Latent Trait Shared Parameter Mixed-Models

for Missing Ecological Momentary Assessment Data

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Ecological Momentary Assessment

- Ecological Momentary Assessment (EMA) - Real-time data capture
- Addiction, depression, anxiety, and mood studies
- Capture outcomes as they occur in normal settings
- Hand-held computer beeps at random times throughout the day
- Long series of outcomes collected with intermittent missingness

Statement of the Problem

- Most EMA studies ignore the missing data
- Shared parameter models - one approach if missingness is non-random
- Latent class pattern-mixture models used with intermittent missing data (Lin 04, Roy 03, Beunckens 08)
- Solution: Latent Trait Shared Parameter Mixed-Model for EMA to handle missing data

Latent Trait Theory

- Also known as Item Response Theory
- IQ tests, Law School Admissions tests
- Each subject has latent trait or "ability" - intelligence
- Latent trait determined by number of correct questions answered, question difficulty, and discrimination parameter

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Item Response Theory Models

Logistic Model written as:

$$P(R_{ij} = 1|\theta_i) = \frac{1}{1 + \exp[-a_j(\theta_i - b_j)]} \quad (1)$$

- a_j: slope parameter for item j
- $a_j = a$ in One-Parameter Model (Rasch)
- b_j: difficulty parameter for item j
- $R_{ij} = 1$ subject i responds to prompt j
- θ_i Latent trait or "ability" of each subject assume $\theta_i \sim$ N(0,1)
- Higher $\theta_i \rightarrow$ higher "ability"

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EMA Data collection

- Hedeker, Mermelstein, and Demirtas (08)
- 452 subjects in high school (9th or 10th grade)
- Positive affect (PA) and negative affect (NA) measured over 7-day period with 30 to 40 responses per subject
- Covariates include: Gender, Smoker, Negative Mood Regulation (NMR), Grade Point Average (GPA)
- Alone indicator separated into between-subject (AloneBS) and within-subjects (AloneWS) components

Time-Bins

Form 5 time-bins per day:

Time-bin	Description
3am - 8:59am	Early Morning
9am - 2:59pm	Mid-Day
3pm - 5:59pm	Afternoon
6pm - 8:59pm	Evening
9pm - 2:59am	Late Evening

Data collection over 1 week: $m_i = 5 \times 7 = 35$

Response Indicator Setup

All subjects have a unique response vector R_{ij} ,where:

- $j = 1, \ldots, 35$ time-intervals =(7 days x 5 periods)
- $R_{ij} = 1$ participant responded
- $R_{ij} = 0$ participant did not respond
- $R_{ij} = .$ no prompt generated

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Example data - rows (days) are stacked

$$\text{Response vector } R_{ij} = \begin{bmatrix} . & 1 & . & . & 0 \\ 1 & 1 & . & . & 0 \\ 1 & 0 & . & . & 0 \\ 1 & 1 & . & . & 0 \\ 0 & . & 1 & 1 & 1 \\ . & . & . & . & 0 \\ . & . & . & 0 & 1 \end{bmatrix}$$

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Latent-Trait Shared Parameter Mixed-Model (LTSPMM)

Will combine information about the response process and missingness process

Why?

- Latent class approach usually has predefined idea about group composition
- Roy (Biometrics 08) notes that statistical tests for number of classes may also be difficult
- Use additional information from R_{ij} and the latent trait θ_i

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Estimated LTSPMM - Longitudinal Model

$$y_{ij} = x'_{ij}\beta + \gamma\theta_i + \sigma_\upsilon \xi_i + e_{ij}$$
$$i = 1, 2, \ldots, N \text{ subjects}, \; j = 1, 2, \ldots, n_i \text{ times}$$

- y_{ij} : Negative Affect or Positive Affect, for subject i at time j
- x_{ij} : Subject covariates, β : vector of regression coefficients
- γ: regression coefficient for latent trait θ_i
- θ_i: latent trait for subject $i \sim N(0, 1)$
- σ_υ: random intercept standard deviation
- ξ_i: random intercept coefficient $\sim N(0, 1)$
- e_{ij} : error terms $\sim N(0, \sigma_e^2)$

Estimated LTSPMM - Response Process

$$logit(R_{ij}) = c_j + a_j\theta_i$$

- One- or Two-Parameter Latent Trait Model
- $j = 1, 2, \ldots, m_i$ time-bins
- $c_j = -a_j b_j$: item-intercept parameter
- a_j: discrimination parameter
- $R_{ij} = 1, 0, \text{“.”}$
  $R_{ij} = 1 \Rightarrow$ subject i answered prompt in time-bin j
  $R_{ij} = 0 \Rightarrow$ subject i did not answer prompt in time-bin j
  $R_{ij} = \text{“.”} \Rightarrow$ no prompt in time-bin j

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Estimation in SAS

- SAS NLMIXED used to estimate models in a joint fashion
- Marginal Maximum Likelihood solved using Adaptive Gaussian Quadrature
- 4 models estimated (NA or PA, MAR and One-Parameter LTSPMM)
- Explore ordinal outcomes
- Main interest: how are mood outcomes (PA or NA) influenced by model covariates?

Estimates (Standard Errors)

	Negative Affect MAR	Negative Affect LT(1P)	Positive Affect MAR	Positive Affect LT(1P)
Smoker	0.390*	0.392*	-0.248	-0.245
	(0.159)	(0.157)	(0.141)	(0.140)
Male	-0.428**	**-0.487****	0.221	**0.269**
	(0.165)	(0.165)	(0.147)	(0.147)
NMR	-0.980***	-0.987***	0.818***	0.828***
	(0.120)	(0.119)	(0.107)	(0.106)
GPA	0.336**	0.399***	-0.181	-0.222*
	(0.108)	(0.108)	(0.097)	(0.097)

*** p<0.001,** p<0.01,* p<0.05

Estimates (Standard Errors)

	Negative Affect MAR	Negative Affect LT(1P)	Positive Affect MAR	Positive Affect LT(1P)
Alone (WS)	0.391***	0.391***	-0.584***	-0.584***
	(0.036)	(0.036)	(0.036)	(0.036)
Alone (BS)	1.094***	-1.121***	-1.695***	-1.713***
	(0.405)	(0.401)	(0.360)	(0.358)
Grade10	0.051	0.035	0.037	0.051
	(0.157)	(0.155)	(0.139)	(0.139)
NoveltySeek	0.223	0.216	0.062	0.075
	(0.121)	(0.120)	(0.108)	(0.107)
γ		**-0.301****		**0.232***
		(0.105)		(0.094)

*** p<0.001,** p<0.01,* p<0.05

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Model Results

- Coefficient estimates for gender drastically different across the two models (males have higher average mood)
- One-parameter models: Positive Affect and Negative Affect are influenced by the latent "ability" to respond
- NA $\rightarrow \hat{\gamma}$ is negative and significant! If an individual is more responsive, their NA has a tendency to decrease (feel better)
- PA $\rightarrow \hat{\gamma}$ is positive and significant! If an individual is more responsive, their PA has a tendency to increase (feel better)

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Results Summary

- LTSPMM offers technique to model missing EMA data
- Joint model linking longitudinal model with response model
- Ordinal model adds to previous research with continuous outcomes
- Future work:

1. Explore two-parameter latent trait model for response process
2. Simulation of MAR and latent trait model fits
3. Markov chain Monte Carlo approach?

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