Long-range dependence in low-frequency earthquake catalogs

Ariane Ducellier
University of Washington

1. Long-range dependence (LRD) [1, 2]

Autocorrelation: X_t is called a process with long-range dependence if there exists a real number $\delta \in (0, 1)$ and a constant c_ρ such that: $\lim_{\tau \to \infty} \frac{\rho_{X,\tau}}{c_\rho \tau^{-\delta}} = 1$ where $\rho_{X,\tau}$ is the autocorrelation sequence of X_t at time lag τ.

Spectral density: X_t is called a process with long-range dependence if there exists a real number $\delta \in (0, 1)$ and a constant c_S such that: $\lim_{f \to 0} \frac{S_X(f)}{c_S |f|^{-\delta}} = 1$ where $S_X(f)$ is the spectral density of X_t at frequency f.

2. Low-frequency earthquakes (LFEs)

- Small magnitude earthquakes ($M \sim 0 - 2$).
- Reduced amplitudes at frequencies greater than 10 Hz.
- Earthquake source located close to the plate interface.
- Grouped into families: All LFEs from a given family originate from the same small patch.
- Dozens of LFEs within a few hours or days, followed by weeks or months of quiet.

5. Summary

- LRD: Slow rate of decay of the statistical dependence between two points with increasing time interval between the points.
- Evidence of LRD in LFE catalogs.
- Earthquake occurrence times $\rightarrow$ Discrete time series = Number of earthquakes per unit of time.
- Graphical methods to estimate either the Hurst parameter H or the fractional differencing parameter d.

3. Estimators [3]

Aggregated time series: $X^{(m)}(k) = \frac{1}{m} \sum_{i=(k-1)m+1}^{km} X_i$

First absolute moment: $AM^{(m)} = \frac{m}{N} \sum_{k=1}^{\frac{N}{m}} \left| X^{(m)}(k) - \overline{X} \right|$

Sample variance: $\widehat{Var} X^{(m)} = \frac{m}{N} \sum_{k=1}^{\frac{N}{m}} \left(X^{(m)}(k) - \overline{X} \right)^2$

Average $VR^{(m)}$ over k of the sample variance of the residuals of the linear regression of the partial sums of the time series.

$$\frac{1}{m} \sum_{t=1}^{m} \left(Y_k^{(m)}(t) - a - bt \right)^2 \text{ and } Y_k^{(m)}(t) = \sum_{i=(k-1)m+1}^{(k-1)m+t} X_i$$

R/S statistics: $R/S_k^{(n)} = \frac{R_k^{(n)}}{S_k^{(n)}}, k = 0, \cdots, K-1$ with:

$$R_k^{(n)} = \max_{1 \leq t \leq n} \left[Y_k(t) - \frac{t-1}{n} Y_k(n) \right] - \min_{1 \leq t \leq n} \left[Y_k(t) - \frac{t-1}{n} Y_k(n) \right]$$

$S_k^{(n)}$ = Square root of sample variance of $X(i)$ and $Y_k(t) = \sum_{i=\left[\frac{N}{K}\right]k+1}^{\left[\frac{N}{K}\right]k+t} X_i$

Periodogram: $I(f) = \frac{1}{2\pi N} \left| \sum_{t=1}^{N} X_t e^{itf} \right|^2$ where f is the frequency.

Asymptotic behavior of the graphical estimators

Estimator	Asymptotic behavior	For
$AM^{(m)}$	m^{H-1}	Large m
$\widehat{Var} X^{(m)}$	m^{2d-1}	Large N/m and m
$VR^{(m)}$	m^{2d+1}	Large m
$R/S_k^{(n)}$	$n^{d+\frac{1}{2}}$	Large n
$I(f)$	$\lvert f \rvert^{-2d}$	$\nu \to 0$

4. Results

Distribution of the value of H or d for all the time series in each LFE catalog for the five methods of estimation. For better comparison between the distributions of H and d, I plotted $H - 0.5$ instead of H.

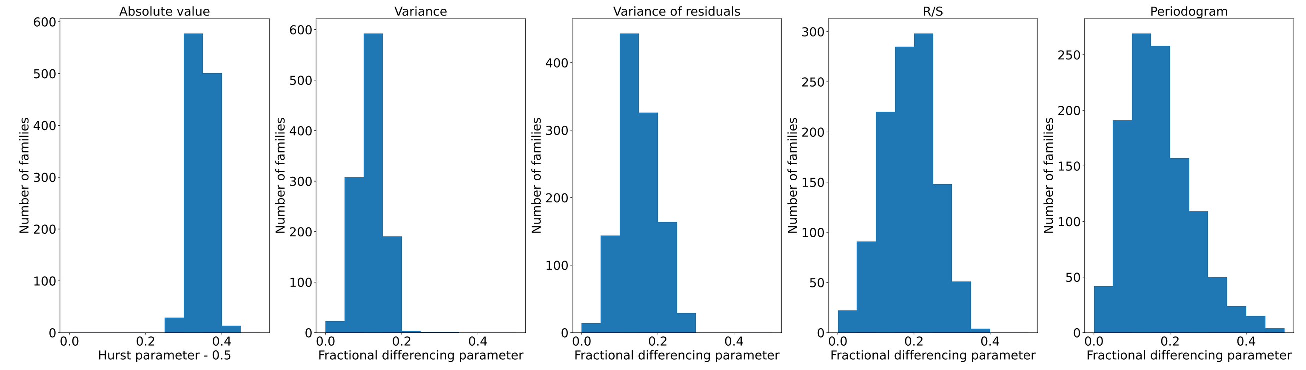

Figure 1: 1120 time series from the LFE catalog of Frank *et al.* (2014) in Mexico.

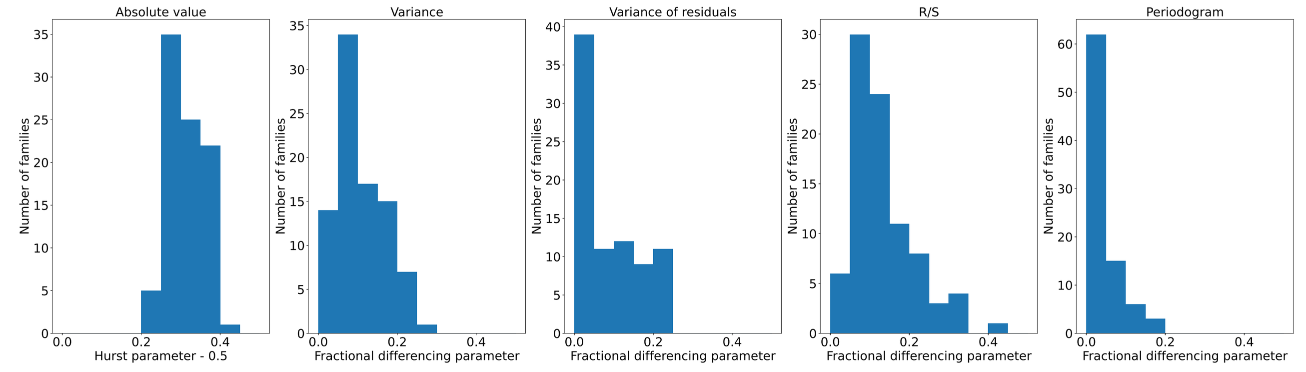

Figure 2: 88 time series from the LFE catalog of Shelly (2017) in the San Andreas Fault.

6. References and Acknowledgements

ariane.ducellier.pro@gmail.com

I would like to thank William Frank and David Shelly for providing their catalogs of low-frequency earthquakes. I also would like to thank Pr Donald Percival for introducing me to long-range dependence during his excellent class on wavelets at University of Washington. This work was funded by the grant from the National Science Foundation EAR-1358512.

[1] J. Beran. *Statistics for Long-Memory Processes*. Chapman and Hall, New York, NY, USA, 1994.

[2] D. Percival and A. Walden. *Wavelet Methods for Time Series Analysis*. Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press, New York, NY, USA, 2000.

[3] M. Taqqu and V. Teverovsky. On estimating the intensity of long-range dependence in finite and infinite variance time series. In R. J. Adler, R. E. Feldman, and M. S. Taqqu, editors, *A Practical Guide to Heavy Tails: Statistical Techniques and Applications*. Birkhäuser, Boston, MA, USA, 1998.