Effect of the Number and Size of the Initial Samples on the Performance of the S Chart

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Abstract
Most papers on the effect of parameter estimation over control chart performance focus on the marginal distribution of the run length (RL). However, once a process parameter is estimated, the estimation error becomes a constant (albeit unknown), and the RL follows its conditional distribution given the (unknown) value of the error. With this in mind, our focus is rather the conditional distribution of the RL — specifically, of the in-control RL of the S chart, which is geometrically distributed with parameter α (false-alarm probability). We obtain the distribution function of α as a function of the number and size of the initial samples; resulting probability intervals for α (or quantiles of its probability distribution); and, for several sample sizes, the minimum number of initial samples that guarantees (with a specified probability) that the actual value of α will not exceed its nominal value by more than a pre-specified percentage. The calculations are straightforward and carried out in spreadsheet software. We also draw some conclusions in terms of guidance for the user.

Key Words: S control chart; False-alarm probability; Performance; Parameter estimation; Phase I; Number of samples

1. Introduction

Most research works on the effect of parameter estimation over the performance of control charts have studied the marginal distribution of the run length (RL). A comprehensive literature review is found in Jensen et al. (2006). In this paper we focus Shewhart's S chart. The control limits of this chart (whether "3-sigma" or probability limits) are a function of the estimate ($\hat{\sigma}$) of the process standard deviation σ. The run length and the probability of a signal are functions of the control limit(s), so they are functions of $\hat{\sigma}$ and hence random variables themselves.

Previous authors obtained the mean, standard deviation and/or some quantiles of the marginal (unconditioned) in-control RL distribution (see Chen, 1998; Maravelakis *et al*., 2002; Zhang *et al*., 2005), which, of course, has a much larger spread and a different shape than the theoretical RL distribution (the RL distribution in the ideal, σ known, case). Yang and Hillier (1970) had also studied the effect of estimation of the parameters on the S and S^2 charts, proposing exact factors for computing 3-sigma control limits. These are based on the idea of making the unconditioned ARLs equal the nominal ones. Quesenberry's (1991) proposal of *Q* charts follows somewhat alongside the same lines.

However, once any particular *S* chart is constructed, their control limits have a fixed value and the in-control RL follows a geometric distribution with a constant (albeit unknown) parameter α_{TRUE}. Although α_{TRUE} depends on the value of the estimate of σ and is therefore a random variable, for any given chart it assumes a fixed (unknown) value. It is a single realization of the random variable. The RLs that the chart will produce when in use do not follow the marginal RL distribution but rather the conditional distribution of the RL given the unknown value of $\hat{\sigma}$. The actual ARL and SDRL of the chart are not the mean and standard deviation of the marginal RL distribution either.

With this in mind, we are interested in the actual RL distribution. Since it is geometric in shape, it is completely characterized by its parameter, the actual false-alarm probability, α_{TRUE}. As said before, α_{TRUE} is a random variable. We are then interested in its distribution, quantiles and probabilities that it exceeds a given maximum tolerated value.

For space limitations, we will restrict ourselves to the case of the one-sided *S* chart with a probability UCL (that is, intended to give a pre-specified false-alarm probability α). The case of the chart with “3-sigma” limits is analogous and rewriting the expressions for that case is straightforward.

2. The model

For greater generality, let’s define

$$\gamma = \sigma / \sigma_0 \tag{1}$$

where σ_0 is the true (unknown) in-control process standard deviation and σ is the current (also unknown) true process standard deviation. When the process is in control, $\gamma = 1$. If the process dispersion increases, $\gamma > 1$.

Also, let’s define the *error factor of the estimate* as

$$k = \hat{\sigma}_0 / \sigma_0 \tag{2}$$

where $\hat{\sigma}_0$ denotes the estimated value of σ_0.

Given the estimate $\hat{\sigma}_0$, the *UCL* for a specified (or nominal) probability of signal α_{NOM} is established as

$$UCL = \sqrt{\frac{\chi^2_{n-1,\alpha_{NOM}} \hat{\sigma}_0^2}{n-1}} \tag{3}$$

So, the true probability of signal is $P(S > UCL) = P\left(S^2 > \frac{\chi^2_{n-1,\alpha_{NOM}} k^2 \sigma_0^2}{n-1} \right)$ since $\hat{\sigma}_0 = k\sigma_0$. And, because $(n-1)S^2/\sigma^2$ is a χ^2_{n-1} variable, and using the fact that $\frac{\sigma_0^2}{\sigma^2} = \frac{1}{\gamma^2}$, the probability of signal becomes

$$P(S > UCL) = P\left(\chi^2_{n-1} > \frac{k^2}{\gamma^2} \chi^2_{n-1.\alpha_{NOM}} \right). \tag{4}$$

The (true) false-alarm probability is obtained making $\gamma = 1$ in the above expression:

$$\alpha_{TRUE} = P\left(\chi^2_{n-1} > k^2 \chi^2_{n-1.\alpha_{NOM}}\right) \tag{5}$$

and is a function of *k*, the error factor of the estimate, which is the value of a random variable *K*.

Considering that the $\hat{\sigma}_0$ estimator used is $\bar{S}/c_4 = \left(\frac{1}{m} \sum_{j=1}^{m} S_j \right) \Big/ c_4$, obtained from *m* initial (independent) samples with the process in control, and considering that *m* is reasonably large for the Central Limit Theorem to be applicable, then $\bar{S}$ can be considered normally distributed with mean equal to $c_4\sigma_0$ and variance equal to $\sigma_0^2\left(1-c_4^2\right)/m$, where the constant $c_4 = \Gamma\left(\frac{n}{2}\right)\sqrt{\frac{2}{n-1}} \Big/ \Gamma\left(\frac{n-1}{2}\right)$ is a function of the sample size *n*, tabulated in virtually any main text on SPC, SQC, sampling distributions and also in many books on Statistics. As a consequence, $k = \bar{S}/(c_4\sigma_0)$ can be considered normally distributed with mean equal to 1 and variance equal to $\left(1-c_4^2\right)/\left(mc_4^2\right)$. So, denoting by $\Phi(\cdot)$ the standard normal distribution function,

$$F(k) = \Phi\left(\frac{(k-1)mc_4^2}{1-c_4^2} \right). \tag{6}$$

Note that the false-alarm probability is a decreasing function of *k*, that is, denoting $\alpha^* = P\left(\chi^2_{n-1} > (k^*)^2 \chi^2_{n-1.\alpha_{NOM}}\right)$ then

$$P(\alpha > \alpha^*) = P(k < k^*) = F(k^*) = \Phi\left(\frac{(k^*-1)mc_4^2}{1-c_4^2} \right) \tag{7}$$

where

$$k^* = \sqrt{\chi^2_{n-1.\alpha^*} \big/ \chi^2_{n-1.\alpha_{NOM}}}\ . \tag{8}$$

This enables tabulating (and also plotting) the exceedance probability $\left[1 - F_{\alpha^*}(\alpha^*)\right]$ versus α^*, parameterized by the sample size *n* and the number of initial samples, *m*.

It is possible, as well, to obtain prediction (probability) intervals for α_{TRUE}; in other words, percentiles of the probability distribution of the true false-alarm risk. The value of α_{TRUE} that is exceeded with a specified probability *p* (that is, the $(1-p)$-quantile of α_{TRUE}, which we will denote by α_p), is given by

$$\alpha_p = 1 - F_{\chi^2_{n-1}}\left(k_p^2 \chi^2_{n-1,\alpha_{NOM}}\right) \tag{9}$$

where, in turn,

$$k_p = 1 + \left(\Phi^{-1}(p)\right)\sqrt{\frac{1-c_4^2}{mc_4^2}} \,. \tag{10}$$

Finally, given the sample size, *n*, and the desired (or nominal) value α_{NOM}, a relevant question for the practitioner is the minimum number *m* of initial samples that guarantees with a specified probability $1-p$ (say, 0.9) that α_{TRUE} does not exceed α_{NOM} by more than a given percentage ε (say, 20%).

Specifically, given the values of *n*, α_{NOM}, ε and of the acceptable risk *p* that $\alpha_{TRUE} > \left(1+\frac{\varepsilon}{100}\right)\alpha_{NOM}$, the minimum number of in-control samples required for the estimation of σ is:

$$m = \frac{1-c_4^2}{c_4^2}\left(\frac{\Phi^{-1}(p)}{k_p - 1}\right)^2 \tag{11}$$

(which depends on *n* via c_4) and where

$$k_p = \sqrt{\chi^2_{n-1.\alpha_{TOL}} \Big/ \chi^2_{n-1.\alpha_{NOM}}} \,, \tag{12}$$

with $\alpha_{TOL} = \left(1+\frac{\varepsilon}{100}\right)\alpha_{NOM}$.

3. Results

For space limitation, we will present the results for α_{NOM} = 0.005 (nominal ARL_0 = 200). Recall we are considering only one-sided charts with a probability-based *UCL*. Results for other values of α_{NOM} and also for *S* charts with a 3-sigma *UCL* (and also for other values of some of the problem parameters) are available from the authors. Anyway, it is easy to perform the calculations in a spreadsheet, using the formulae here presented, for any case of interest.

The 0.90 and 0.95 quantiles of the distribution of α_{TRUE} are given in Tables 1 and 2 and corresponding Figures 1 and 2. The numbers of initial samples (assumed independent and with the process in control) that guarantee with probability $(1-p)$ of 0.95, 0.90 and 0.85 that α_{TRUE} will not exceed α_{NOM} by more than $\varepsilon = 10$ and 20% (that is, that α_{TRUE} will not exceed 0.0055 and 0.0060 — which correspond to ARL_0 values of 182 and 167, respectively) are given in Table 3. All 6 combinations of these values of p and ε are considered. Figures 3, 4 and 5 exhibit the $m \times n$ curves for these 3 values of p, and for some other values of ε (up to 50%).

The results in tables 1 to 3 (and corresponding figures) show that the numbers of initial samples that guarantee the desired "precision" of the control limits are much larger than the 25 to 50 samples traditionally recommended in many manuals and textbooks. This is not novel at all, and coincides with the findings of previous researchers (see the references in this paper). Nevertheless, the figures we obtained are also different from — as a matter of fact, even higher — than the ones obtained by those previous authors. The discrepancy is explained by the difference of criteria adopted. Those authors were considering the marginal distribution of the RL — calculating, for example, the unconditioned ARL_0 (the expected value of the marginal distribution of the RL). We believe it more useful (or interesting) to the end-user to limit the risk that the true ARL_0 be smaller than a minimum specified limit (or that the true false-alarm risk is greater than a maximum specified limit, which is the same thing).

Of course one cannot wait many hundreds or even thousands or samples are taken before starting the monitoring. The practical conclusion to be drawn from these results is that there must be an intermediate phase between Phase I and Phase II of SPC, in which the process is being monitored but the limits should be recalculated periodically.

Table 1 : 0.95-quantiles of α_{TRUE} as a funcion of *m* and *n*, for $\alpha_{NOM} = 0.005$

	a, P (α > a) = 0,05 e $\alpha_{nominal}$ = 0,005													
n	***m = 10***	***15***	***20***	***25***	***30***	***40***	***50***	***60***	***70***	***80***	***90***	***100***	***150***	***200***
2	0,0884	0,0566	0,0427	0,0349	0,0300	0,0241	0,0207	0,0184	0,0168	0,0156	0,0147	0,0140	0,0117	0,0105
3	0,0603	0,0405	0,0315	0,0264	0,0232	0,0191	0,0168	0,0152	0,0140	0,0132	0,0125	0,0120	0,0102	0,0093
4	0,0499	0,0343	0,0272	0,0231	0,0204	0,0171	0,0152	0,0138	0,0129	0,0122	0,0116	0,0111	0,0096	0,0089
5	0,0443	0,0310	0,0249	0,0213	0,0189	0,0160	0,0143	0,0131	0,0122	0,0116	0,0110	0,0106	0,0093	0,0086
6	0,0408	0,0289	0,0233	0,0201	0,0179	0,0153	0,0137	0,0126	0,0118	0,0112	0,0107	0,0103	0,0091	0,0084
7	0,0384	0,0274	0,0223	0,0193	0,0173	0,0148	0,0132	0,0122	0,0115	0,0109	0,0104	0,0101	0,0089	0,0082
8	0,0366	0,0263	0,0215	0,0186	0,0167	0,0144	0,0129	0,0119	0,0112	0,0107	0,0102	0,0099	0,0088	0,0081
9	0,0352	0,0255	0,0209	0,0181	0,0163	0,0141	0,0127	0,0117	0,0110	0,0105	0,0101	0,0097	0,0086	0,0081
10	0,0341	0,0248	0,0204	0,0177	0,0160	0,0138	0,0125	0,0116	0,0109	0,0104	0,0100	0,0096	0,0086	0,0080
11	0,0332	0,0242	0,0199	0,0174	0,0157	0,0136	0,0123	0,0114	0,0108	0,0103	0,0099	0,0095	0,0085	0,0079
12	0,0324	0,0237	0,0196	0,0171	0,0155	0,0134	0,0121	0,0113	0,0106	0,0102	0,0098	0,0094	0,0084	0,0079
13	0,0317	0,0233	0,0193	0,0169	0,0153	0,0133	0,0120	0,0112	0,0106	0,0101	0,0097	0,0094	0,0084	0,0078
14	0,0312	0,0230	0,0190	0,0167	0,0151	0,0131	0,0119	0,0111	0,0105	0,0100	0,0096	0,0093	0,0083	0,0078
15	0,0307	0,0226	0,0188	0,0165	0,0150	0,0130	0,0118	0,0110	0,0104	0,0099	0,0096	0,0093	0,0083	0,0078
16	0,0302	0,0224	0,0186	0,0163	0,0148	0,0129	0,0117	0,0109	0,0103	0,0099	0,0095	0,0092	0,0083	0,0077
17	0,0298	0,0221	0,0184	0,0162	0,0147	0,0128	0,0117	0,0109	0,0103	0,0098	0,0095	0,0092	0,0082	0,0077
18	0,0295	0,0219	0,0182	0,0160	0,0146	0,0127	0,0116	0,0108	0,0102	0,0098	0,0094	0,0091	0,0082	0,0077
19	0,0291	0,0217	0,0181	0,0159	0,0145	0,0127	0,0115	0,0107	0,0102	0,0097	0,0094	0,0091	0,0082	0,0077
20	0,0288	0,0215	0,0179	0,0158	0,0144	0,0126	0,0115	0,0107	0,0101	0,0097	0,0093	0,0091	0,0081	0,0076
21	0,0286	0,0213	0,0178	0,0157	0,0143	0,0125	0,0114	0,0106	0,0101	0,0097	0,0093	0,0090	0,0081	0,0076
22	0,0283	0,0212	0,0177	0,0156	0,0142	0,0125	0,0114	0,0106	0,0101	0,0096	0,0093	0,0090	0,0081	0,0076
23	0,0281	0,0210	0,0176	0,0155	0,0142	0,0124	0,0113	0,0106	0,0100	0,0096	0,0093	0,0090	0,0081	0,0076
24	0,0279	0,0209	0,0175	0,0155	0,0141	0,0123	0,0113	0,0105	0,0100	0,0096	0,0092	0,0089	0,0081	0,0076
25	0,0277	0,0208	0,0174	0,0154	0,0140	0,0123	0,0112	0,0105	0,0100	0,0095	0,0092	0,0089	0,0080	0,0076
26	0,0275	0,0207	0,0173	0,0153	0,0140	0,0123	0,0112	0,0105	0,0099	0,0095	0,0092	0,0089	0,0080	0,0075
27	0,0273	0,0205	0,0172	0,0152	0,0139	0,0122	0,0112	0,0104	0,0099	0,0095	0,0092	0,0089	0,0080	0,0075
28	0,0272	0,0204	0,0172	0,0152	0,0139	0,0122	0,0111	0,0104	0,0099	0,0095	0,0091	0,0089	0,0080	0,0075
29	0,0270	0,0203	0,0171	0,0151	0,0138	0,0121	0,0111	0,0104	0,0099	0,0094	0,0091	0,0088	0,0080	0,0075
30	0,0269	0,0203	0,0170	0,0151	0,0138	0,0121	0,0111	0,0104	0,0098	0,0094	0,0091	0,0088	0,0080	0,0075

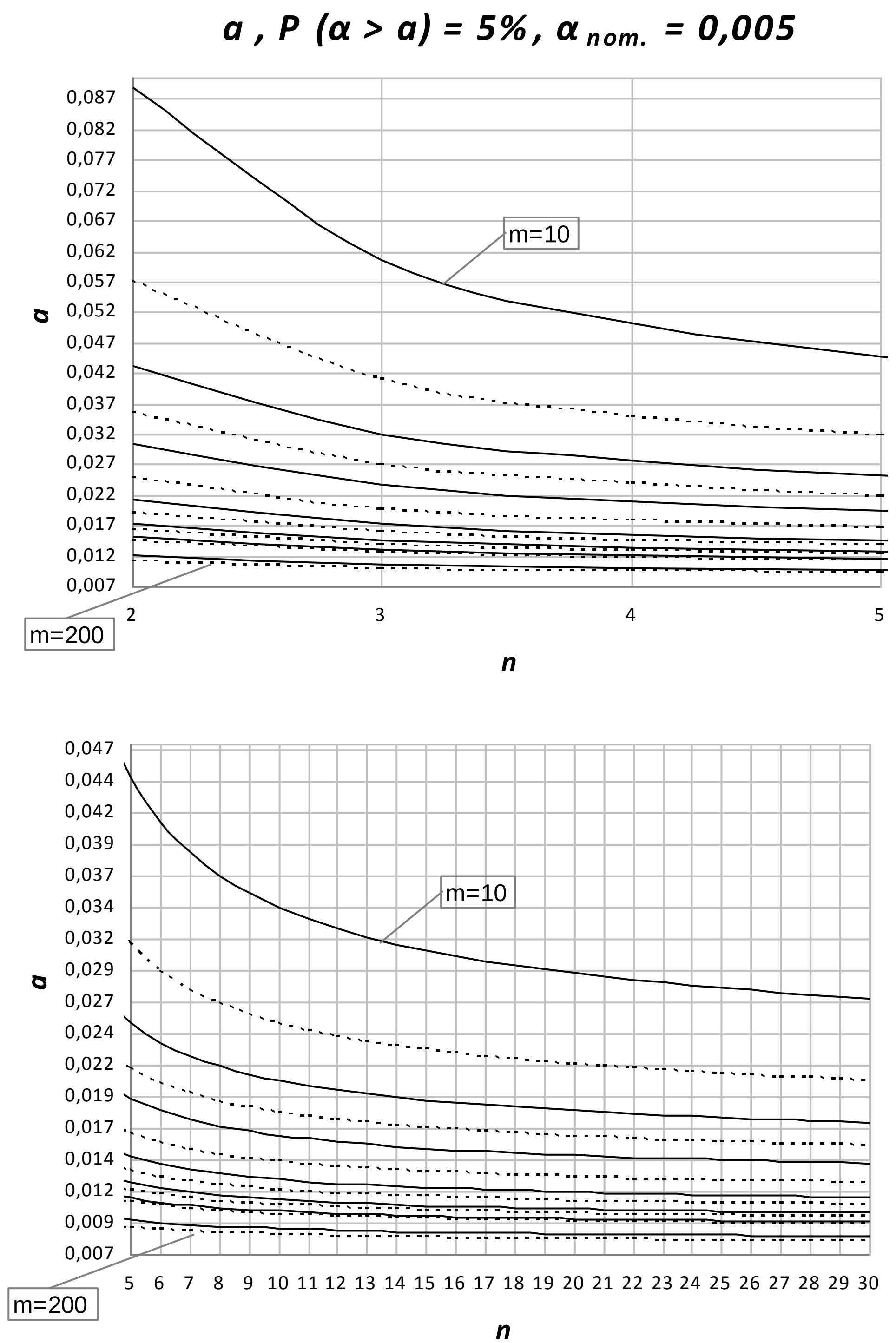

Figure 1 : 0.95-quantiles of α_{TRUE} as a funcion of *m* and *n*, for $\alpha_{NOM} = 0.005$ (corresponds to the data in Table 1)

Table 2 : 0.90-quantiles of α_{TRUE} as a funcion of *m* and *n*, for $\alpha_{NOM} = 0.005$

	a, P (α > a) = 0,1 e $\alpha_{nominal}$ ***= 0,005***													
n	***m = 10***	***15***	***20***	***25***	***30***	***40***	***50***	***60***	***70***	***80***	***90***	***100***	***150***	***200***
2	0,0515	0,0353	0,0279	0,0236	0,0208	0,0174	0,0154	0,0140	0,0131	0,0123	0,0117	0,0112	0,0097	0,0089
3	0,0372	0,0267	0,0217	0,0188	0,0169	0,0145	0,0130	0,0120	0,0113	0,0107	0,0103	0,0099	0,0088	0,0082
4	0,0317	0,0233	0,0193	0,0169	0,0153	0,0132	0,0120	0,0112	0,0105	0,0101	0,0097	0,0094	0,0084	0,0078
5	0,0288	0,0214	0,0179	0,0158	0,0143	0,0125	0,0114	0,0107	0,0101	0,0097	0,0093	0,0090	0,0081	0,0076
6	0,0269	0,0202	0,0170	0,0151	0,0137	0,0121	0,0110	0,0103	0,0098	0,0094	0,0091	0,0088	0,0080	0,0075
7	0,0256	0,0194	0,0164	0,0145	0,0133	0,0117	0,0108	0,0101	0,0096	0,0092	0,0089	0,0087	0,0078	0,0074
8	0,0246	0,0188	0,0159	0,0142	0,0130	0,0115	0,0106	0,0099	0,0094	0,0091	0,0088	0,0085	0,0078	0,0073
9	0,0238	0,0183	0,0155	0,0139	0,0127	0,0113	0,0104	0,0098	0,0093	0,0090	0,0087	0,0084	0,0077	0,0073
10	0,0232	0,0179	0,0152	0,0136	0,0125	0,0111	0,0103	0,0097	0,0092	0,0089	0,0086	0,0084	0,0076	0,0072
11	0,0227	0,0175	0,0150	0,0134	0,0124	0,0110	0,0102	0,0096	0,0091	0,0088	0,0085	0,0083	0,0076	0,0072
12	0,0222	0,0172	0,0147	0,0132	0,0122	0,0109	0,0101	0,0095	0,0091	0,0087	0,0085	0,0082	0,0075	0,0071
13	0,0219	0,0170	0,0146	0,0131	0,0121	0,0108	0,0100	0,0094	0,0090	0,0087	0,0084	0,0082	0,0075	0,0071
14	0,0215	0,0168	0,0144	0,0130	0,0120	0,0107	0,0099	0,0093	0,0089	0,0086	0,0084	0,0081	0,0075	0,0071
15	0,0213	0,0166	0,0143	0,0128	0,0119	0,0106	0,0098	0,0093	0,0089	0,0086	0,0083	0,0081	0,0074	0,0071
16	0,0210	0,0164	0,0141	0,0127	0,0118	0,0106	0,0098	0,0092	0,0088	0,0085	0,0083	0,0081	0,0074	0,0070
17	0,0208	0,0163	0,0140	0,0126	0,0117	0,0105	0,0097	0,0092	0,0088	0,0085	0,0083	0,0080	0,0074	0,0070
18	0,0206	0,0161	0,0139	0,0126	0,0116	0,0104	0,0097	0,0092	0,0088	0,0085	0,0082	0,0080	0,0074	0,0070
19	0,0204	0,0160	0,0138	0,0125	0,0116	0,0104	0,0096	0,0091	0,0087	0,0084	0,0082	0,0080	0,0073	0,0070
20	0,0202	0,0159	0,0137	0,0124	0,0115	0,0103	0,0096	0,0091	0,0087	0,0084	0,0082	0,0080	0,0073	0,0070
21	0,0201	0,0158	0,0137	0,0124	0,0115	0,0103	0,0096	0,0091	0,0087	0,0084	0,0081	0,0079	0,0073	0,0070
22	0,0199	0,0157	0,0136	0,0123	0,0114	0,0103	0,0095	0,0090	0,0087	0,0084	0,0081	0,0079	0,0073	0,0069
23	0,0198	0,0156	0,0135	0,0122	0,0114	0,0102	0,0095	0,0090	0,0086	0,0083	0,0081	0,0079	0,0073	0,0069
24	0,0197	0,0156	0,0135	0,0122	0,0113	0,0102	0,0095	0,0090	0,0086	0,0083	0,0081	0,0079	0,0073	0,0069
25	0,0196	0,0155	0,0134	0,0121	0,0113	0,0102	0,0095	0,0090	0,0086	0,0083	0,0081	0,0079	0,0073	0,0069
26	0,0195	0,0154	0,0134	0,0121	0,0112	0,0101	0,0094	0,0089	0,0086	0,0083	0,0081	0,0079	0,0072	0,0069
27	0,0194	0,0153	0,0133	0,0121	0,0112	0,0101	0,0094	0,0089	0,0086	0,0083	0,0080	0,0078	0,0072	0,0069
28	0,0193	0,0153	0,0133	0,0120	0,0112	0,0101	0,0094	0,0089	0,0085	0,0083	0,0080	0,0078	0,0072	0,0069
29	0,0192	0,0152	0,0132	0,0120	0,0111	0,0101	0,0094	0,0089	0,0085	0,0082	0,0080	0,0078	0,0072	0,0069
30	0,0191	0,0152	0,0132	0,0120	0,0111	0,0100	0,0093	0,0089	0,0085	0,0082	0,0080	0,0078	0,0072	0,0069

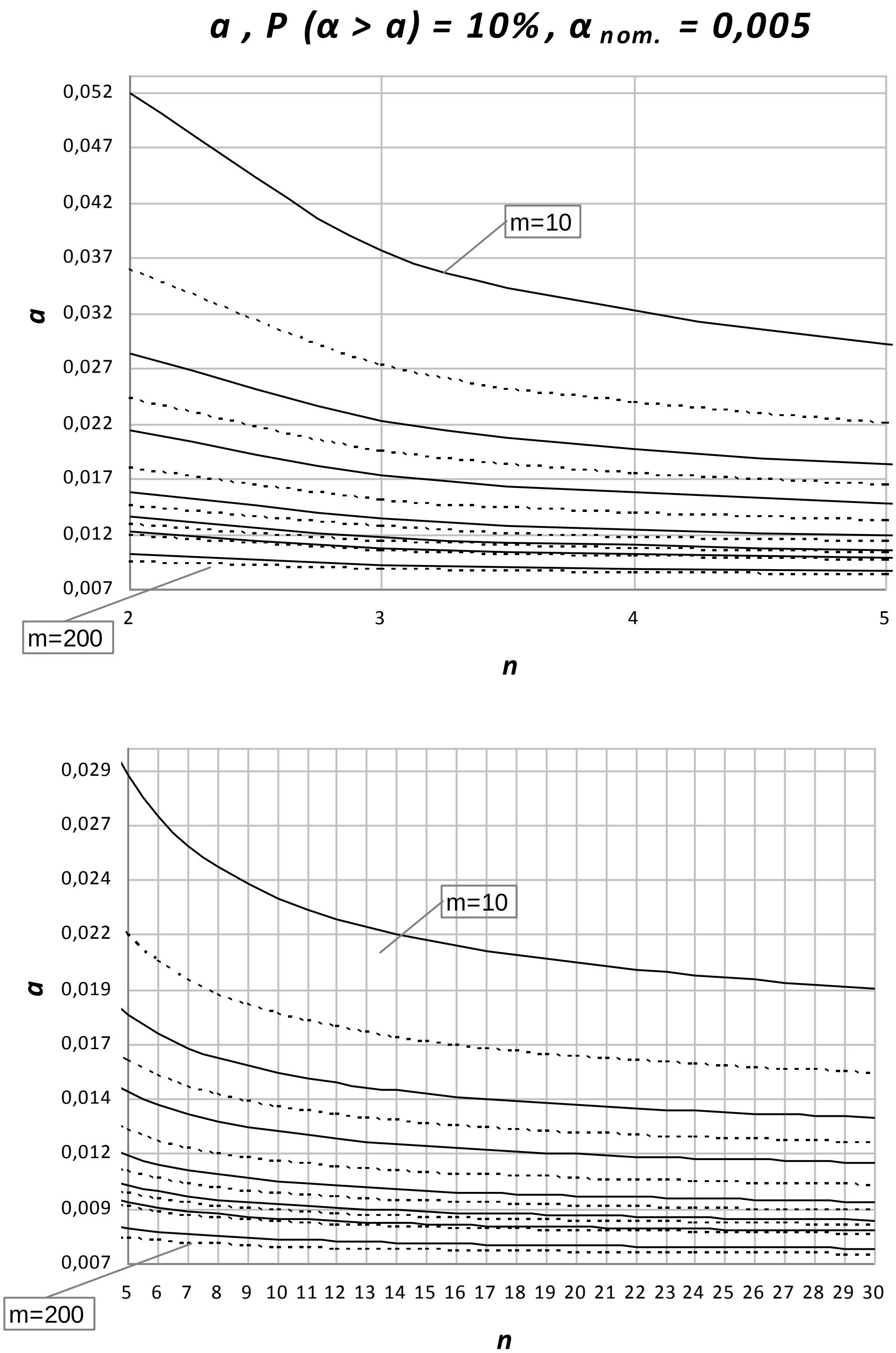

Figure 2 : 0.90-quantiles of α_{TRUE} as a funcion of *m* and *n*, for $\alpha_{NOM} = 0.005$ (corresponds to the data in Table 2)

Table 3 : minimum number of initial samples required, *m*, as a function of *n*, with ε = 10 and 20%, *p* = 0.05, 0.10 and 0.15, and α_{NOM} = 0.005

	ε = 10%			ε = 20%		
		p			*p*	
n	0.05	0.10	0.15	0.05	0.10	0.15
2	12792	7765	5079	3466	2104	1377
3	9056	5498	3596	2455	1490	975
4	7527	4570	2989	2040	1239	810
5	6672	4050	2649	1808	1098	718
6	6116	3713	2429	1658	1007	659
7	5722	3474	2272	1551	942	616
8	5425	3293	2154	1470	893	584
9	5192	3152	2062	1407	854	559
10	5003	3037	1987	1356	823	539
11	4846	2942	1925	1314	798	522
12	4714	2862	1872	1278	776	580
13	4600	2793	1827	1247	757	495
14	4501	2733	1787	1220	741	485
15	4414	2680	1753	1196	726	475
16	4336	2633	1722	1175	714	467
17	4267	2590	1694	1156	702	459
18	4204	2552	1670	1139	692	453
19	4144	2518	1647	1124	682	447
20	4096	2486	1626	1110	674	441
25	3891	2362	1545	1054	640	419
30	3745	2274	1487	1015	616	403

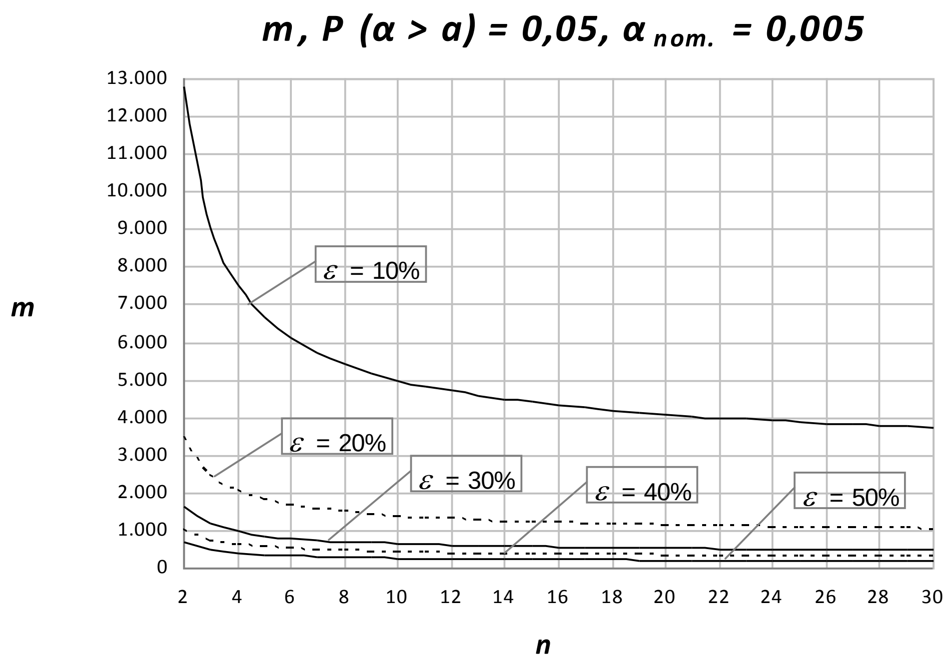

Figure 3 : minimum number of initial samples required, *m*, as a function of *n*, with ε = 10, 20, 30, 40 e 50%, p = 0.05 and α_{NOM} = 0.005

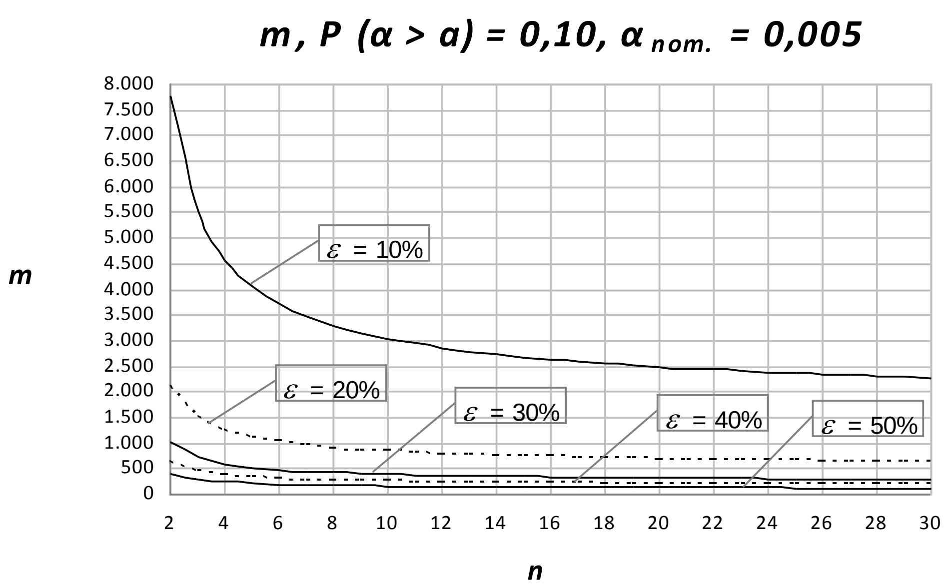

Figure 4 : minimum number of initial samples required, *m*, as a function of *n*, with ε = 10, 20, 30, 40 e 50%, p = 0.10 and α_{NOM} = 0.005

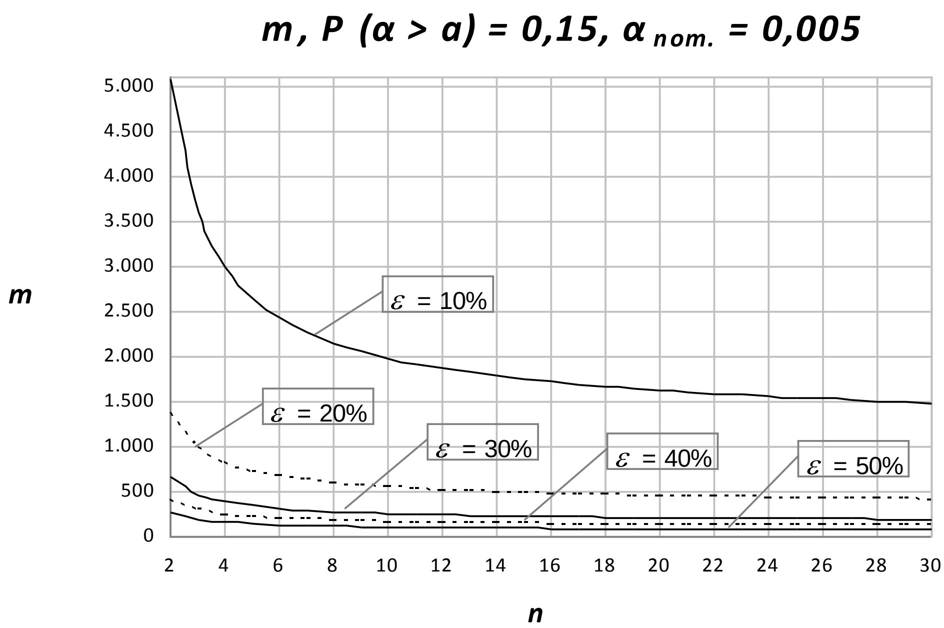

Figure 5 : minimum number of initial samples required, *m*, as a function of *n*, with ε = 10, 20, 30, 40 e 50%, p = 0.15 and α_{NOM} = 0.005

Finally, Table 4 presents the product *mn* corresponding to each cell of Table 3. This is the number of initial *items* to be inspected before the control limit can be "frozen". Also, if if there is a limitation on the average number of items that we are willing to inspected per unit of time so that the time between samples is proportional to the sample size, then *mn* is proportional to the *time* until the limits can be frozen. Table 4 shows the advantage of small over large samples if this is the case.

This effect can be seen from tables 1 and 2 as well: picking up pairs of (*m*, *n*) values with the same product — for instance, (200, 2), (100, 4), (50, 8) and (25, 16) — the 0.90 and 0.95 quantiles of α_{TRUE} are smaller for the pairs with smaller *n*.

One cannot disconsider, however, that the choice of the sample size should take into account other criteria as well, such as the operational convenience and the out-of-control performance of the control chart, to mention only two of them. In addition, research the authors of this paper have done on the effect of the number and size of the initial samples on the estimation of the process capability (still unpublished) has shown the opposite effect: the estimation of process capability favors larger samples over smaller ones. A possible reason for the fact is the higher loss of degrees of freedom in the case of (more) smaller samples; this also happens with the estimation of σ for control charting, but in the latter case the sample size has also an effect on the false-alarm risk, which does not exist in the estimation of the process capability[1].

[1] We are grateful to Subha Chakraborti for this explanation.

Table 4 : minimum number of initial items required, $m \times n$, as a function of n, with ε = 10 and 20%, p = 0.05, 0.10 and 0.15, and α_{NOM} = 0.005

	ε = 10%			ε = 20%		
	p			*p*		
n	0.05	0.10	0.15	0.05	0.10	0.15
2	25584	15530	10158	6932	4208	2754
3	27168	16494	10788	7365	4470	2925
4	30108	18280	11956	8160	4956	3240
5	33360	20250	13245	9040	5490	3590
6	36696	22278	14574	9948	6042	3954
7	40054	24318	15904	10857	6594	4312
8	43400	26344	17232	11760	7144	4672
9	46728	28368	18558	12663	7686	5031
10	50030	30370	19870	13560	8230	5390
11	53306	32362	21175	14454	8778	5742
12	56568	34344	22464	15336	9312	6960
13	59800	36309	23751	16211	9841	6435
14	63014	38262	25018	17080	10374	6790
15	66210	40200	26295	17940	10890	7125
16	69376	42128	27552	18800	11424	7472
17	72539	44030	28798	19652	11934	7803
18	75672	45936	30060	20502	12456	8154
19	78736	47842	31293	21356	12958	8493
20	81920	49720	32520	22200	13480	8820
25	97275	59050	38625	26350	16000	10475
30	112350	68220	44610	30450	18480	12090

4. Conclusions

The number of initial samples that guarantees that there is a reasonable probability (say 90 or 95%) that the actual false-alarm probability of the S chart is not much larger than the specified one (say, just 5, 10 or 20% larger) is much greater than the traditionally recommended 25, 30 or 50 samples. For example, if the specified risk is of 0.005 (ARL0 = 200), with samples of size 4, 1239 initial samples are required for estimating and establishing the UCL of the one-sided S chart with a 90% probability that the true false-alarm risk is at most 20% greater than 0.005 (or, equivalently, that ARL0 $\geqslant$ 166.7).

The numbers of initial samples required obtained in this analysis are also greater than the numbers obtained by previous authors — who, however, based their analyses on different criteria; specifically, they considered the marginal distribution of the RL. In our opinion, however, the quantiles of the true false-alarm risk distribution are more meaningful for the practitioner, since the distribution of the the distribution of the RLs of any S chart is indeed the conditional distribution on the value of the estimate of .

Of course one cannot wait until, say, 1239 samples are taken to start monitoring the process. The recommendation is that the control limits are revised periodically until the required number of samples is attained. We might say that there should be an intermediate phase between Phase I (using retrospective data to check process stability, estimate process parameters and set the control limits) and Phase II (monitoring the process).In such phase, the process is being monitored but the control limits are not yet "frozen".

Tables and graphs are available from the authors, with the results of the analysis for a wide range of cases. Anyway, the formulae given in this paper enable anyone to straightforwardly calculate the number of samples required (or the quantiles of the true false-alarm risk) for each particular case of interest.

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