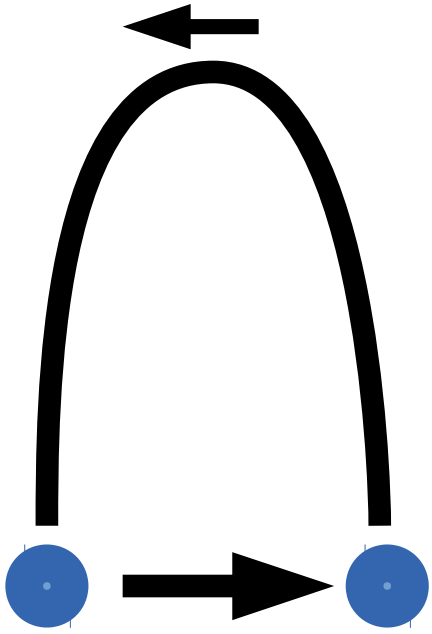
Juggling: Introducing the theory

When juggling two, the most common technique is to throw high from one hand, catch in the other, then pass back to the first hand to throw again. This is simple to understand, and fairly simple to do with just two.

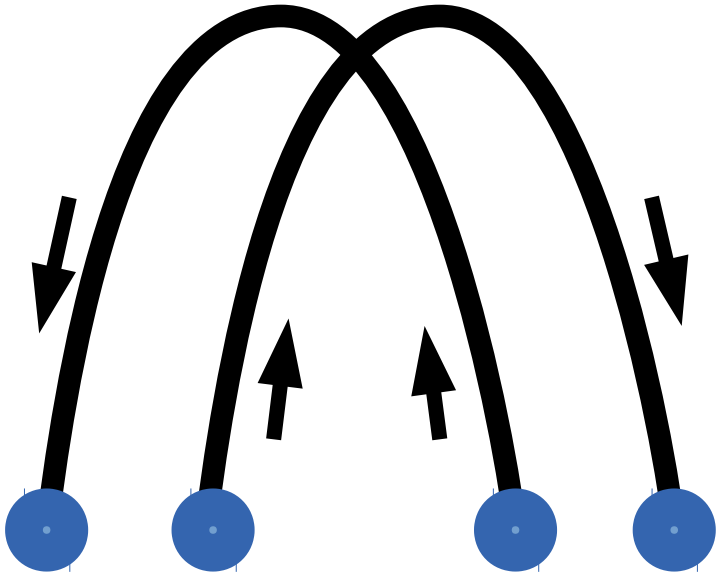


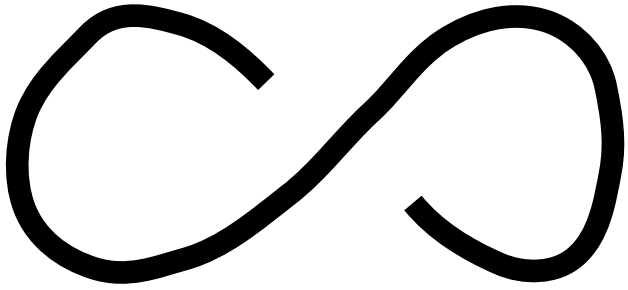
If you try it, though, it's guaranteed that you'll do the throwing with your good hand. It's remarkably difficult to do it the other way round. This is the secret of juggling, though:

It's the throw that matters.

Juggling three the easy way ...

However, the easy way to juggle three is not in that cyclic pattern. Instead, imagine holding one in each hand, with one coming down. As it comes down, throw the one you already have. Throw from the middle, catch on the outside, and you will end up with this double-arch pattern. You can almost see it as a figure-of-eight on its side, or as a twisted circle:

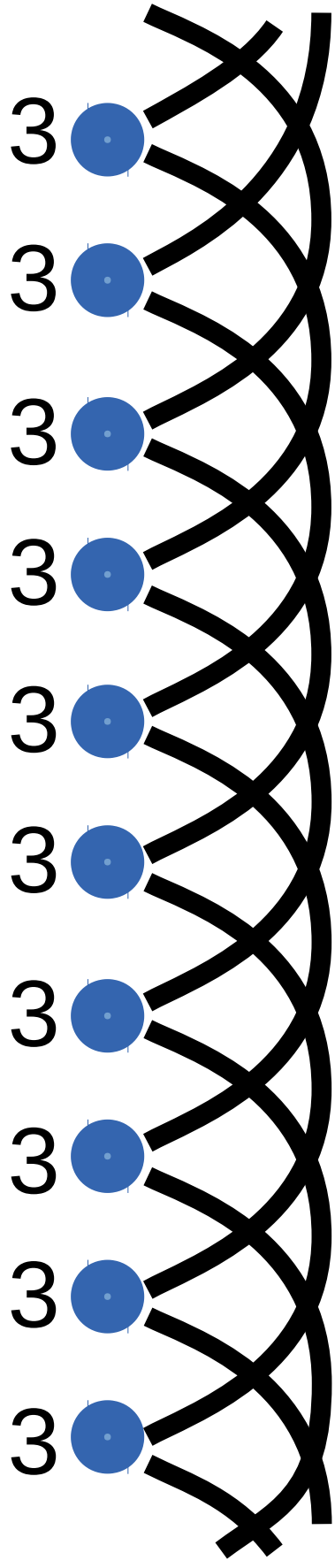




So here's what happens when we do “3” throws constantly. Each ball goes forward three places to be thrown again three beats later. The balls all take it in turns, coming down in the same order as they were thrown.

But as we've seen in the video, that doesn't have to be the case. An interesting thing to ask is just how many variations we can find.

So we draw the same chart, but remove some of the “3” throws, and ask what can replace them.

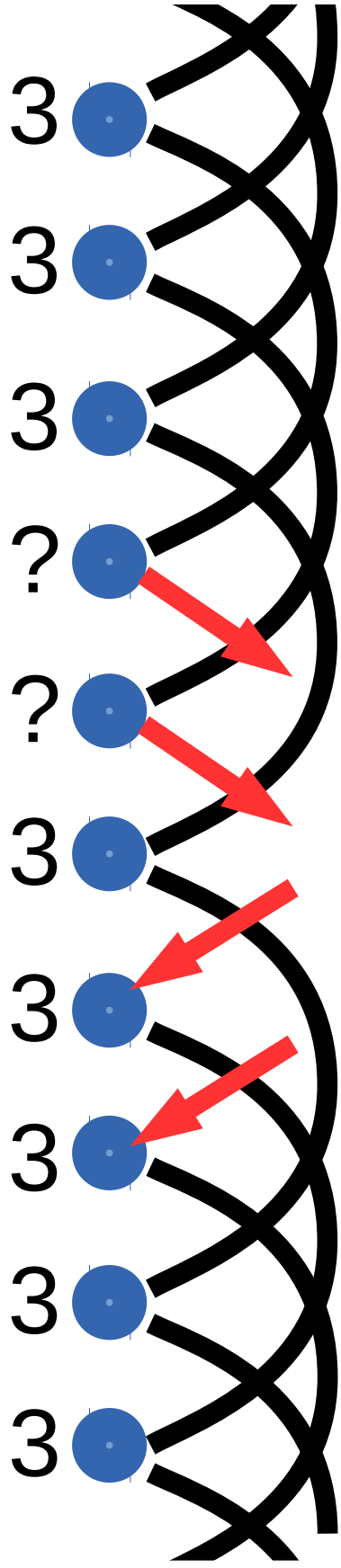


On this diagram we've got three balls and we've started juggling in the standard three ball cascade. Then we've taken out two of the throws, and we ask, what could we do instead?

Obviously the two blank spots could be “3”s, but what other option (or options!) might there be?

Well, with two balls in the air and two landings places, there are only two possible orderings for the balls to come down. Check them out, and you get either “3 3” or “4 2”.

Do you get that?



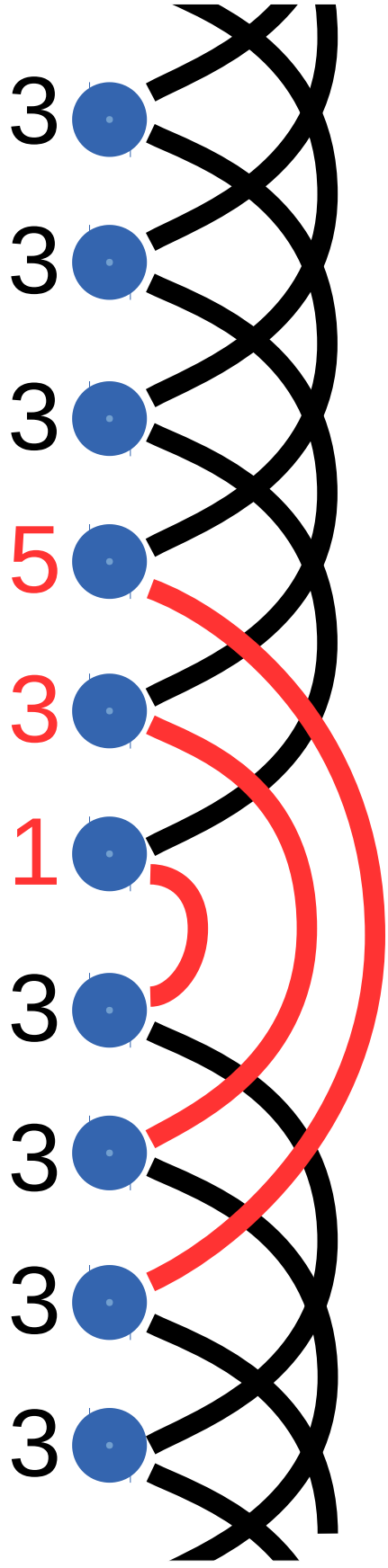
Here's one of the examples where we have three balls changing places. The first one thrown comes in last, the last one thrown comes in first, and the middle one is left as it always was.

This particular sequence is notoriously difficult, and when done constantly as:

“... 5 3 1 5 3 1 5 3 1 ...”

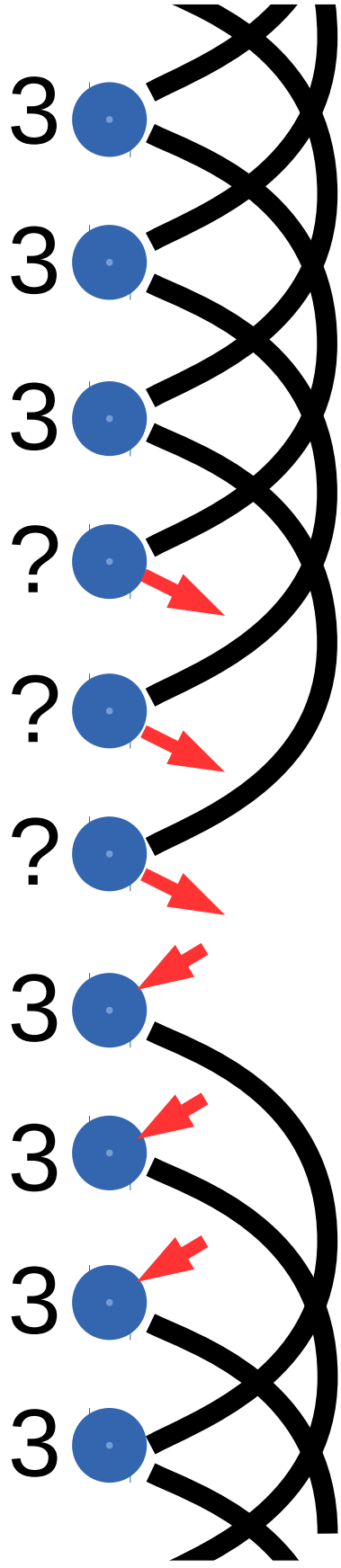
We get a ball circling in one direction, a ball circling in the other direction, and one ball bobbing back and forth in the classic cascade.

Very difficult.



So now it starts to get more interesting. We have three launches, and we ask how many different ways there are of landing. As before, they could all come down in the same order, and if we join them up like that then we get “3 3 3”. That's reassuring, but not very interesting. We can also get the first two swapping and the third coming down third. That gives us “4 2 3”.

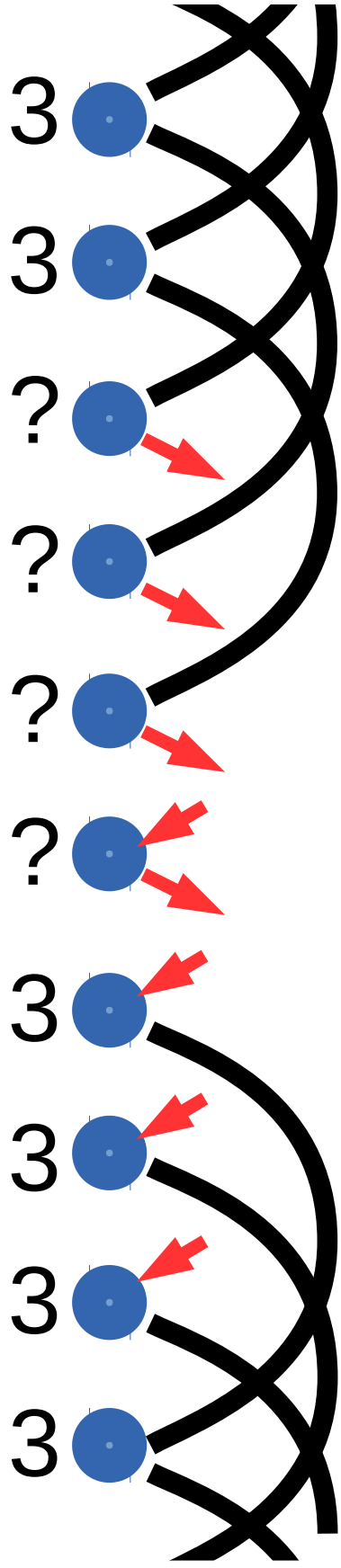
But there are other options – six options in total, including the ones already mentioned. Can you find them all?



And so we continue – we can remove another “3” and look at the options. We now have four launches for which we can choose our landing places. It even looks like we can launch and land at the same time!

What does that even mean? Have a look at “4 4 4 0”.

One of the videos will explain. There are now 24 possible landing sequences including “3 3 3 3”, “4 2 3 3”, “3 4 2 3”, “3 3 4 2”, “4 2 4 2”, and 19 more. If you want to find them all you'll need a methodical way to list them.



How to create juggling tricks

It turns out that the critical thing in all this is that you don't have two balls coming down at the same time. If we have a sequence of numbers that avoids that then it should be a juggling sequence. We can deliberately create such a thing. We start with the landing times, which must all be different. Then we subtract off the time of the throw to get the sequence we want.

3	1	4	2	: Landing time
1	2	3	4	: Launch time
2	-1	1	-2	: Variation
3	3	3	3	: Number of balls
5	2	4	1	: Juggling pattern

Can you draw the bouncing ball diagram?

A final mystery ...

So we've seen how to draw a diagram of juggling tricks, just like the one here at right of "... 5 1 5 1 5 1 ...".

We've also seen how to use a table to create juggleable sequences. The problem is, the table does not generate the valid trick "... 5 1 5 1 ..."

The trick is valid, the table works, why does the table not create the trick? The clue is that "5 1" is "excited" and can't be done directly from a three ball cascade. Try diagramming "... 3 3 3 3 5 1 ..." and you'll see what happens.

The table's not enough.