Using an Approximate Bayesian Bootstrap to Multiply Impute Nonignorable Missing Data

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Reasons for Missing Data

Rubin (1976) Classifies the reasons for missing data as either *ignorable* or *nonignorable*.

In a dataset where variables X are fully observed and variables Y have missing values:

- Ignorable Missingness: Missing Y are only randomly different from observed Y when conditioning on X
- Nonignorable Missingness: Missing Y are systematically different from observed Y even when conditioning on X
- E.g. Missing Y are typically 20% larger than observed Y with the same values of X

Hot-deck Imputation

- Hot-deck imputation is an imputation technique that replaces missing values in a data set (donees) with observed values (donors)
- Most hot-deck imputation methods attempt to match donors and donees based on observed covariates
- *Predictive mean matching* (PMM), Rubin (1986), Little (1988)

Appoximate Bayesian Bootstrap

- Rubin (1987) suggests an Approximate Bayesian Bootstrap (ABB) to incorporate appropriate uncertainty into hot-deck procedures
- To incorporate an ABB into a hot-deck imputation model, the steps are:
  1. Draw a bootstrap sample of the observed values
  2. Impute missing values by drawing donors from this bootstrap sample
  3. For multiple imputation, repeat steps 1 and 2 m times, so that m bootstrap samples are drawn
- Applies an improper prior on the donor selection probabilities

Nonignorable ABB

- Rubin and Schenker (1991) discuss how an ABB can be modified to handle nonignorable missing data. Instead of drawing n_{obs} cases of Y_{obs} randomly with replacement (i.e. with equal probability), they suggest
- Draw n_{obs} cases of Y_{obs} with probability proportional to Y_{obs}^c so that the probability of selection for for $y_i \in Y_{obs}$ is

$$\frac{y_i^c}{\sum_{j=1}^{n_{obs}} y_j^c}. \tag{1}$$

- This skews the nonrespondents to have typically larger (when $c > 0$ and $y_j > 0$) values of Y than respondents.

Nonignorable ABB Terminology

We refer to ABBs where values of Y_{obs} are drawn with probability proportional to Y_{obs}^c where

- $c = -1$: Inverse-to-size ABB
- $c = 1$: Proportional-to-size ABB
- $c = 2$: Proportional-to-size-squared ABB
- $c = 3$: Proportional-to-size-cubed ABB

Additional Nonignorable ABBs

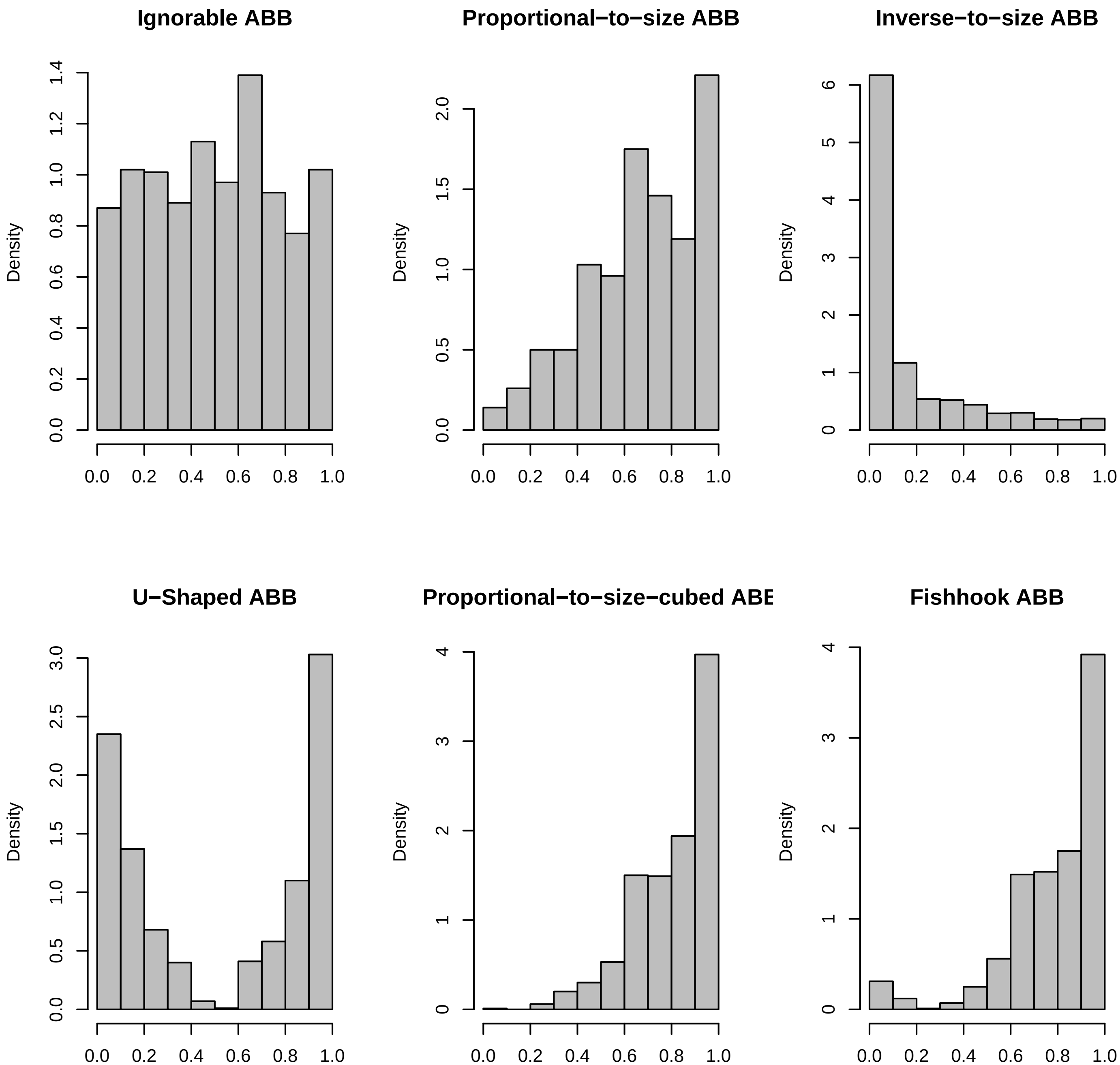
- A nonignorable ABB where n_{obs} cases of Y_{obs} are drawn with probability proportional to Y_{obs} centered around a quantile of Y_{obs}.
- E.g. center observed Y around its median. The implication of drawing with probability proportional to the distance from the median is to favor values for the non-respondents with either larger *or* smaller values than respondents with the same set of covariates (when $c > 0$).
- We refer to this approach as a "U-Shaped ABB" because observations in the extremes of the distribution of Y_{obs} have greater weight than observations in between that are close to the median

Additional Nonignorable ABBs cont'd

- We refer to the ABB that centers the donor sizes around the 1st quantile as a "Fishhook ABB," because this ABB mostly favors large values but retains a U-shaped pattern featuring a slight upturn in the weight given to the smallest observed values.

Nonignorable ABB Illustration

- Simulated $n = 1000$ observations from a uniform distribution
- Drew a ABB sample and recorded the number of times an observation was included in the sample
- Histogram using ABB weights
- Six different ABB types



Ignorable ABB
Proportional−to−size ABB
Inverse−to−size ABB
U−Shaped ABB
Proportional−to−size−cubed ABE
Fishhook ABB
Density

Prior Distributions

- Histograms can be considered prior distributions on the donor selection probabilities.
- Ignorable ABB is akin to a flat prior, and gives each donor equal prior probability of selection
- Other ABB types are informative priors, and give more or less weight to a donor based on the donor size.
- We will see that inferences are affected by the choice of ABB.

Motivating Example: The WECare Study

- The WECare study was a longitudinal depression treatment study of low income women
- Women were measured for depression every month for 6 months, every other month for 6 more months
- Here, we focus on the slope of the medication treatment over 6 months
- Missingness in months 1-6 ranges from 24% to 38%

Experiment Based on Motivating Example

- We multiple imputed the WECare data using the predictive mean matching hot-deck approach of Siddique and Belin (2008).
- A *closeness parameter* k determines the size of the donor pool: As $k \to \infty$ this procedure amounts to a nearest-neighbor hot-deck where the donor whose predicted mean is closest to the donee is always chosen. Conversely, when k equals 0, each donor has equal probability of selection, which is equivalent to a simple random hot-deck.

Experiment Cont'd

We imputed the WECare data many times with the following ABBs.

1. An Ignorable ABB that treats donors as a priori equally likely
2. An Inverse-to-size ABB
3. A Proportional-to-size ABB
4. A Proportional-to-size-cubed ABB
5. A U-Shaped ABB
6. A Fishhook ABB
7. A Mixture ABB where a different ABB is implemented for each of the 5 imputed data sets.

Each ABB was imputed using closeness parameter values 0-10 (we only show for values of 2)

Mixture ABB

- The mixture ABB implements a different ABB for each dataset.
- Motivation is to average over the uncertainty regarding the missing data mechanism
- We will see that this approach provides better coverage than other ABB approaches
- For the WECare Experiment, the following ABBs were used in the mixture ABB
  1. An Inverse-to-size ABB (c=-1)
  2. An Ignorable ABB (c=0)
  3. A Proportional-to-size ABB (c=1)
  4. A Proportional-to-size-squared ABB (c=2)
  5. A Proportional-to-size-cubed ABB (c=3)

Table of Medication Intervention Slopes by ABB Type based on one replication with a closeness parameter value of 2

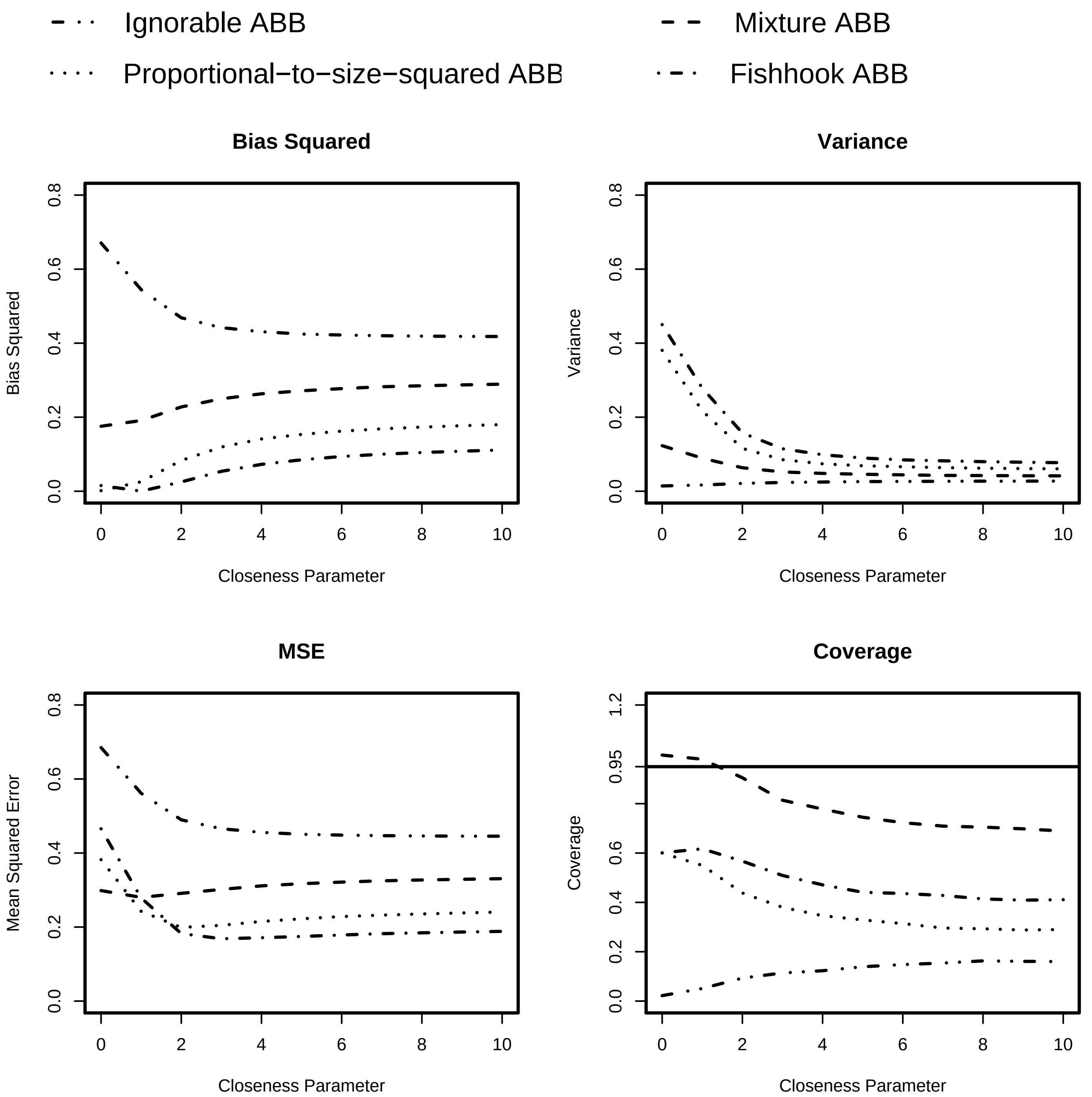
ABB Type	Estimate	Std. Error	t value	$Pr(> \|t\|)$
Ignorable	-2.18	0.47	-4.66	<0.0001
Inverse-to-size	-2.79	0.48	-5.86	<0.0001
Proportional-to-size	-1.83	0.45	-4.05	0.0002
Proportional-to-size-cubed	-1.42	0.43	-3.30	0.0012
U-Shaped	-1.22	0.74	-1.65	0.1319
Fishhook	-1.34	0.52	-2.61	0.0141
Mixture	-2.13	0.70	-3.04	0.0156

Simulation Study

- Bivariate normal-lognormal (Y_1, Y_2) data was simulated where $Y_1 \sim N(1.0, 0.13)$, $Y_2 \sim LN(1.7, 4.7)$, and $\text{corr}(Y_1, Y_2) = 0.54$. ($Y_2 = e^{Z_2}$ where $Z_2 \sim N(0, 1)$)
- 50% nonignorable missingness was applied to the lognormal variable (Y_2) so that larger values tended to be missing
- $N = 100$
- Hot-deck of Siddique and Belin (2008) was used to create five multiply imputed data sets
- Bias, variance, MSE, and coverage were calculated for the mean of Y_2
- 1000 replications

Simulation Study Cont'd

- Data were imputed using closeness parameter values between 0-10
- Investigate the effect of donor pool size on ABB performance
- Four different ABBs
  1. Nonignorable ABB
  2. Proportional-to-size-squared ABB
  3. Mixture ABB
  4. Fishhook ABB


Ignorable ABB
Mixture ABB
Proportional−to−size−squared ABB
Fishhook ABB
Bias Squared
Variance
MSE
Coverage
Bias Squared
Variance
Mean Squared Error
Coverage
Closeness Parameter
0.0
0.2
0.4
0.6
0.8
0.95
1.2
0
2
4
6
8
10

Implementation Guidelines

- Nonignorable ABB proved to be effective, but not all performed equally well
- Mixture ABB outperformed those that used the same ABB for each imputed dataset
- Committing to one ABB when the missing data mechanism is unknown may provide overly precise estimates
- Similarly, avoid strong nonignorability assumptions. I.e. ABB that chooses donors with probability proportional to donor size raised to 100 will always choose largest donor value=single imputation.

Implementation Guidelines Cont'd

- Recommendation is to use Mixture ABB approach
- Accounts for appropriate uncertainly
- Nominal coverage in simulation study
- Large closeness parameter values reduce effectiveness of Nonignorable ABB, use small values: 1 or 2
- Mixture ABB can be designed to favor larger values: $c = -1, 0, 1, 2, 3$, smaller values: $c = -3, -2, -1, 0, 1$ or just to add additional uncertainty: $c = -2, -1, 0, 1, 2$
- Can also incorporate U-Shaped and Fishhook ABBs if deemed plausible mechanisms

End

Extra Slides

Computation Details

For all nonignorable ABBs described above, when values of Y_{obs} are less than or equal to 0, the values of Y_{obs} need to be transformed to ensure that the selection probabilities in the nonignorable ABB are positive and (in the case where Y_{obs} are drawn with probability proportional to Y_{obs}^c, $c > 0$) that the selection probability for $y_i \in Y_{obs}$ is greater than the selection probability for $y_j \in Y_{obs}$ when $y_i > y_j$. Define α and β to be the smallest and second smallest values of Y_{obs} respectively where $\alpha \neq \beta$. Transform $y_i \in Y_{obs}$ using $y_i + |\alpha| + |\alpha - \beta|$. Then Equation 1 is rewritten as

$$\frac{(y_i + |\alpha| + |\alpha - \beta|)^c}{\sum_{j=1}^{n_{obs}} (y_j + |\alpha| + |\alpha - \beta|)^c}. \qquad (2)$$

Transformation only for calculating the selection probabilities. The original values of Y_{obs} are used for imputation.