

## Arctic Sea Ice and Linear Equations Teacher Guide

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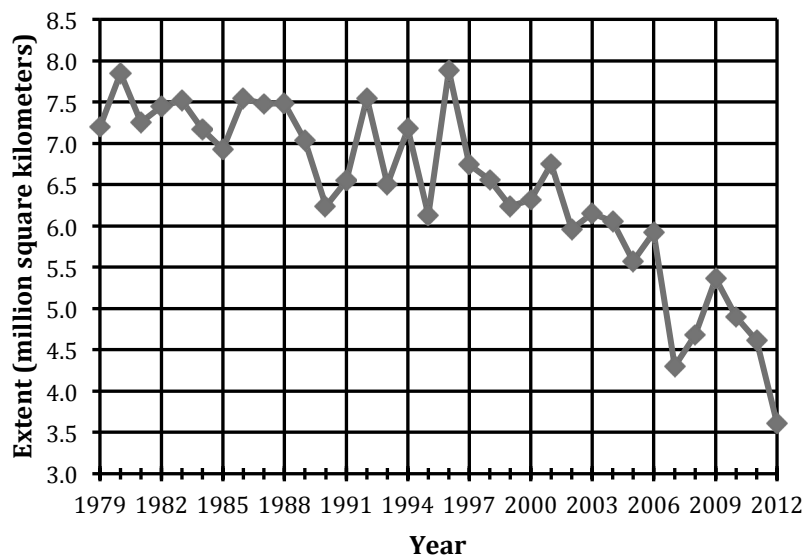
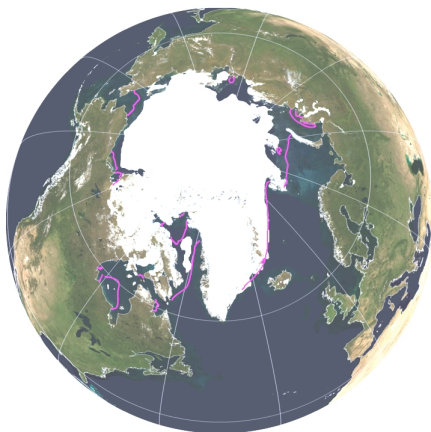


Figure 1: Arctic Sea Ice

Figure 2: Average September Arctic Sea Ice Extent, 1979 to 2012

Lesson Overview: We examine the extent of sea ice in the Arctic (see Figure 1) using images from NASA and data from the National Snow and Ice Data Center, U.C., Boulder [1] (see Figure 2). Students will use a line of best fit to model this data and then use the mathematical model to predict future sea ice levels. Specifically they will predict when the Arctic will start becoming ice free.

Level: Algebra 1 and above.

Common Core State Standards for Mathematics [2]: Standard for Mathematical Practice 4 - Model with mathematics and High School: Algebra >> Creating Equations standards.

Associated Materials: The spreadsheet Sea Ice Data contains the data used to create the graph as well as the linear fit to the data.

### Mathematical Content:

1. Graphs. Graphs provide information about the real world. In this case they show the average extent of the Arctic sea ice in September (measured in millions of square kilometers) as a function of time (measured in years). Graphs can be generated by data that is given in a table.
2. Line of best fit. Given a set of data points, one can find a line that fits the data as well as mathematically possible.
3. Mathematical modeling. The line of best fit is described by a formula or function. This function provides what mathematicians call a mathematical model of the extent of sea ice. One can use a mathematical model to predict the future – in this case the future extent of the Arctic sea ice.
4. Properties of graphs. Graphs can be increasing or decreasing and be concave up or down. These properties have important contextual implications.
5. Slope and rate of change. The slope of a line can be interpreted as the rate of change of the function.
6. Units. Units are important in the mathematics of real world problems and keeping track of units can help one better understand mathematical concepts. The units of rate of change in this problem are million  $\text{km}^2$  per year = million  $\text{km}^2/\text{year}$  which has a crucial meaning in the context of the problem. It tells how much less ice there will be in each successive year.

### Sustainability Content:

European explorers, beginning with Cabot's 1497 attempt to sail to the Orient from England, searched for the *Northwest Passage*, a route through the Arctic Ocean along the coast of Canada. See Figure 1. The Norwegian explorer Roald Amundsen was the first to complete the journey, though it took from 1903 to 1906. In 1957, the U.S. Coast Guard Cutter *Storis* became the first U.S. vessel to circumnavigate the North American continent, a 22,000 mile trek.

The problem is the Arctic Ocean is covered by a sea ice pack nearly all the time---the passage is closed. Since the beginning of the Industrial Revolution, global temperature averages have risen overall causing more of the ice pack to melt in the summer. NASA's *National Snow and Ice Data Center* at the University of Colorado, Boulder, has collected data provided by satellites, overflights, submarines, and other observations measuring the amount of sea ice in the Arctic Ocean for several decades. The amount of ice is at a minimum in September, the end of summer. Figure 2 shows the September average extent of total Arctic sea ice area in millions of square kilometers versus the year from 1979 to 2011. Clearly the extent of ice is decreasing.

Melting sea ice provides an example of a positive feedback loop (also called non-linear feedback). As the ice melts, it leaves more ocean open. Ice is very reflective giving the arctic region a high *albedo*; ice reflects up to 70% of the sun's energy. The ocean is darker, reflecting only 6% of the sun's energy, so as the ice pack retreats, the area's albedo gets lower. More energy is absorbed by ocean water than by sea ice increasing the temperature, causing more ice to melt leading to more open water, creating a positive feedback loop.

## Lesson Plan Notes

This lesson can be taught in a variety of ways ranging from a teacher centered presentation to having students working through the assignment on their own or in groups and then presenting their findings to the class.

0. Lesson launch. Engage students in a discussion of Arctic sea ice. What do students know about this? What questions do they have? Students might know about the [soda company initiative to save polar bears](#). Show some pictures or a short video about the issue (for example, the video [A New Climate State: Arctic Sea Ice 2012 \[3\]](#)). One might show some of the video at the start of class as an introduction and more at the end as a wrap up.
1. The Graph. Understanding and paying attention to the units is important. Often in math problems, there are no units. So students do not always pay attention when there are units. The horizontal axis measures time in years. When answering the questions in the lesson, the students should always include the units. The vertical axis gives the average extent of Arctic sea ice in September. The units are square kilometers measured in millions (million km<sup>2</sup>).

An extension question, would be:

Convert your answer from units of km<sup>2</sup> into units of miles<sup>2</sup> using the conversion factor 1 mile = 1.6 km.

Students are likely to forget that that they will need to square the conversion factor since they are dealing with area:

$$\# \text{ km}^2 \times \left( \frac{1 \text{ mile}}{1.6 \text{ km}} \right)^2$$

2. Finding the line of best fit. When doing this by hand, there will be a lot of variation in the answers. As long as the students come up with something reasonable, that is fine. One can have an animated discussion with students explaining why they drew the line they did and seeing the variation among the results. This would provide a teachable moment for the need to have a precise definition of line of best fit.
3. Slope of line. The students will need to calculate the slope from the drawing by calculating  $m = \frac{\Delta y}{\Delta t}$ . The units for slope are given by the units for the ratio  $m = \frac{\Delta y}{\Delta t} = \frac{\text{millions km}^2}{\text{year}}$ . Many students are not used to dealing with slope in the context of units and could find this question challenging.
4. In Excel, we used the Chart – Add Trendline feature and found that the line of best fit is given by  $y = -0.0921 t + 190.12$ .

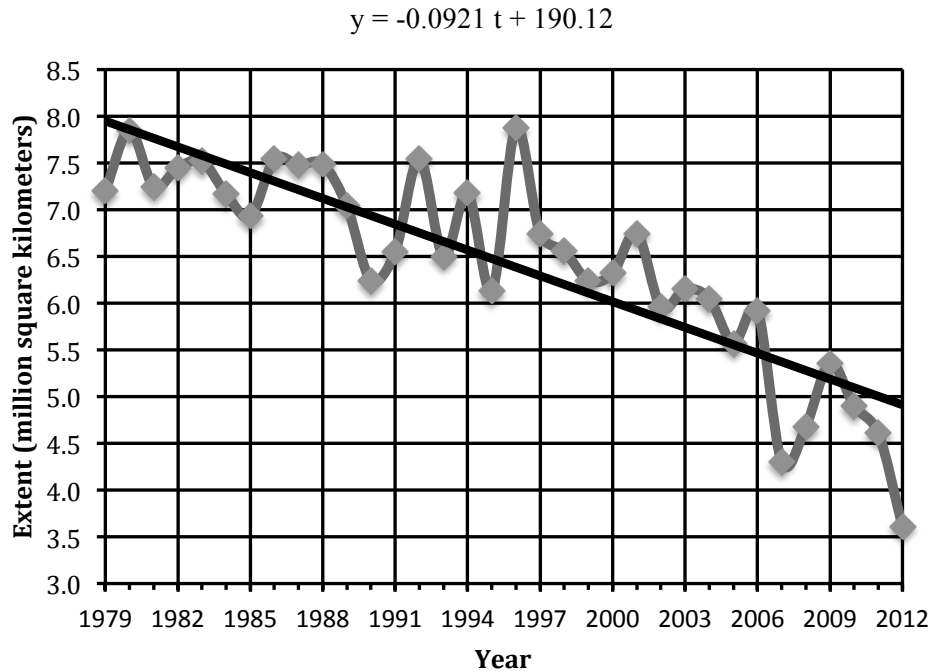


Figure 3: Best fit curve to Average September Arctic Sea Ice Extent

In 2003, the line gives  $y = -0.0921 (2003) + 190.12 = 5.64$  million  $\text{km}^2$  and in 2012,  $y = -0.0921 (2012) + 190.12 = 4.81$  million  $\text{km}^2$ .

5 - 6. The slope is  $m = -0.0921$ , approximately  $-0.09$ , with units of million  $\text{km}^2$  per year. This means that each year, the amount of sea ice in September will be  $0.09$  million  $\text{km}^2$  less than the year before.

7. Use mathematics to predict the future: Using the model students need to find  $y$  when  $t = 2020$ :

$$y = -0.0921 (2020) + 190.12 = 4.08 \text{ million } \text{km}^2.$$

To determine when the Arctic will be ice free, the students must solve the equation

$$y = -0.0921 t + 190.12 = 0$$

which gives  $t = 2064$ .

8. The data in the last three years are all below the line of best fit, with the most recent values of 2012 being significantly below the line. So the best fit line could be a poor predictor of the future because it weights earlier years equally with recent years. But the recent years suggest that the behavior of the sea ice is changing. The positive feedback loop might be accelerating the melting. The problem of sea ice melt is very complex due to the non-linear feedbacks involved; the scientific community has not reached consensus on when they think the ice free stage will arrive [4].

9. Some pros of having an ice free passage through the Arctic sea: it will make shipping through the Northwest Passage possible, which will reduce cost and travel time for

goods. It might also give access to more natural resources. Cons include that will make it difficult for polar bears to survive. And it is an indicator that serious climate changes are taking place on the earth and these changes will have far reaching consequences.

#### Extension Questions:

1. Using the data for just the past several years (ex. 2009-2012), predict what the extent of Arctic sea ice will be in 2013 and 2014 and predict when the Arctic will be ice free in September. Using just the more recent data will lead to a significantly shorter estimate.
2. In Figure 4 (see next page) we give the extent of Arctic sea ice in March when the ice extent is near its maximum. In Figure 5, we give the linear best fit. Note that here we the units for time are years after 1970. For this second set of data, have the students repeat questions 1 – 8. The students can then discuss the differences in their predictions for the two different times of year.

#### References:

1. National Snow and Ice Data Center, U.C., Boulder, [at http://nsidc.org](http://nsidc.org).
2. Common Core Standards at <http://www.corestandards.org/Math> ;  
<http://www.corestandards.org/Math/Practice/MP4> ;  
<http://www.corestandards.org/Math/Content/HSA/CED>
3. *A New Climate State: Arctic Sea Ice 2012*, at  
[http://www.youtube.com/watch?v=ZYaubXBfVqo&feature=player\\_embedded](http://www.youtube.com/watch?v=ZYaubXBfVqo&feature=player_embedded)
4. When will the Arctic be ice-free in the summer? Maybe four years. Or 40. Brad Plume at Wonkblog, <http://www.washingtonpost.com/blogs/wonkblog/wp/2012/09/20/when-will-the-arctic-be-ice-free-maybe-four-years-or-40/>

#### Acknowledgements:

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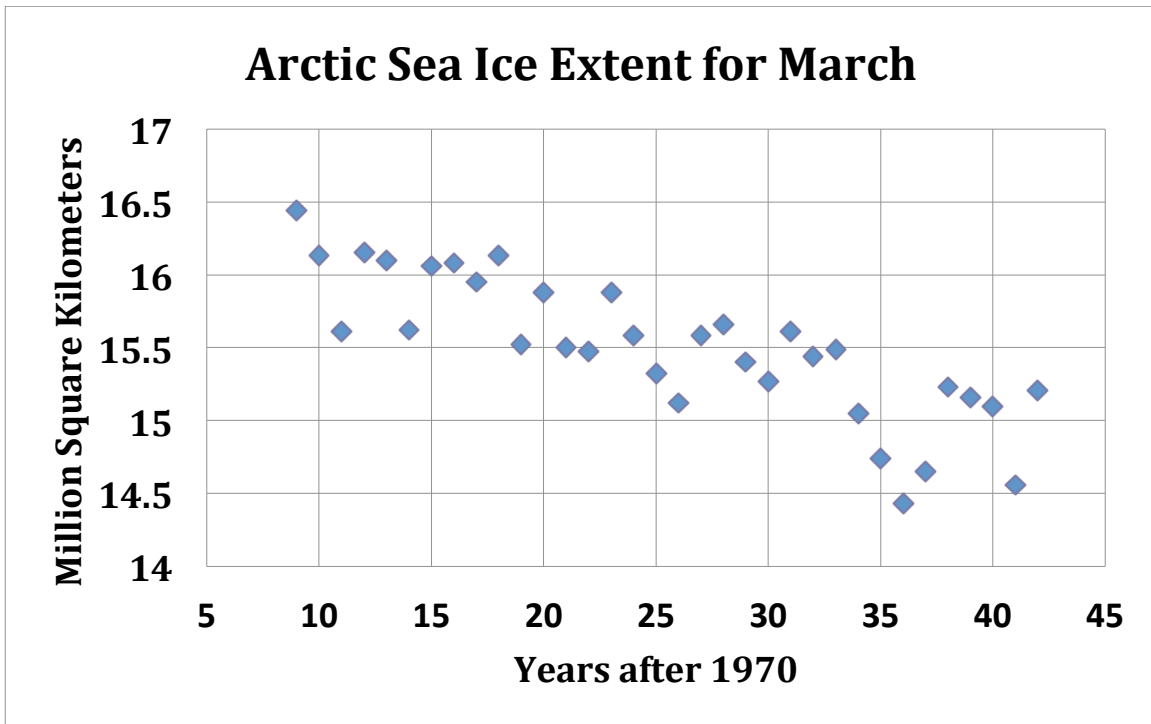


Figure 4: Average March Arctic Sea Ice Extent, 1979 to 2012.

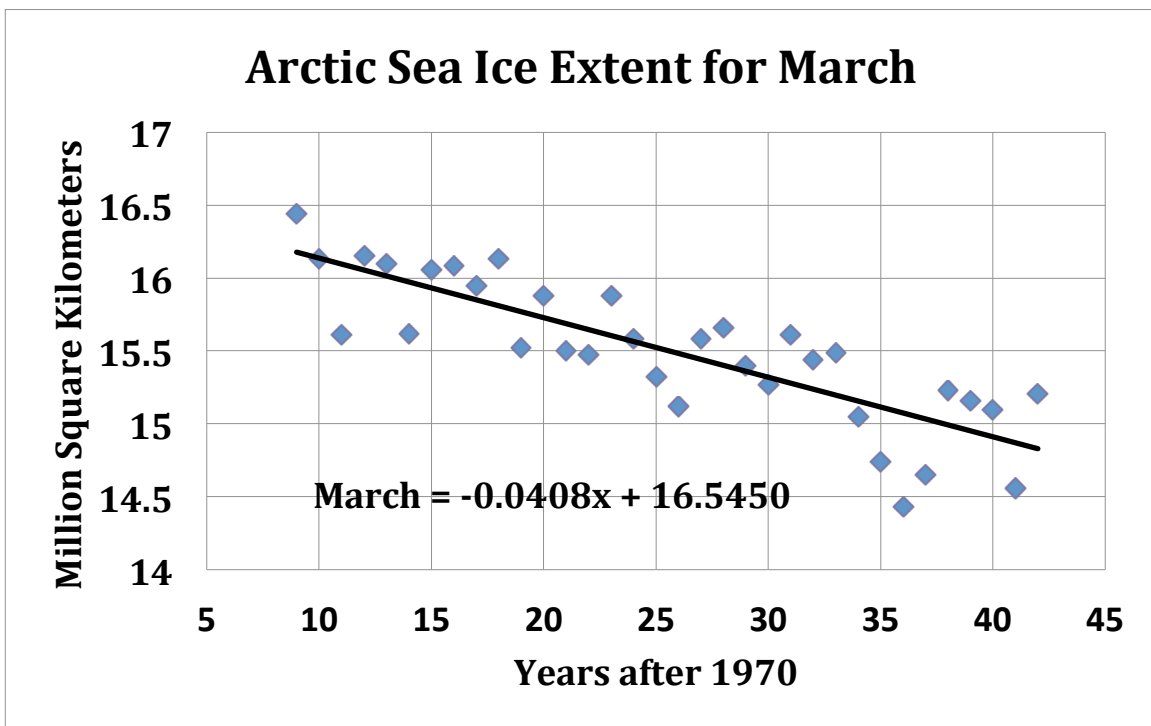


Figure 5: Best fit curve to Average March Arctic Sea Ice Extent.