Nonparametric Tests for Umbrella Alternative in a Mixed Design for a Known Peak

Boampong Adu Asare

Department of Engineering, United Tribes Technical College
Bismarck North Dakota

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Abstract

We proposed six nonparametric tests for testing the umbrella alternative with known peak when the data are mixtures of a randomized complete block design, a completely randomized design, and a balanced incomplete block design. The proposed tests consist of various combinations of the usual and modified Mack-Wolfe test, the usual and modified Kim-Kim test, and the usual and modified Magel-Ndungu test respectively.

The tests consist of the following combinations: Nonmodification MackWolfe test, nonmodification Kim-Kim test, and nonmodification Magel-Ndungu test, Distance (modified) Mack-Wolfe test, distance Kim-Kim test, and distance Magel-Ndungu test, Squared distance (modified) Mack-Wolfe test, squared distance KimKim test, and squared distance Magel-Ndungu test. The proposed test statistics are standardized.

Goal

Standardized the proposed tests. There are two types of standardized tests.

These are standardized first and standardized last.

Results of these standardized tests are compared to each other.

Introduction

- Nonparametric tests are methods of statistical analysis that do not require a distribution to meet the required assumptions to be analyzed especially if the data is not normally distributed. Due to this reason, they are sometimes referred to as distribution-free tests.
- The nonparametric procedures have few assumptions on the nature of the data. For instance, they are regarded to be computationally easier. This is desirable for most places where either access to sophisticated software is limited or there is a lack of trained personnel. More importantly, nonparametric procedures can deal with missing observations by offering different experiment designs and corresponding tests.

Introduction

- **Completely Randomized Design (CRD)**
- In a CRD subjects are randomly assigned to treatments. Each subject is exposed to only one treatment. The number of subjects assigned to each treatment does not have to be the same for all treatments. The treatment effect is then the difference among the groups formed by the treatments. The subject is an individual or object to which the treatment is applied. The effect is the difference in the outcome attributed to the treatment.

- **Randomized Complete Block Design (RCBD)**
- An RCBD introduces blocking to the CRD. Instead of randomly assigning subjects to treatments, an RCBD exposes each subject to all treatments. The order in which the treatment is exposed can be random though. Treatment effects are then measured within each block. Blocking is a technique where homogeneous subjects are grouped forming a block and effects are compared within that block. An RCBD is sometimes expensive since there cannot be missing observations

Introduction

- **Balanced Incomplete Block Design (BIBD)**
- A Balanced Incomplete Block Design is a form of Incomplete Block Design where missing observations form a balanced pattern. It introduces a fair comparison of treatments in the presence of missing observations

- **Incomplete Block Design (IBD)**
- This is where missing observations are selected at random and therefore a balanced pattern is not formed. In other words, an IBD does not allocate all treatments to every block.

Introduction-Mixed Design

- A researcher at times may wish to test for umbrella alternatives when a blocking factor is introduced. The researcher may use a Randomized Complete Block Design (RCBD) instead of a Completely Randomized Design (CRD). For example, in testing the effectiveness of a drug, a blocking factor could be the location and the experimental units are the patients. Blocking is a technique where homogeneous subjects are grouped forming a block and effects are compared within that block. Kim-Kim (1992) proposed a nonparametric procedure for testing umbrella alternatives for a randomized block design which is an extension of Mack-Wolfe (1981).

- In performing an experiment and conducting a test of the hypothesis, the researcher needs to decide on the design structure of the test to be used. Some researchers normally may begin with one experimental design. While the experiment is being implemented, it is realistic for the researcher to expect that some conditions around the experiment may change. Though the researcher may have started with one experimental design, the design may change before the completion of the experiment due to a variety of reasons.

Introduction-Mixed Design

- One such reasons is that a situation may arise when it may not be possible to continue the experiment using a full RCBD. When a researcher after a point realizes that the design is too expensive to continue or ends up with incomplete blocks due to participants dropping out, the researcher at this point may switch to a completely randomized design.

- In the example of the effectiveness of the drug, the researcher may switch from applying the drug in each location in the facility to randomly assigning drugs to patients or may continue conducting the remainder of the experiment by implementing the BIBD so that every treatment does not need to be applied to all observations within each block.

- The data obtained will then be organized in what is known as a mixture of RCBD, BIBD and CRD designs. For example, we have five locations that have five patients each, as in the first part which formed the RCBD portion. In the second part, we have five locations that have three patients each, which formed the BIBD portion. The third part has two patients for each drug. In Figure 3 below, P denotes Patient and X denotes no observation is taken

Introduction-Mixed Design

Layout for RCBD, BIBD and CRD

	Drug1	Drug2	Drug3	Drug4	Drug5	
Location1	P	P	P	P	P	RCBD Portion
Location2	P	P	P	P	P	
Location3	P	P	P	P	P	No replication
Location4	P	P	P	P	P	
Location5	P	P	P	P	P	
Location6	P	X	P	X	P	BIBD Portion
Location7	X	P	P	P	X	
Location8	P	P	X	X	P	
Location9	X	X	P	P	P	
Location10	P	P	X	P	X	
Location11	P	X	X	X	X	CRD Portion
Location12	X	X	X	P	X	
Location13	X	P	X	X	X	
Location14	X	X	X	X	P	
Location15	X	X	P	X	X	
Location16	X	X	X	X	P	
Location17	P	X	X	X	X	
Location18	X	X	P	X	X	
Location19	X	X	X	P	X	
Location20	X	P	X	X	X	

Layout for BIBD and CRD

	Drug1	Drug2	Drug3	Drug4	Drug5	
Location1	P	X	P	X	P	BIBD Portion
Location2	X	P	P	P	X	
Location3	P	P	X	X	P	
Location4	X	X	P	P	P	
Location5	P	P	X	P	X	
Location6	P	X	X	X	X	CRD Portion
Location7	X	X	X	P	X	
Location8	X	P	X	X	X	
Location9	X	X	X	X	P	
Location10	X	X	P	X	X	
Location11	X	X	X	X	P	
Location12	P	X	X	X	X	
Location13	X	X	P	X	X	
Location14	X	X	X	P	X	
Location15	X	P	X	X	X	

Introduction-Umbrella Alternative

- Many researchers in recent times have developed interest for the Umbrella Alternative particularly when it comes to testing the effect of a drug on animals.
- For instance, in dose-response studies, animals are assigned to k groups corresponding to k doses of an experimental drug. The effect of the drug on these animals is likely to increase or decrease with increasing and decreasing doses. The drug's effect on these animals may be an increasing function of dosage to a certain level called the peak or the turning point and then its effect decreases with further increasing doses. An umbrella alternative, in this case, is considered the most appropriate hypothesis for these kinds of studies.
- The null hypothesis in these instances is given by:
- $H_0: \mu_1 = \mu_2 = \cdots = \mu_k$ treatments effects assumed to be equal
- Versus alternative (1)
- $H_a: \mu_1 \leq \mu_2 \leq \cdots \leq \mu_{p-1} \leq \mu_p \geq \mu_{p+1} \geq \cdots \geq \mu_k$ with at least one strict inequality
- Where μ_i is the location parameter of the i^{th} sample, p is called the peak or the turning point.

Simulation

Studies in Monte Carlo simulation were conducted using SAS 9.4 to vary the design and to estimate the test statistic powers to each other. Three underlying distributions are included in the study. These are normal, exponential and t-distribution with 3 degrees of freedom With peak p, assumed to be known, three, four and five populations are considered.

For 3 treatment case, peak is at 2 For 4 treatments case, peaks considered are 2 and 3 For 5 treatments, 2, 3 and 4 peaks are considered Data are generated from a mixed design consisting of BIBD, CRD and RCBD. Equal sample sizes of 5, 6, 12, 10, 15 and 18 were considered for the completely randomized design portion.
A variety of location parameter configurations are considered for 3, 4 and 5 populations. Missing observations were created using the Uniform distribution for the IBD portion. Simulated observations were individually assigned a probability of missing by this call function RAND ('Uniform'). This produced a random number between zero and one from the Uniform distribution. The number of blocks for the balanced incomplete block design and randomized complete block design portions considered were equal, two thirds, twice and thrice the sample size for each treatment. The powers for all the tests are estimated based on 5,000 iterations. An estimate α level of 0.05 is used at the initial stage.

Methodology

Six nonparametric tests for testing the umbrella alternative with known peak when the data are mixtures of a randomized complete block design, a completely randomized design and a balanced incomplete block design were proposed. The proposed tests consist of various combinations of the usual and modified Mack-Wolfe test, the usual and modified Kim-Kim test, and the usual and modified Magel-Ndungu test respectively.

The tests consist of the following combinations;

Nonmodification Mack-Wolfe test, nonmodification Kim-Kim test, and nonmodification Magel-Ndungu test

Distance (modified) Mack-Wolfe test, distance Kim-Kim test, and distance Magel-Ndungu test,

Squared distance (modified) Mack-Wolfe test, squared distance Kim-Kim test, and squared distance Magel-Ndungu test.

The proposed test statistics are standardized. There are two types of standardized tests. These are standardized first and standardized last. Results of these standardized tests are compared to each other.

First proposed test for umbrella known peak. Non-Modification for three mixed designs: Mack-Wolfe (1981) for CRD, Kim-Kim (1992) for RCBD and Magel- Ndungu(2011) for (BIBD)

- **Standardized Version:**
- **First:** The first proposed KA_{mp} test is given by
- **Standardized** $$KA_{mp} = A_p^* + KA_n^* + M_n^* \quad (36)$$
- where A_p^* is the standardized version of the usual Mack-Wolfe test for CRD, KA_n^* is the standard version of the
- Kim and Kim (1992) for RCBD and M_n^* the standardized version of Magel-Ndungu (2011). Under H_0, A_p^*, KA_n^*, and
- M_n^* have an asymptotic standard normal distribution, thus the expected value and variance of KA_{mp} are given by
- $$E_0(KA_{mp}) = E_0(A_p^*) + E_0(KA_n^*) + (M_n^*) = 0 \quad (37)$$
- And
- $$var_0(KA_{mp}) = var_0(A_p^*) + var_0(KA_n^*) + var_0(M_n^*) = 1 + 1 + 1 = 3 \quad (38)$$
- The standardized version of the first proposed test is given by
- $$KA_{pm} = \frac{KA_{mp} - E_0(KA_{mp})}{\sqrt{var_0(KA_{mp})}} = \frac{KA_{mp} - 0}{\sqrt{3}} \quad (39)$$
- Under H_0, KA_{pm} has an asymptotic standard normal distribution. The null hypothesis is rejected when $KA_{pm} \geq Z_\alpha$

Second proposed test for umbrella known peak. Non-Modification for three mixed designs of (Mack-Wolfe (1981) for CRD, Kim-Kim (1992) for RCBD and Magel-Ndungu (2011) for BIBD

- Unstandardized Version:
- **Standardized Last:** The second proposed A_{mp}^{*} test is given by
- $$KA_{mp}^{*} = A_P + KA_n + M_n \qquad (40)$$
- where A_P is the usual Mack-Wolfe (1981) test for CRD, KA_n is the Kim and Kim (1992) test for RCBD and M_n is
- Magel-Ndungu (2011)test for BIBD. Under H_0, the expected value and variance of KA_{mp}^{*} are the sum of the means and
- variances for the Mack-Wolfe, Kim- Kim and Magel-Ndungu tests. They are given by
- $$E_0(KA_{mp}^{*}) = E_0(A_P) + E_0(KA_n) + E_0(M_n) \qquad (41)$$
- And
- $$var_0(KA_{mp}^{*}) = var_0(A_P) + var_0(KA_n) + var_0(M_n) \qquad (42)$$
- where $E_0(A_P)$, $E_0(KA_n), E_0(M_n), var_0(A_P), var_0(KA_n)$ and $var_0(M_n)$ are the expected values and variances of the
- usual Mack-Wolfe (1981) test for CRD, Kim and Kim (1992) for RCBD, and the Magel-Ndungu(2011) for BIBD respectively.
- The standardized version of the second proposed test is given by
- $$KA_{pm}^{*} = \frac{KA_{mp}^{*} - E_0(KA_{mp}^{*})}{\sqrt{var_0(KA_{mp}^{*})}} \qquad (43)$$
- Under H_0,KA_{pm}^{*} has an asymptotic standard normal distribution. The null hypothesis is rejected when $KA_{pm}^{*} \geq Z_\alpha$.

Significance

- To check the power of the test statistics in a mixed design having BIBD, IBD, RCBD, and CRD portions; IBD, RCBD, and CRD portions; RCBD and CRD portions; and BIBD and CRD portions by applying the modification to the Mack-Wolfe (1981), Kim-Kim (1992) and Magel-Ndungu (2011) test statistics.

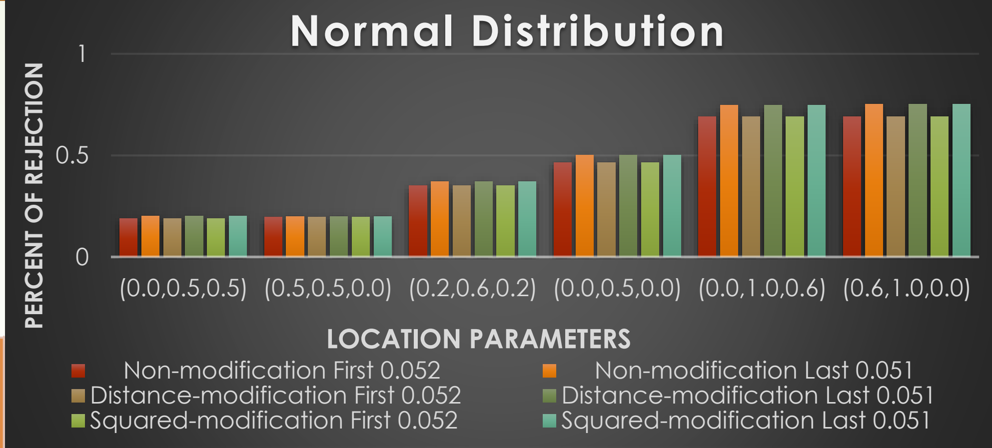
Results of 3 treatments with peak at 2: 6 BIBD, 6 CRD & 6 CRBD with one missing observation

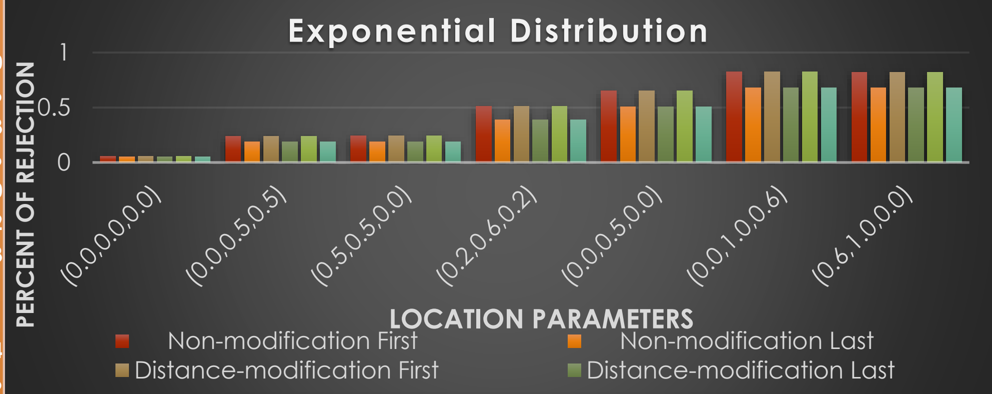
- With 3 treatments at peak 2, there were no difference between the results of the three modification tests(Non-modification, Distance modification, and Distance Squared Modification) for the mixed design of BIBD, CRD and RCBD.

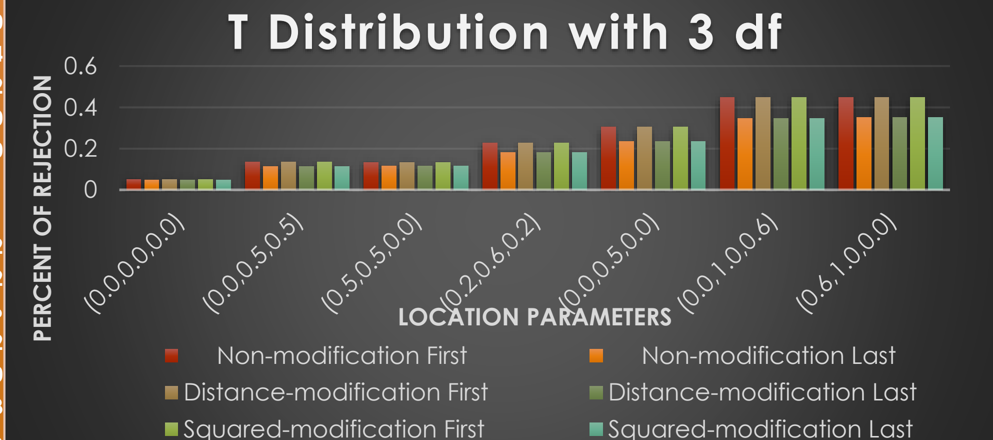
Location Parameters	Non-modification First	Non-modification Last	Distance-modification First	Distance-modification Last	Squared-modification First	Squared-modification Last
(0.0,0.0,0.0)	0.0494	0.0480	0.0494	0.0480	0.0494	0.0480
(0.0,0.5,0.5)	0.1698	0.1446	0.1698	0.1446	0.1698	0.1446
(0.5,0.5,0.0)	0.1752	0.1468	0.1752	0.1468	0.1752	0.1468
(0.2,0.6,0.2)	0.3024	0.2406	0.3024	0.2406	0.3024	0.2406
(0.0,0.5,0.0)	0.4066	0.3110	0.4066	0.3110	0.4066	0.3110
(0.0,1.0,0.6)	0.6014	0.4732	0.6014	0.4732	0.6014	0.4732
(0.6,1.0,0.0)	0.6084	0.4668	0.6084	0.4668	0.6084	0.4668

Location Parameters	Non-modification First	Non-modification Last	Distance-modification First	Distance-modification Last	Squared-modification First	Squared-modification Last
(0.0,0.0,0.0)	0.0548	0.0534	0.0548	0.0534	0.0548	0.0534
(0.0,0.5,0.5)	0.2416	0.1866	0.2416	0.1866	0.2416	0.1866
(0.5,0.5,0.0)	0.2444	0.1890	0.2444	0.1890	0.2444	0.1890
(0.2,0.6,0.2)	0.5104	0.3874	0.5104	0.3874	0.5104	0.3874
(0.0,0.5,0.0)	0.6520	0.5092	0.6520	0.5092	0.6520	0.5092
(0.0,1.0,0.6)	0.8242	0.6810	0.8242	0.6810	0.8242	0.6810
(0.6,1.0,0.0)	0.8214	0.6810	0.8214	0.6810	0.8214	0.6810

Location Parameters	Non-modification First	Non-modification Last	Distance-modification First	Distance-modification Last	Squared-modification First	Squared-modification Last
(0.0,0.0,0.0)	0.0514	0.0472	0.0514	0.0472	0.0514	0.0472
(0.0,0.5,0.5)	0.1354	0.1142	0.1354	0.1142	0.1354	0.1142
(0.5,0.5,0.0)	0.1338	0.1156	0.1338	0.1156	0.1338	0.1156
(0.2,0.6,0.2)	0.2276	0.1822	0.2276	0.1822	0.2276	0.1822
(0.0,0.5,0.0)	0.3046	0.2340	0.3046	0.2340	0.3046	0.2340
(0.0,1.0,0.6)	0.4488	0.3468	0.4488	0.3468	0.4488	0.3468
(0.6,1.0,0.0)	0.4480	0.3516	0.4480	0.3516	0.4480	0.3516



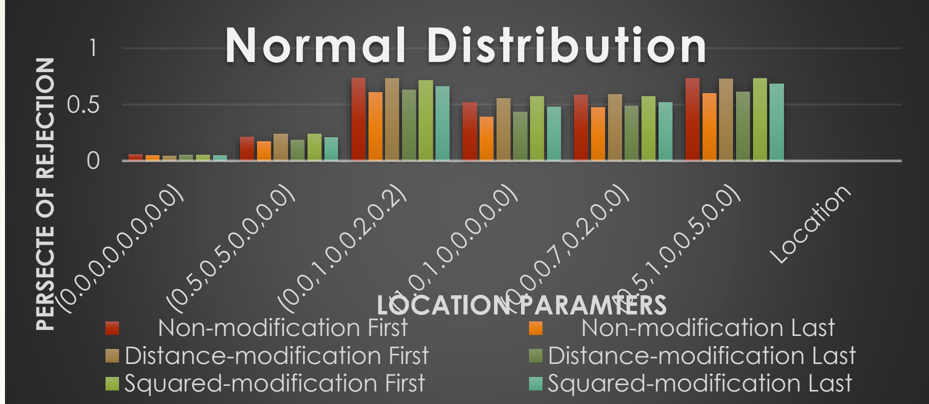


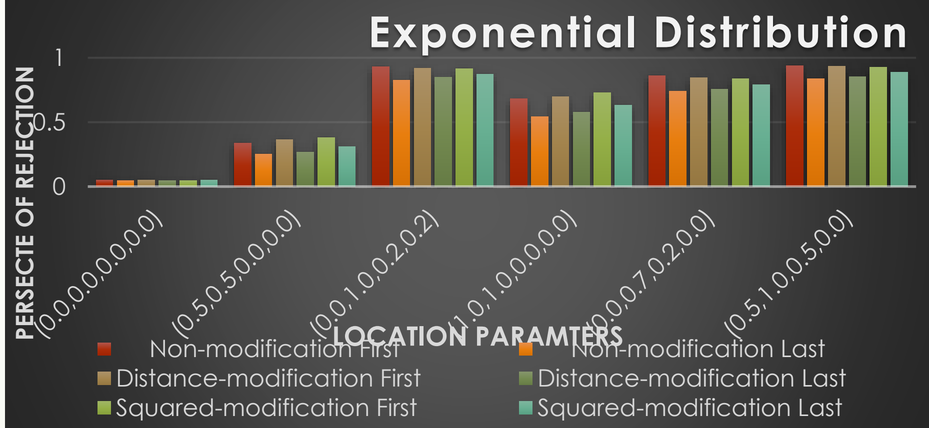


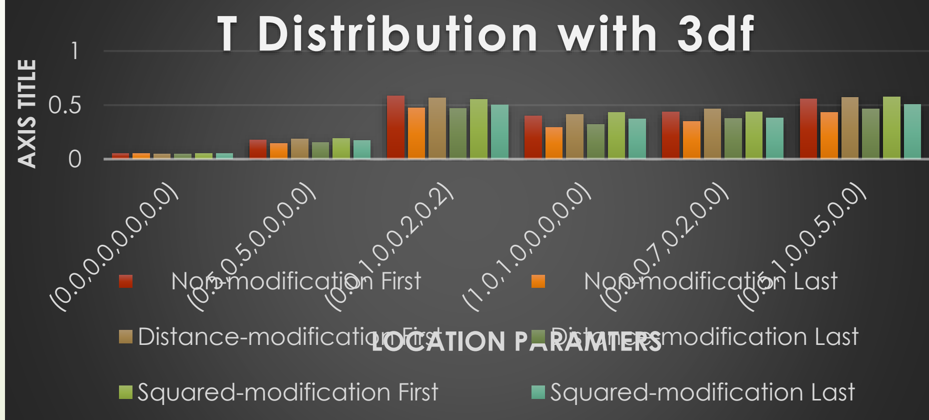
Results of 4 treatments with peak at 2: 6 BIBD, 6 CRD & 6 CRBD with two missing observations

With 4 treatments at peaks 2 and 3, and with 5 treatments at peaks 2, 3 and 4, generally, the results among the three modification tests vary from one configuration to the other and from one distribution to the other. Therefore, It was difficult to indicate which modification test provided the highest values of the estimated powers test statistics.

Location	Non-modification		Distance-modification		Squared-modification	
Parameters	First	Last	First	Last	First	Last
(0.0,0.0,0.0,0.0)	0.0556	0.0486	0.0428	0.0528	0.0528	0.0494
(0.5,0.5,0.0,0.0)	0.2132	0.1728	0.2384	0.1890	0.2396	0.2092
(0.0,1.0,0.2,0.2)	0.7358	0.6090	0.7346	0.6326	0.7180	0.6620
(1.0,1.0,0.0,0.0)	0.5178	0.3904	0.5564	0.4340	0.5708	0.4790
(0.0,0.7,0.2,0.0)	0.5870	0.4756	0.5902	0.4898	0.5732	0.5214
(0.5,1.0,0.5,0.0)	0.7312	0.5982	0.7282	0.6156	0.7312	0.6846
Location	Non-modification		Distance-modification		Squared-modification	
Parameters	First	Last	First	Last	First	Last
(0.0,0.0,0.0,0.0)	0.0534	0.0472	0.0532	0.047	0.0474	0.0528
(0.5,0.5,0.0,0.0)	0.3402	0.2544	0.3670	0.2698	0.3816	0.3102
(0.0,1.0,0.2,0.2)	0.9320	0.8280	0.9204	0.8486	0.9168	0.8748
(1.0,1.0,0.0,0.0)	0.6822	0.5432	0.6984	0.5794	0.7280	0.6332
(0.0,0.7,0.2,0.0)	0.8636	0.7402	0.8462	0.7582	0.8394	0.7912
(0.5,1.0,0.5,0.0)	0.9394	0.8396	0.9336	0.8530	0.9276	0.8884
Location	Non-modification		Distance-modification		Squared-modification	
Parameters	First	Last	First	Last	First	Last
(0.0,0.0,0.0,0.0)	0.0504	0.0530	0.0484	0.0470	0.0522	0.0504
(0.5,0.5,0.0,0.0)	0.1754	0.1476	0.1896	0.1554	0.1902	0.1732
(0.0,1.0,0.2,0.2)	0.5876	0.4766	0.5684	0.4692	0.5556	0.5010
(1.0,1.0,0.0,0.0)	0.4024	0.2948	0.4126	0.3224	0.4318	0.3706
(0.0,0.7,0.2,0.0)	0.4400	0.3498	0.4642	0.3770	0.4396	0.3822
(0.5,1.0,0.5,0.0)	0.5584	0.4306	0.5722	0.4638	0.5742	0.5044







Conclusion

Overall, in all the mixed designs except the mixed design of BIDB and CRD, it is

- recommended using the standardized first since it had the highest powers for the test statistics. This means that standardizing the sum of the standardized test statistics of Magel-Ndungu (2011), Mack-Wolfe (1981), and Kim-Kim (1992) was better than standardizing the sum of the unstandardized test statistics.
- In the case of the mixed design of BIDB and CRD, it is recommended using the standardized last when the sample size is n times higher than the number of blocks b in the BIBD portion since it provided the highest powers test statistics, where $n \geq 2$. This shows that standardizing the sum of the unstandardized test statistics of Magel-Ndungu (2011) and Mack-Wolfe (1981 had better results than standardizing the sum of the standardized test statistics.
- **Mixed Design of BIBD, CRD and RCBD Results**
- **For 3 treatments at peak two**, there were no difference between the results of the three modification tests(Non-modification, Distance modification, and Distance Squared Modification). The results were exactly the same.
- **With 4 treatments at peaks 2 and 3, and with 5 treatments at peaks at 2, 3 and 4,** generally, the results among the three distinct modification tests vary from one configuration to the other and from one distribution to the other. It was, therefore, difficult to indicate which modification test provided the highest values of the estimated powers test statistics.

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Future Work

- In future, for the known umbrella peak, one may consider the same mixed designs in this research study, but with higher sample size in the CRD portion by using different proportions between the number of blocks in the RCBD, BIBD, and IBD portions.
- Also, one may consider other designs such as a combination of CRD and incomplete block designs for the for the known umbrella peak and a combination between two or more mixed designs for the unknown umbrella peak.

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Thank you for your time!