558.6	-32262.0	6521.4
2002	181298.9	-14838.6	166460.3	-78715.7	102583.2

The first observation of the distribution results is that using either model, a larger portion of the discrepancy is distributed to final expenditures. The second observation is that there is no consistent ranking of the relative shares of the distribution between expenditures and VA from the two models. Moreover, distributions under the two models follow very similar patterns from 1998 to 2001. However, the two distributions become quite diverged in 2002 when the actual estimates of reliabilities were used to reconcile the accounts. Particularly, distribution determined by relative variances rather than by relative absolute values of initial estimates results in much larger share of the aggregate statistical discrepancy being distributed to the industry VA because of the relatively lower reliabilities of initial VA estimates in the GDP-by-industry accounts (Chen, 2010).

Table 3-b: Mean Distribution of Statistical Discrepancy by Expenditure Categories and by Industry VA, 1998-2001 (In million $)

	$S^1(\alpha^*;\omega_1 = \lvert\hat{\alpha}_{i,t}\rvert)$		$S^2(\alpha^*; \omega_2 = \hat{\alpha}_{i,t}^2)$	
Year	SD(VA)	SD(Final Use)	SD(VA)	SD(Final Use)
1998	299.7	2949.3	348.3	3192.2
1999	361.1	2594.3	384.8	2712.5
2000	152.7	5380.9	0.1	4617.8
2001	100.3	-2481.7	147.1	-2248.1

Reconciled estimates also allow the comparison of the two distributions at disaggregated levels. Table 3-b above shows that relative sizes of the two distributions do not seem to be consistently determined by the weighting schemes used in the two models. However, the dispersions are generally smaller in the distribution from model $S^2(\alpha^*; \omega_2)$.

3.3 Assessment of the Two Alternative Models

The two-step joint benchmarking-reconciliation models can be compared statistically using some summary indices of the adjustments made to the original estimates prior to any adjustments. Given that our objective is to restore all accounting constraints in the time series of the accounts, we would like to compare the effects of the two alternative models on the original estimates. Three criteria should help assess the results from the two procedures: 1) the reconciled estimates should result in smaller adjustments to the level of the original series; 2) the reconciled estimates should result in a smaller adjustment to the period-to-period movement of the original series; and 3) highly volatile series should be altered more than less volatile series.

Various simple summary indices have been used in the literature to assess different reconciliation procedures. In this study, we consider two summary indices: root mean-squared percentage adjustments (MSPA) to the level, which is better to assess the models under criterion 1), and the root mean-squared adjustments (MSA) to the percentage growth rates, which is better to assess the models under criterion 2).

MSPA for the i^{th} series and for the entire system are defined as

$$\text{MSPA}_i = 100\sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{\alpha_{i,t}^*-\alpha_{it}^0}{\alpha_{it}^0}\right)^2}, \quad \text{MSPA} = 100\sqrt{\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\frac{\alpha_{i,t}^*-\alpha_{it}^0}{\alpha_{it}^0}\right)^2}.$$

MSPAs for all series in each variable and MSPA for all series in the system of accounts are compared in Table 4. The comparison in the upper panel shows that MSPAs for each variable are very close under the two models, and MSPA for the entire system of series suggests that reconciliation using $S^2(\alpha^*; \omega_2)$ results in slightly smaller adjustments to the level of the original estimates in the system of annual IO accounts.

Table 4: Assessment of the Two Reconciliation Models

Mean Squared Percentage Adjustments to the Level (MSPA)					
Model	MSPA_X	MSPA_Z	MSPA_V	MSPA_Y	$\text{MSPA}(\alpha^*, \alpha^0)$
$S^1(\alpha^*;\omega_1=\lvert\hat{\alpha}_{i,t}\rvert)$	25.07	58.26	0.33	63.01	53.75
$S^2(\alpha^*; \omega_2=\hat{\alpha}_{i,t}^2)$	25.31	55.22	0.33	63.33	51.31
Mean Squared Percentage Adjustments to the Growth Rates (MSA)					
Model	MSA_X	MSA_Z	MSA_V	MSA_Y	$\text{MSA}(\alpha^*, \alpha^0)$
$S^1(\alpha^*;\omega_1=\lvert\hat{\alpha}_{i,t}\rvert)$	30.88	23.13	14.64	25.77	68.89
$S^2(\alpha^*; \omega_2=\hat{\alpha}_{i,t}^2)$	31.77	33.01	15.08	26.79	74.27

Note: In Table 4, X=gross output, Z=intermediate inputs, V=VA, and Y=final uses.

Alternatively, MSA for the i^{th} series and for the entire system are defined as

$$\text{MSA}_i = 100\sqrt{\frac{1}{T-1}\sum_{t=2}^{T}\left(g_{\alpha_{i,t}^*} - g_{\alpha_{it}^0}\right)^2}, \quad \text{MSA} = 100\sqrt{\frac{1}{N(T-1)}\sum_{i}^{N}\sum_{t=2}^{T}\left(g_{\alpha_{i,t}^*} - g_{\alpha_{it}^0}\right)^2},$$

where $g_{\alpha^*_{i,t}} = \frac{\alpha^*_{i,t} - \alpha^*_{it-1}}{\alpha^*_{it-1}}$ and $g_{\alpha^0_{i,t}} = \frac{\alpha^0_{i,t} - \alpha^0_{it-1}}{\alpha^0_{it-1}}$. However, comparison in the lower panel of Table 4 seems to suggest that reconciliation using $S^1(\alpha^*;\ \omega_1)$ results in smaller adjustments to the period-to-period growth rates of the original estimates for each variable and for the entire system of series.

Although the statistical concept underlining $S^2(\alpha^*;\ \omega_2 = \hat{\alpha}^2_{i,t})$ is more consistent with the principal of the GLS estimation, because it considers variances of initial estimates as the proper measures of uncertainties, the computed MSPA and MSA do not both favor $S^2(\alpha^*;\ \omega_2 = \hat{\alpha}^2_{i,t})$ as the preferred procedure for reconciliation. The comparative results from this application suggest further investigation is needed to render conclusive assessment.

4. Conclusion

In this study, we have illustrated a two-step benchmarking-reconciliation approach to restore temporal and contemporaneous accounting constraints in the time series of the system of U.S. industry accounts. In particular, MPFD method was used for benchmarking, and two alternative procedures are implemented for reconciliation in the second step of the two-step process. The results show that both two-step benchmarking-reconciliation procedures can effectively restore all accounting constraints in a large system of accounts, and the two-step approach is computational efficient, as it allows reconciliation to be conducted independent of prediction errors from benchmarking and allows the problem of reconciling a larger system of accounts to be conducted period-by-period, thus greatly reducing the computational requirements.

The reconciliation results and the computed summary indices do not seem to clearly favor one procedure over the other, although $S^2(\alpha^*;\ \omega_2 = \hat{\alpha}^2_{i,t})$ is more consistent with the principal underlying GLS. This might be due to the type of variable being benchmarked in this application being *stock* variable according to the mathematical terminology in benchmarking. It means that estimates in the 2002 Quinquennial benchmark accounts are not the temporal *sums* of benchmarked estimates in the annual IO accounts from 1998 to 2002, but should be equal to the benchmarked estimates in the 2002 annual IO accounts. This is a special case in joint benchmarking and reconciliation, because it may induce additional difficulty in preserving the period-to-period movements in the reconciliation process. However, restoring both temporal and contemporaneous constraints in annual accounts with respect to Quinquennial benchmarks is a commonly encountered application for national accounting systems. Thus, to reach a definitive conclusion on the best method for joint benchmarking and reconciliation in such an application, other methods including the simultaneous method are to be considered.

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Figure 1: Percentage Corrections from Benchmarking at Aggregate Level, 1998~2002

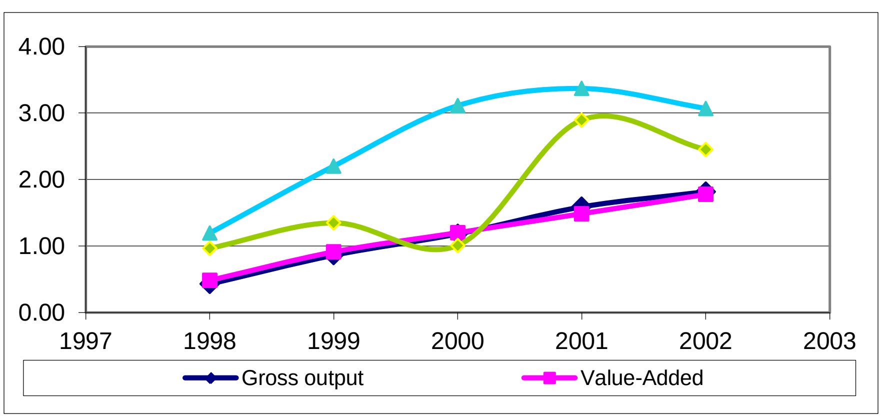

Figure 1: Percentage Adjustment from Reconciliation at Aggregate Level, 1998~2001

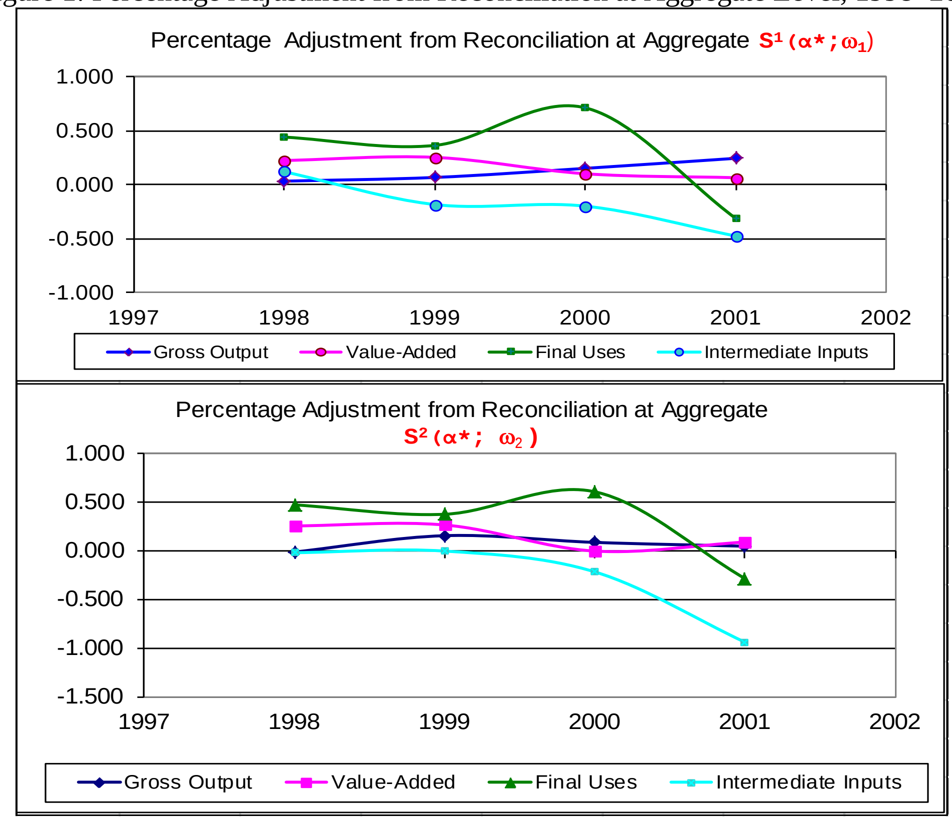

Figure 2: Mean Percentage Adjustments by Industry and Expenditure Category from the Two Reconciliation Models