GLR Charts for Monitoring Multiple Proportions

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Abstract

This paper develops a multinomial generalized likelihood ratio(GLR) chart for detecting shifts in category probabilities. This chart can be used when all items from the process are inspected continuously and classified into more than two categories. It is shown that the multinomial GLR chart has a very significant advantage relative to some other charts when the direction of the out-of-control shift in the parameter vector can not be specified. Some charts such as the multinomial cumulative sum(CUSUM) chart give a good performance when the shifts in parameters can be specified, but give a very poor performance when the shifts are not in anticipated directions or the shift direction is unknown. Because there may not be too many applications with multiple categories where the shifts in parameters can be specified or there is only one specific direction of interest, the multinomial GLR chart provides a very attractive option for detecting shifts in category probabilities.

KEY WORDS: Average Run Length; Generalized Likelihood Ratio Chart; Multinomial Distribution; Multiple Categories; Statistical Process Control.

Introduction

In a production process, the quality of a product is usually defined as defective or non-defective according to some special requirement based on the customer's need. Then the quality of the products can be expressed as a sequence of Bernoulli random variable X_i, with value "1" being the product is defective and value "0" being non-defective. The long-run in-control proportion of defectives is defined as $p_0 = \Pr(X_i = 1)$. When an abrupt change occurs, an increase shift in p_0 refers to the process deterioration while a decrease shift corresponds to improvement of the process. It is usually the process deterioration being the interest of practitioners since high quality of products is in demand to satisfy the customers. The traditional approach for monitoring the proportion of defective items *p* is the Shewhart *p*-chart which plots the proportion of defective items for each sample. An alternative control chart for monitoring proportion is the CUSUM chart. Gan (1993) proposed a binomial CUSUM chart which plots the binomial CUSUM statistic for each sample, and Reynolds and Stoumbos (1999) proposed a Bernoulli CUSUM chart which is a special case of the binomial CUSUM chart when sample size $n = 1$. It is shown by Reynolds and Stoumbos (1999) that the Bernoulli CUSUM chart can detect the shift in *p* much faster comparing with the Shewhart *p*-chart and the binomial CUSUM chart. For a thoroughly review of control chart for monitoring proportion, see Szarka and Woodall (2011).

The Shewhart *p*-chart, the binomial CUSUM chart, and the Bernoulli CUSUM chart are all used for monitoring the proportion of defective items, where the quality of an item is only classified into two categories: defective or non-defective. If in practice, the quality is classified into more than two categories, such as defective, moderate defective, minor defective, and non-defective each with proportion $p_1, p_2, p_3,$ and p_4 respectively, then monitoring a single proportion of defective p_1 cannot be equivalent to monitoring the whole process as there are three other categories except the defective category. This is a very common case in reality not only in production process, but also in health-care area.

To monitor multiple proportions, one approach is to use different Shewhart *p*-charts, different binomial CUSUM charts, or different Bernoulli CUSUM charts for the proportion of each category. However, as the number of categories increases, an increased number of individual charts used for monitoring process will be cumbersome, for more details see Duncan (1950). Instead of multiple Shewhart *p*-charts, Ducan (1950), Marcucci (1985), and Nelson (1987) all considered one control chart based on chi-square distribution to monitor process where items can be classified into multiple categories. The charts mentioned above all based on the fact that items are grouped into samples. Ryan et al. (2011) proposed a multinomial CUSUM chart to monitor the process where items can be classified into more than two categories when the items are inspected individually and the direction of the out-of-control shift in parameter vector can be specified. It is shown in their paper that the multinomial CUSUM chart can detect as fast as, and in most of the cases even faster than using multiple Bernoulli CUSUM charts when the proportion vector shifts to a specified direction. When direction is unknown, they suggested using multiple Bernoulli CUSUM charts. In practice, it is sometimes difficult to specify shift direction due to the uncertainty in production process and thus unknown cases for the shift direction are much more interested and reasonable as the assumption.

In this paper, we propose a multinomial GLR chart based on the likelihood ratio statistic with an EWMA type estimator to monitor the proportion vector when the shift direction is unspecified and the inspection is taken place on a single item. The unspecified shift direction is in the sense that both the process deterioration and improvement are considered, i.e. the shift in the proportion of "non-defective" category could decreases or increase and there is no clear pattern of the parameter shift. Even though our multinomial GLR chart is not designed for the specified shift direction, we still compare the performance of the multinomial GLR chart with other existing charts for the completeness of discussion and propose to use the combination of multinomial GLR and CUSUM charts (the multinomial GLR-CUSUM chart) when the shift direction is known.

In literature, GLR charts have been used for monitoring the mean of observations from a normal distribution and provided a better overall performance than the traditional Shewhart type and CUSUM charts, see Reynolds and Lou (2010). It is shown that the GLR chart can detect a wide range of shift relatively fast without using tuning parameters. Huang et al. (2012) have studied the performance of the binomial GLR chart for monitoring proportion of defective when the items are grouped into samples and the number of defective items follows a binomial distribution. The performance of the binomial GLR chart is at least as good as the Shewhart *p*-chart, the binomial CUSUM chart, and the multiple binomial CUSUM charts with different tuning parameters. In general, the GLR chart is more convenient to design for detecting a wild range of shift size.

The paper will be discussed as follows: in the next section, we will introduce using the multinomial CUSUM chart and the multiple Bernoulli CUSUM charts. This is followed by the derivation of the multinomial GLR chart for monitoring the multinomial data. Following the derivation of the multinomial GLR chart, the performance of the multinomial GLR chart on observations with three categories is compared with that of the multinomial CUSUM chart and the multiple Bernoulli CUSUM charts when the proportion shift direction is unknown. Next, cases in which the proportion shift direction can be specified are discussed.

Multinomial CUSUM Charts and Multiple Bernoulli CUSUM Charts

In this section, we introduce the multinomial CUSUM chart and the multiple Bernoulli CUSUM charts which have been shown to have a good performance on monitoring multiple proportions when the shift direction is specified and unspecified respectively.

Let $X_1, X_2, \ldots$ be a sequence of independent multinomial random variables, where $X_t = i$ if the t^{th} item is classified in the i^{th} category, $t = 1, 2, \ldots,$ $i = 1, 2, \ldots, k.$ The event of $\{X_t = i\}$

can be redefined as $\{X_{t,1}=0,\dots,X_{t,i}=1,\dots,X_{t,k}=0\}$, where

$$X_{t,i}=\begin{cases}1 & \text{if the } t\text{th item is classified in the } i\text{th category}\\ 0 & \text{otherwise}\end{cases}$$

for $i=1,2,\dots,k$. If p_i denotes the probability of being classified into category i for $i=1,2,\dots,k$, we can see that the pdf of X_t is

$$\Pr(X_t=i)=\Pr(X_{t,1}=0,\dots,X_{t,i}=1,\dots,X_{t,k}=0)=p_i\ .$$

Let $p_{0,i}$ and $p_{1,i}$ be the in-control and out-of-control probability of being classified into category *i*, respectively. Then the multinomial CUSUM chart is defined based on the likelihood ratio statistic:

$$S_t=\max(0,S_{t-1}+L_t),\ t=1,2...,$$

where $S_0=0$ and L_t is the log-likelihood ratio which is equal to $\ln(p_{1,i}/p_{0,i})$ when $X_t=i$. The chart signals if $S_t>h_C$, where the value h_C is pre-specified from the desired in-control average run length (ARL) value. This multinomial CUSUM chart proposed by Ryan et al. (2011) is similar to the multinomial CUSUM chart proposed by Steiner et al. (1996). However, the later is based on the situation where continuous observations can be grouped into samples.

Another approach for monitoring multiple proportions is using several Bernoulli CUSUM charts with each monitoring the proportion of one category. Suppose we want to monitor the proportion of category *i*, then the Bernoulli CUSUM statistics are defined as

$$S_{t,i}=\max(0,S_{t-1,i}+L_{t,i}),\quad t=1,2...,\ i=1,2...k,$$

where $S_{0,i}=0$ and

$$L_{t,i}=\begin{cases}\ln\dfrac{1-p_{1,i}}{1-p_{0,i}}, & \text{if } X_{t,i}=0\\[2ex] \ln\dfrac{p_{1,i}}{p_{0,i}}, & \text{if } X_{t,i}=1\end{cases}.$$

For monitoring a process with *k* categories, *k* Bernoulli CUSUM charts can be run simultaneously. It is usually argued that $k-1$ Bernoulli CUSUM charts would be enough for k different categories since the sum of the proportions of each category equals to 1. However, as in Ryan et al. (2011), we would consider k Bernoulli CUSUM charts for each k category respectively for the completeness of discussion. These multiple Bernoulli CUSUM charts have been shown to beat the multinomial CUSUM chart when the shift direction is unspecified by Ryan et al. (2011).

Multinomial GLR Charts

As in the multinomial CUSUM chart, $X_1, X_2, \ldots$ is a sequence of independent multinomial random variables.

Suppose that a special cause occurs at a random time between times τ and $\tau+1$, and its effect appears from time $\tau+1$. The unknown time τ is called the process change point. Let $\boldsymbol{p_0} = (p_{0,1}, p_{0,2}, \ldots, p_{0,k})$ and $\boldsymbol{p_1} = (p_{1,1}, p_{1,2}, \ldots, p_{1,k})$ be the in-control and out-of-control values of $\boldsymbol{p} = (p_1, p_2, \ldots, p_k)$, where $\sum_{j=1}^{k} p_{0,j} = 1$ and $\sum_{j=1}^{k} p_{1,j} = 1$. We assume that the in-control values of $\boldsymbol{p_0} = (p_{0,1}, p_{0,2}, \ldots, p_{0,k})$ are known or have been estimated accurately enough that any error in estimation can be neglected.

Under the null hypothesis that there has been no change in the process, the likelihood function at time can be represented as

$$L(\infty, \boldsymbol{p_0} \mid X_1, X_2, \cdots, X_t) = p_{0,1}^{\sum_{j=1}^{t} x_{j,1}} \cdots p_{0,k}^{\sum_{j=1}^{t} x_{j,k}},$$

where $x_{j,i}$ denotes the observed value of $X_{j,i}$ for $j = 1, 2, \ldots, t$, $i = 1, 2, \ldots, k$. Note that $\sum_{j=1}^{t} x_{j,i}$ is the number of items classified in the i^{th} category among $X_1, X_2, \ldots, X_t$. Consider the alternative hypothesis that specials causes make a change such as $\boldsymbol{p} = \boldsymbol{p_1}$ for $t \geq \tau + 1$. Then the likelihood function at time t is

$$L(\tau, \boldsymbol{p_1} \mid X_1, X_2, \cdots, X_t) = p_{0,1}^{\sum_{j=1}^{\tau} x_{j,1}} p_{1,1}^{\sum_{j=\tau+1}^{t} x_{j,1}} \cdots p_{0,k}^{\sum_{j=1}^{\tau} x_{j,k}} p_{1,k}^{\sum_{j=\tau+1}^{t} x_{j,k}}.$$

The MLE of $p_{1,i}$ is $\hat{p}_{1,i,t} = N_{i,\tau,t} / (t-\tau)$, where $N_{i,\tau,t} = \sum_{j=\tau+1}^{t} x_{j,i}$. However, the possibility that the value of $N_{i,\tau,t}$ is zero can be large for some τ and $\hat{p}_{1,i,t}$ will be zero in this case. To avoid this problem, we suggest a modified EWMA type estimator of $p_{1,i}$, say $\tilde{p}_{1,i,t}$, as

$$\tilde{p}_{1,i,t} = \lambda \hat{p}_{1,i,t} + (1-\lambda) \tilde{p}_{1,i,t-1},$$

where $\tilde{p}_{1,i,0} = p_{0,i}$ for $i = 1, 2, \ldots, k$ and λ is a tuning parameter with $0 < \lambda < 1$.

A log likelihood ratio statistic for testing whether there has been a change in the multiple proportions is

$$\begin{aligned} R_t &= \ln \frac{\max\limits_{0 \leq \tau < t, 0 < p_{1,i} < 1} L(\tau, \boldsymbol{p}_1 \mid X_{1,} X_2, \ldots, X_t)}{L(\infty, \boldsymbol{p}_0 \mid X_{1,} X_2, \ldots, X_t)} \\ &\approx \max_{0 \leq \tau < t} \left\{ N_{1,\tau,t} \ln\left(\frac{\tilde{p}_{1,1,t}}{p_{0,1}} \right) + \cdots + N_{k,\tau,t} \ln\left(\frac{\tilde{p}_{1,k,t}}{p_{0,k}} \right) \right\} \end{aligned}$$

A signal of the multinomial GLR chart is given at sample t if $R_t > h_G$, where the value h_G is pre-specified from the desired in-control ARL value.

The multinomial CUSUM chart is designed for the situation in which the direction of shift is known and the expected out-of-control values of probabilities are specified. This chart gives good performance when the true shifts are in the anticipated direction. In contrast, the multinomial GLR chart can be designed for the cases in which the shift direction is unknown or the shift is not in the anticipated direction. The out-of-control values of probabilities are estimated in the monitoring process, and the multinomial GLR chart gives good performance for any shift direction as shown in the following section.

Comparisons for the Situation Where the Shift Direction Is Unknown

In this section, the performance of the multinomial GLR chart is compared with that of the multinomial CUSUM chart and the multiple Bernoulli CUSUM charts by using their ARL values. We obtain the zero-state ARL values of the multinomial GLR chart by simulations, for comparing the results of Ryan et al. (2011). The steady-state performance would lead similar conclusions since there is no headstart feature in all the charts considered here. Each estimated ARL value is obtained from 100,000 simulations. The value of λ considered is 0.2.

Two cases are investigated in this comparison study. We assume in the process each item can be classified as one of the three different categories: good, fair, and bad. This can be generalized to the situation where the categories are listed as A, B, and C. Compared with other cases in next section, the in-control probabilities for fair and bad items are relatively large in this section.

For case 1, we consider the situation where the parameter shift is not in the anticipated direction. We assume that the in-control probabilities of $\boldsymbol{p}$ is $\boldsymbol{p_0} = (p_{0,1}, p_{0,2}, p_{0,3}) = (0.65, 0.25, 0.10)$ and the in-control ARL is $\text{ARL}_0 = 280$. The multinomial CUSUM chart and the 3-Bernoloulli CUSUM chart are designed for detecting increases in the probabilities of fair and bad items and a decrease in the probability of good items. Specifically, they are optimized for detecting the out-of-control probabilities:

$$\boldsymbol{p_1} = \left(p_{1,1}, p_{1,2}, p_{1,3}\right) = \left(0.4517, 0.2999, 0.2484\right).$$

These probabilities were slightly adjusted to give the exact in-control ARL value for the multinomial CUSUM chart based on Markov Chain method. However, the true parameter shifts may not be in the anticipated direction. The probabilities of good and bad items are both actually increased, while the probability of fair items is in fact decreased as shown in Table 1. The ARL comparisons of the multinomial GLR chart, the multinomial CUSUM chart, and the 3-Bernoulli CUSUM chart are also shown in Table 1. The values of h_1, h_2, and h_3 in the 3-Bernoulli CUSUM chart denote control limits that focus on the good, fair, and bad categories, respectively. Distribution (Dist.) 1 denotes the in-control case.

Comparing the multinomial CUSUM chart with the 3-Bernoulli CUSUM chart, the former is better for small shifts while the later is better for large shifts. It is recommended by Ryan et al. (2011) that when the shifts are not in the anticipated direction, it's better to use the 3-Bernoulli CUSUM chart than the multinomial CUSUM chart.

The multinomial GLR chart is uniformly better than the 3-Bernoulli CUSUM chart. Compared with the multinomial CUSUM chart, the multinomial GLR chart is only a little worse for very small shifts, but much better for the moderate and large shifts. This result shows that the multinomial GLR chart is preferred over other two charts for case 1.

Table 1. ARL Comparisons for Case 1

Dist.	p_1 Pr(Good)	p_2 Pr(Fair)	p_3 Pr(Bad)	Multinomial GLR $h_G = 5.787$	Multinomial CUSUM $h_C = 2.95$	3-Bernoulli CUSUM $h_1 = 3.706$ $h_2 = 1.7896$ $h_3 = 3.506$
1	0.65	0.25	0.10	280.09	279.96	281.00
2	0.66	0.22	0.12	215.69	193.53	244.93
3	0.67	0.20	0.13	165.34	170.57	204.23
4	0.68	0.18	0.14	123.60	152.01	163.29
5	0.69	0.15	0.16	77.86	117.63	106.13
6	0.72	0.10	0.18	44.39	104.19	74.23
7	0.73	0.07	0.20	32.08	85.56	55.56
8	0.74	0.05	0.21	26.64	79.98	48.98
9	0.75	0.02	0.23	20.40	67.91	39.37

For case 2, we consider the situation in which various parameter shifts are in misspecified directions. The multinomial CUSUM chart and the 3-Bernoulli CUSUM chart are designed in the same way as case 1. The true parameter shifts direction is unknown and therefore considered as random, which means it could increase or decrease, and there is no clear pattern for the shifts as shown in Table 2. Notice that although the direction of shifts is random for each component, there is a constraint that the sum of those probabilities must add up to 1. The ARL results of all three charts are also listed in Table 2.

It should be noted that the multinomial CUSUM chart and the 3-Bernoulli CUSUM chart are biased control charts for case 2, since the out-of-control ARLs for some shifts are larger than the in-control ARL. This is undesirable for any control chart. When the parameter shifts is not in the designed direction, for example in distribution 2 all three probabilities shifts are in the exact opposite direction of the designed one, the ARL of the multinomial GLR chart is 203.14, whereas the ARL of the multinomial CUSUM chart and the 3-Bernoulli CUSUM chart is 1081.05 and

879.27, respectively. The multinomial GLR chart is almost at least as good as, and for most shifts distributions in Table 2, much better than the multinomial CUSUM chart and the 3-Bernoulli CUSUM chart.

Table 2. ARL Comparisons for Case 2

Dist.	p_1 Pr(Good)	p_2 Pr(Fair)	p_3 Pr(Bad)	Multinomial GLR $h_G = 5.787$	Multinomial CUSUM $h_C = 2.95$	3-Bernoulli CUSUM $h_1 = 3.706$ $h_2 = 1.7896$ $h_3 = 3.506$
1	0.65	0.25	0.10	280.09	279.96	281.00
2	0.70	0.23	0.07	203.14	1081.05	879.27
3	0.80	0.09	0.11	45.69	628.67	445.12
4	0.68	0.23	0.09	256.60	460.61	527.15
5	0.60	0.32	0.08	153.65	317.04	105.41
6	0.70	0.19	0.11	167.53	315.55	405.01
7	0.50	0.45	0.05	39.73	298.70	37.83
8	0.45	0.15	0.40	15.39	12.80	14.24
9	0.40	0.20	0.40	15.19	11.81	14.04

Based on the above results for case 1 and 2, we conclude that when the parameter shifts direction cannot be anticipated, the multinomial GLR chart has much better overall performance than CUSUM type charts. The farther the true shifts away from the designed ones, the more superior performance of the multinomial GLR chart would have.

Comparisons for the Situation Where the Shift Direction Is Known

The multinomial GLR chart has better performance when the shift direction is unknown as shown from the results in the previous section. One natural question is how the performance changes if the shift direction is known. Although the multinomial GLR chart is not designed for such cases, for a complete discussion, we still investigate the performance of the multinomial GLR chart and compare it with that of the multinomial CUSUM chart and the 3-Bernoulli CUSUM chart by using their ARL values.

Two cases are studied in this section. In both case, we concern with detecting increases in the probabilities of fair and bad items, and a decrease in the probabilities of good items. The out-of-control probabilities for each category are shifted in the anticipated direction. For case 3, the in-control probabilities for each category are the same as those in the previous section. The multinomial and 3-Bernoulli CUSUM charts are designed for detecting the probabilities shifts in distribution 6 in Table 3. The ARL values of all the charts compared here are also displayed in

Table 3.

Table 3. ARL Comparisons for Case 3

Dist.	p_1 Pr(Good)	p_2 Pr(Fair)	p_3 Pr(Bad)	Multinomia GLR $h_G = 5.787$	Multinomial GLR and CUSUM $h_G = 6.49$ $h_C = 3.45756$	Multinomial CUSUM $h_C = 2.95$	3-Bernoulli CUSUM $h_1 = 3.706$ $h_2 = 1.7896$ $h_3 = 3.506$
1	0.65	0.25	0.10	280.09	279.87	279.96	281.00
2	0.625	0.255	0.12	228.11	345.77	153.82	171.06
3	0.60	0.26	0.14	162.22	186.98	95.54	110.74
4	0.55	0.27	0.18	80.38	70.58	47.45	56.38
5	0.50	0.28	0.22	47.32	37.08	29.29	35.51
6	0.4517	0.2999	0.2484	33.45	25.33	21.57	26.78
7	0.35	0.35	0.30	19.32	13.48	14.26	17.70
8	0.25	0.40	0.35	12.87	8.76	10.58	13.23
9	0.15	0.45	0.40	9.35	6.06	8.40	10.57
10	0.05	0.50	0.45	7.23	4.06	6.95	8.87

The multinomial CUSUM charts has better performance than the multinomial GLR chart and the 3-Bernoulli CUSUM chart, indicating the multinomial CUSUM chart is preferred as long as the parameter shifts are in the anticipated direction. We will comment on the multinomial GLR-CUSUM chart in case 4.

For case 4, the in-control and out-of-control probabilities for the fair and bad categories are very small, compared with those in case 3. In this case, we set $\text{ARL}_0 = 500$ as in Ryan et al. (2011). This is modeled for the cases with high quality process, where almost all the products are good items. The probabilities of each category for case 4 are shown in Table 4. The multinomial CUSUM chart and the 3-Bernoulli CUSUM chart are optimized for detecting the probabilities in distribution 4. The results for case 4 are also displayed in Table 4.

The multinomial GLR chart has overall better performance than the multinomial CUSUM chart and the 3-Bernoulli CUSUM chart. Only for distribution 2, the multinomial GLR chart is slightly worse than other two charts. For all the other cases, the multinomial GLR chart has better performance. This means that for the high quality process where the probabilities of fair and bad components are very small, the multinomial GLR chart is preferred than CUSUM type charts even though the parameter shifts are in the expected direction.

It should be noted that the multinomial GLR-CUSUM chart gives better performance than multinomial GLR chart for the moderate shifts, while a little worse for the very small shift. This indicates that if we want to detect certain shifts, we can use the multinomial GLR-CUSUM chart. In this combined chart the multinomial CUSUM chart is designed for detecting probabilities in distribution 4. The result shows that the performance is greatly improved for the larger shifts than the designed probabilities, but it is hurt for the smaller shifts than the designed ones.

Table 4. ARL Comparisons for Case 4

Dist.	p_1 Pr(Good)	p_2 Pr(Fair)	p_3 Pr(Bad)	Multinomial GLR $h_G = 3.68921$	Multinomial GLR and CUSUM $h_G = 3.692$ $h_C = 1.03$	Multinomial CUSUM $h_C = 0.8337$	3-Bernoulli CUSUM $h_1 = 1.60$ $h_2 = 1.13$ $h_3 = 1.38$
1	0.994	0.005	0.001	501.57	501.11	499.64	499.94
2	0.99	0.0075	0.0025	229.56	257.94	227.69	227.86
3	0.987	0.009	0.004	149.99	164.80	155.02	155.81
4	0.9848	0.0099	0.0053	117.56	123.74	124.44	125.32
5	0.98	0.015	0.005	90.58	50.07	98.43	99.46
6	0.974	0.02	0.006	64.39	38.50	73.72	74.77
7	0.96	0.03	0.01	35.60	24.85	45.09	45.65
8	0.95	0.035	0.015	25.83	19.98	34.49	34.75
9	0.94	0.04	0.02	20.36	16.68	27.97	28.09
10	0.90	0.06	0.04	10.88	10.02	16.01	16.01
11	0.85	0.09	0.06	6.84	6.68	10.67	10.67
12	0.80	0.11	0.09	5.02	4.99	7.75	7.75
13	0.70	0.17	0.13	3.34	3.34	5.22	5.22
14	0.60	0.24	0.16	2.49	2.50	4.00	4.00
15	0.50	0.30	0.20	1.99	2.00	3.20	3.20
16	0.30	0.40	0.30	1.43	1.42	2.24	2.25

In general, if the practitioners want to detect certain shifts, they can use the multinomial GLR-CUSUM chart and tune the CUSUM chart to detect the shift that is slightly smaller than the interested shift in the expected direction. If there is a particular shift size and direction that is of concern, it is reasonable to use the multinomial GLR-CUSUM chart to improve performance for the size and direction of concern, but still maintain good performance for any other shift situations.

Conclusions

For monitoring multiple proportions when we can inspect continuously, several control charts are studied and their performances are compared on monitoring observations classified into three categories. If the parameter shift direction is unknown, we recommend to use the proposed multinomial GLR chart. If the parameter shift direction is known, for the high quality process the multinomial GLR chart is also preferred than other charts. For other processes with known shift direction we recommend to use the multinomial CUSUM chart or the multinomial GLR-CUSUM chart.

For future work, more general cases of observations classified into different categories can be considered, such as four, five, or even larger categories. As the number of categories increases, we believe that it becomes harder to specify the direction of the parameter shifts and thus the multinomial GLR chart, in which we don't need to specify the shift direction and the out-of-control probabilities, would be more useful.

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