



Math and Global Temperature

Mary Lou Zeeman
Bowdoin College
&
Math and Climate
Research Network



Thanks to:
**many friends
& colleagues**



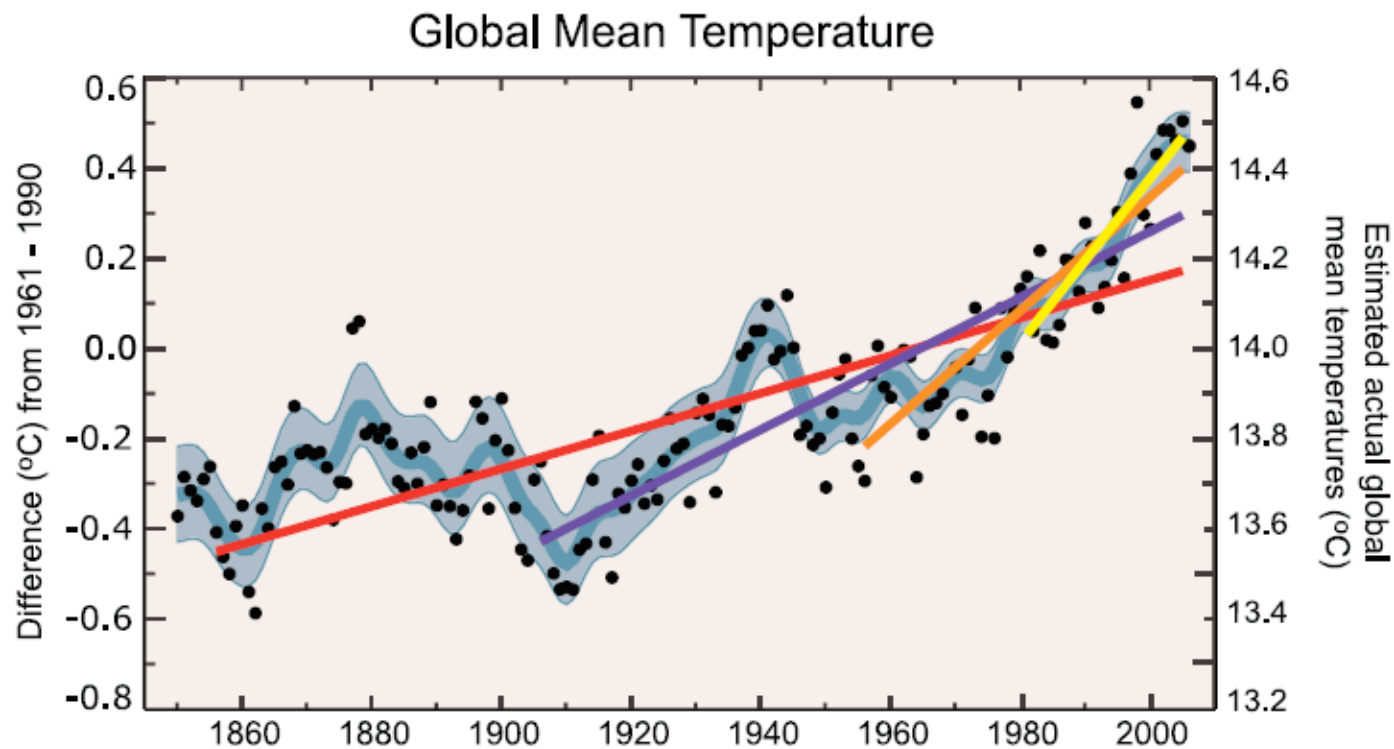
&

Bowdoin



Evidence of a Changing Climate

Intergovernmental Panel on Climate Change (IPCC):
Warming is “unequivocal”



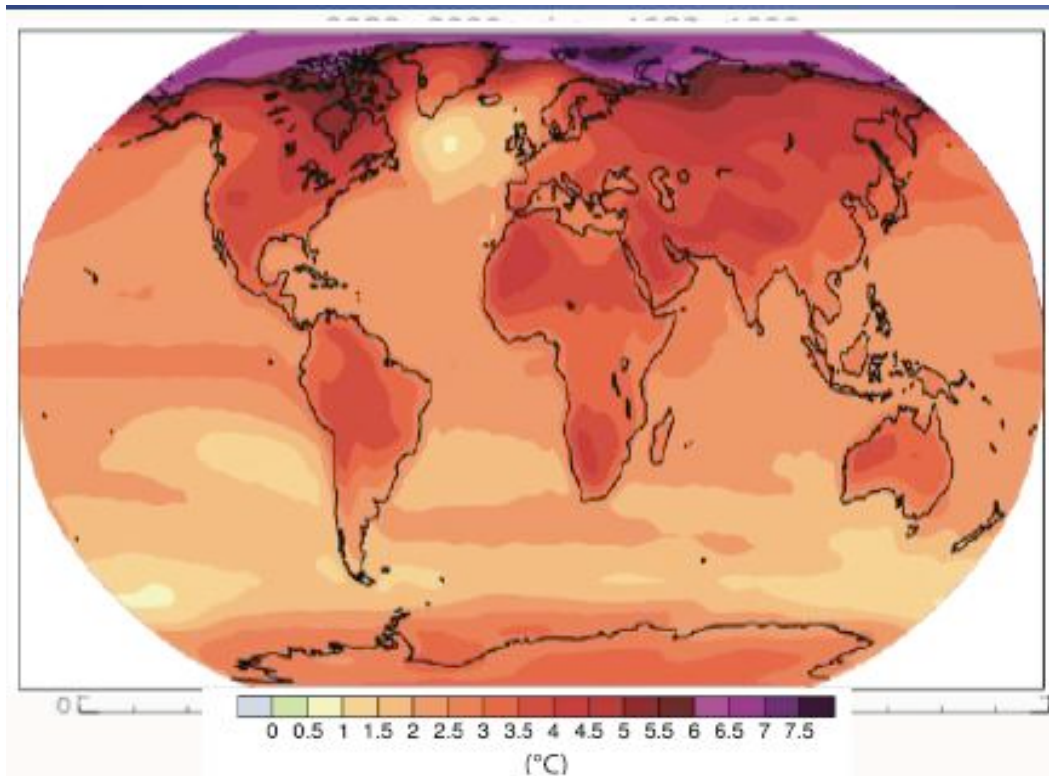
Dots: yearly average

Curve: decadal average

Blue: uncertainty interval



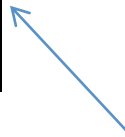
Why do we try to predict?



Warming is NOT uniform

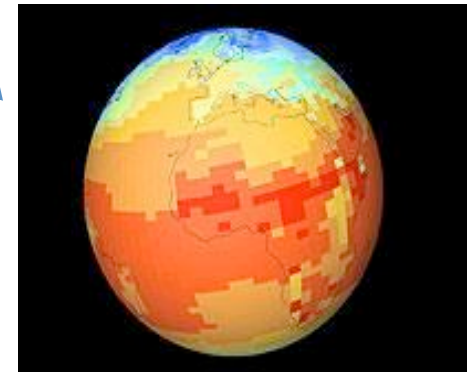


Why use Mathematical Models?



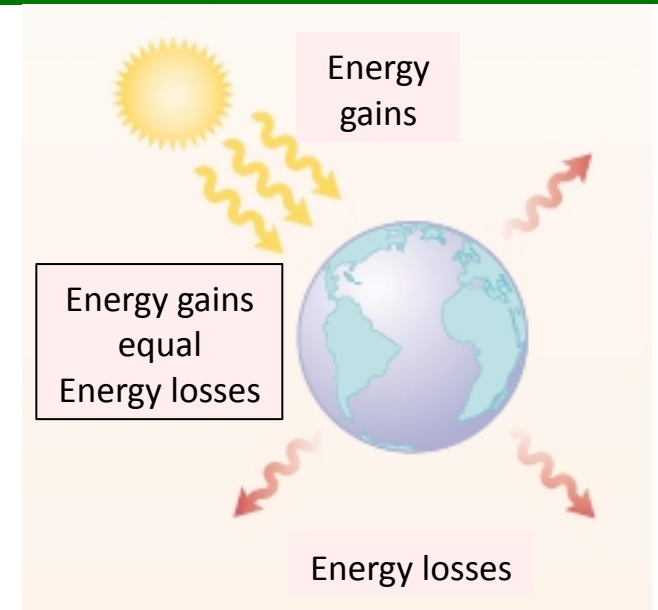
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many





Simplest Energy Balance Model

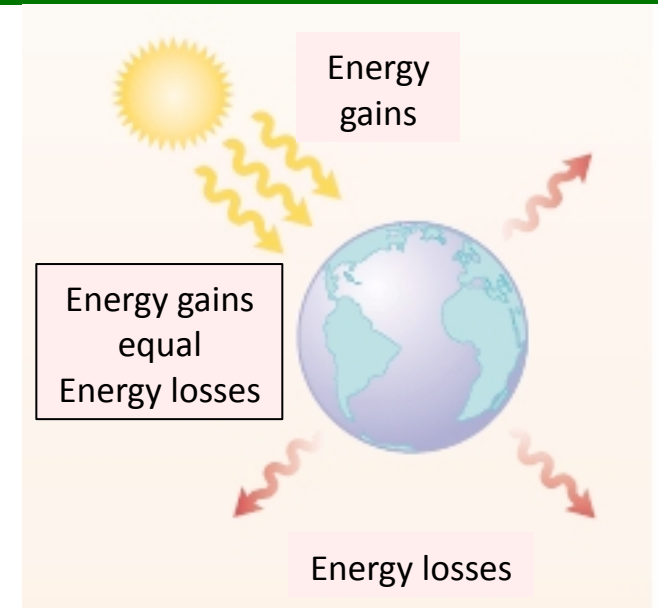


CIMSS/ U Wisc



Simplest Energy Balance Model

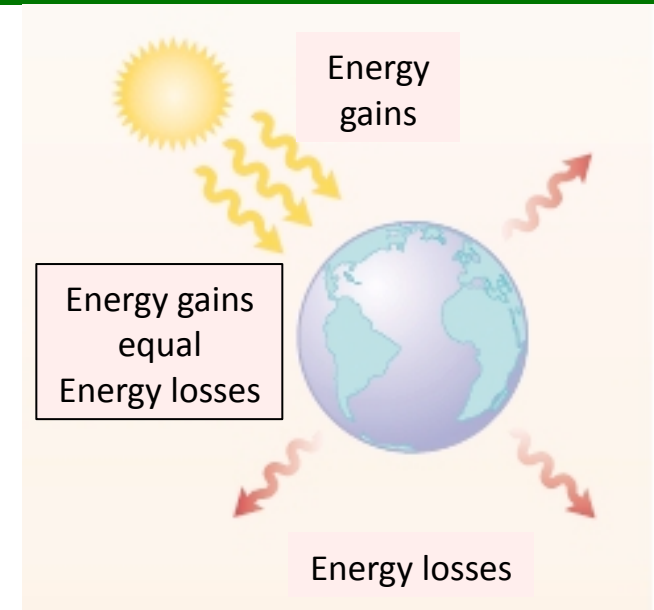
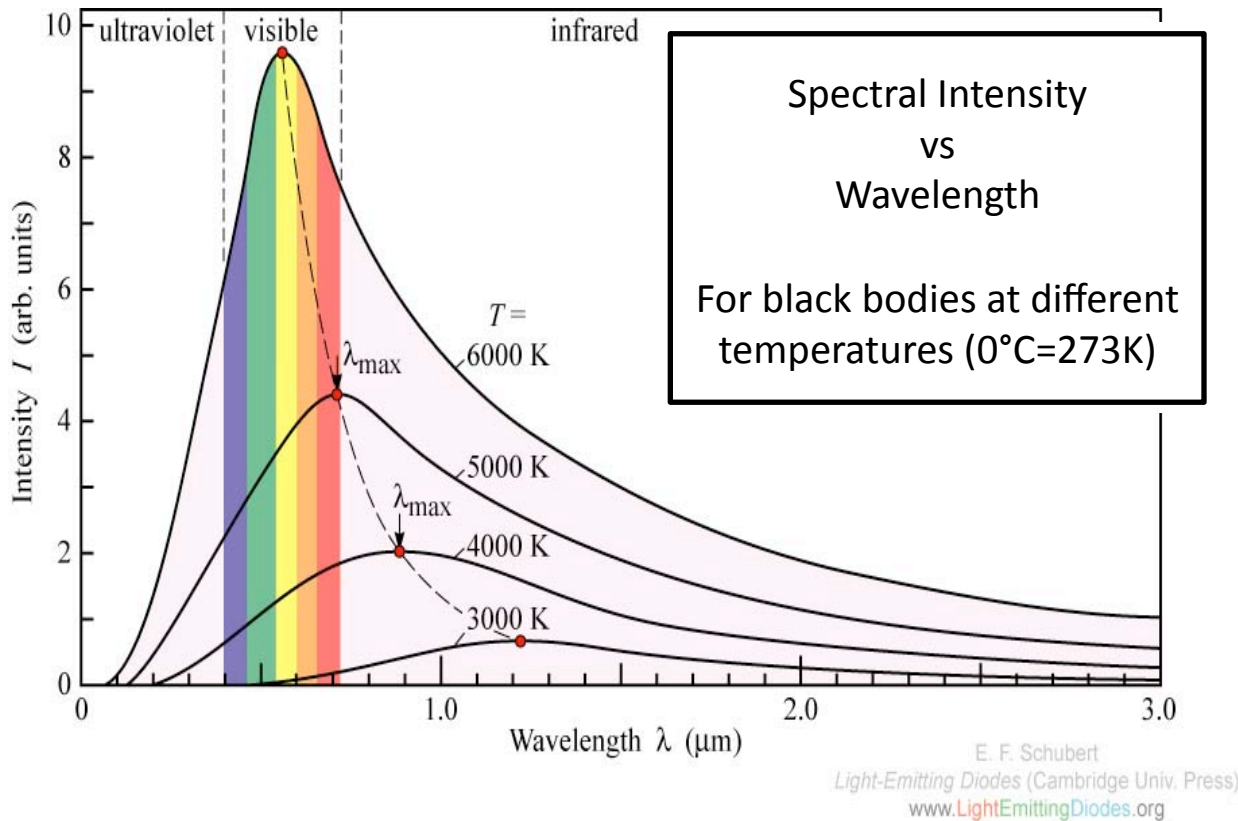
PLAN: Use this model to understand the role of greenhouse gases in regulating our temperature





Understanding Radiated Energy

Black Body Radiation - Planck



Sun: $\sim 6000\text{K}$, emits mainly in ultraviolet (UV) and visible spectrum (shortwave)

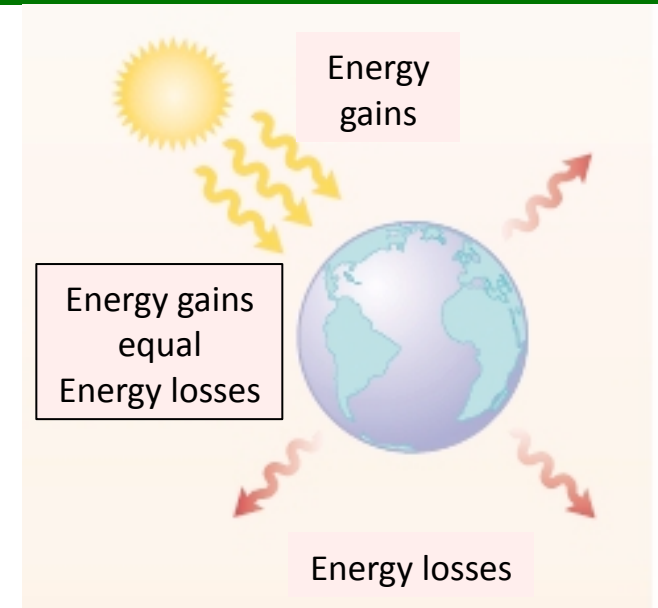
Earth: $\sim 300\text{K}$, emits mainly in infrared (IR) spectrum (longwave)

Total energy flux over all wavelengths for temp T is $\sigma T^4 = (5.67 \times 10^{-8})T^4$ -- Stefan-Boltzmann



Earth's Average Temperature

Question: What is Earth's average global annual temperature T ?
What do you think?

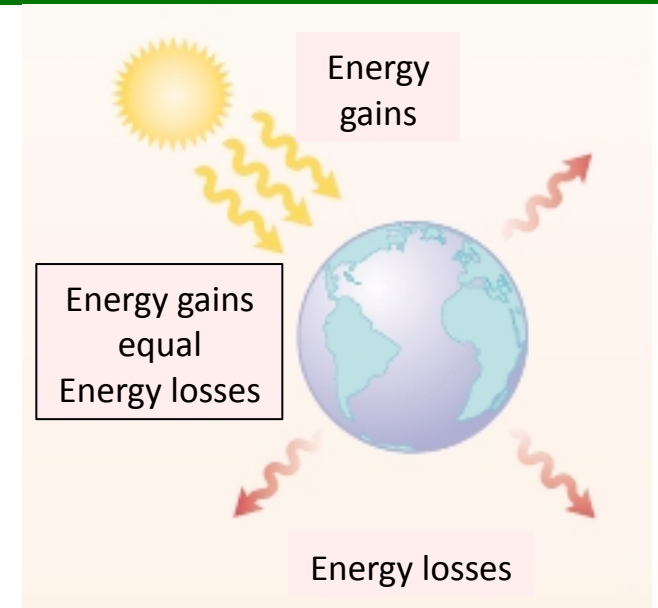




Earth's Average Temperature

Question: What is Earth's average global annual temperature T ?
What do you think?

Answer: 15°C (59°F)



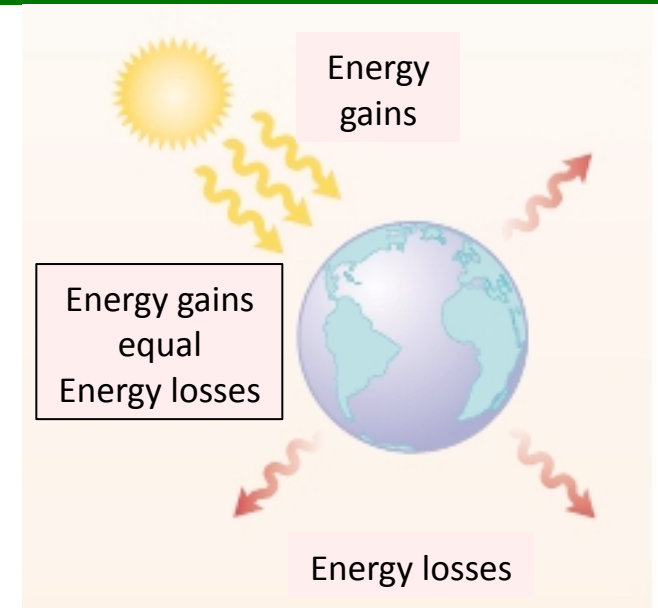


Earth's Average Temperature

Question: What is Earth's average global annual temperature T_e ?
What do you think?

Answer: 15°C (59°F)

Question: Why? What controls it?





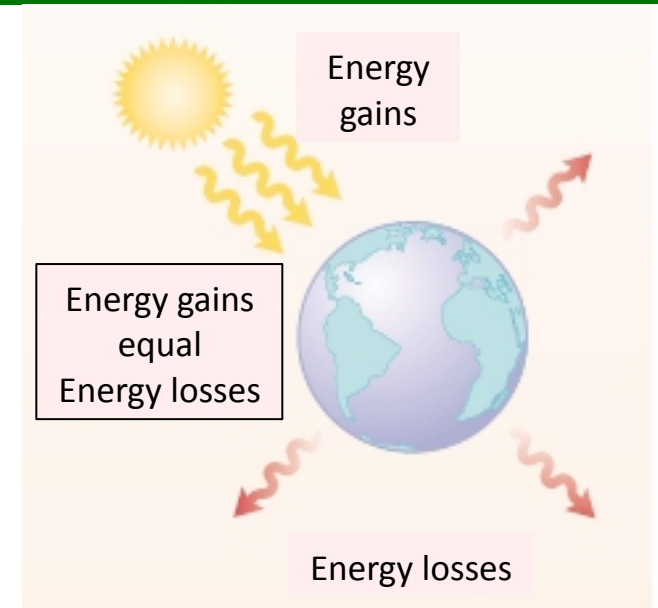
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Question: Why? What controls it?

Answer: Lots of things!





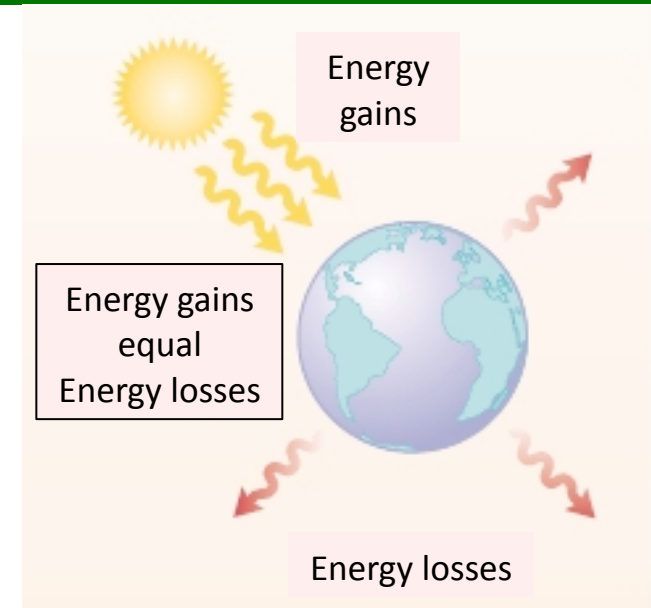
Earth's Average Temperature

Question: What is Earth's average global annual temperature T_e ?
What do you think?

Answer: 15°C (59°F)

Question: Why? What controls it?

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Let's use math to test this hypothesis: Earth's temperature is a consequence of

1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

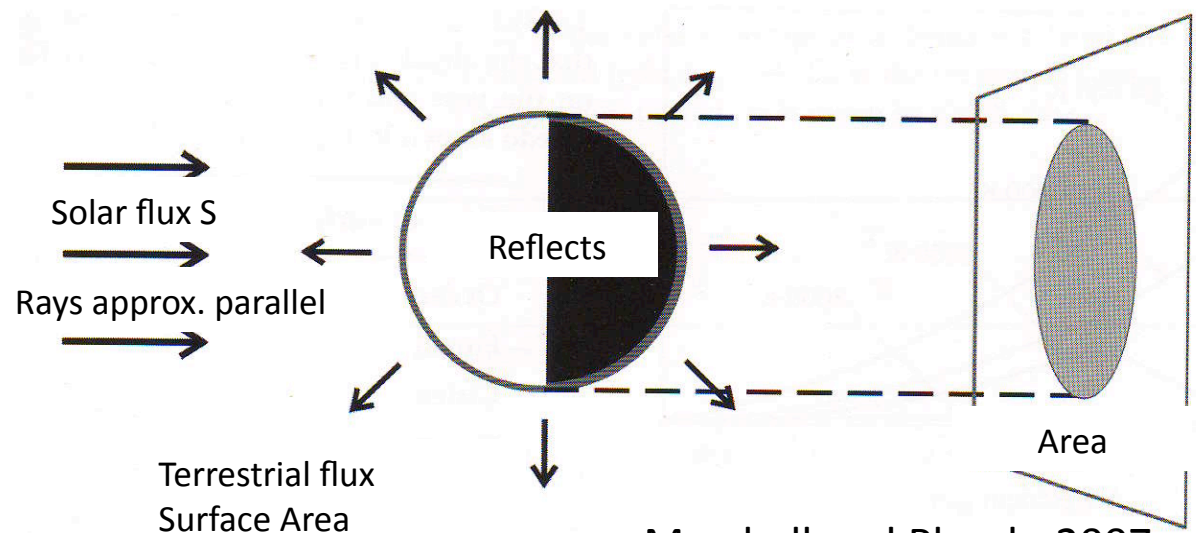
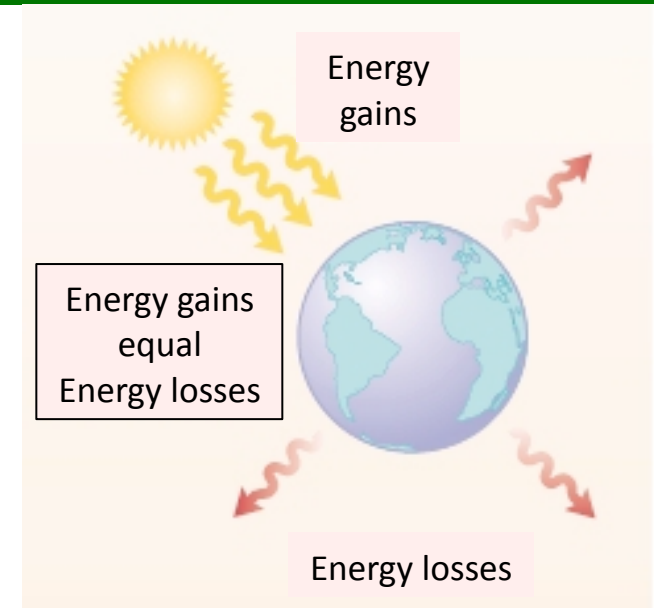
So we pretend the atmosphere is transparent to radiation: "no atmosphere"



Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

1. Blackbody radiation
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Marshall and Plumb, 2007

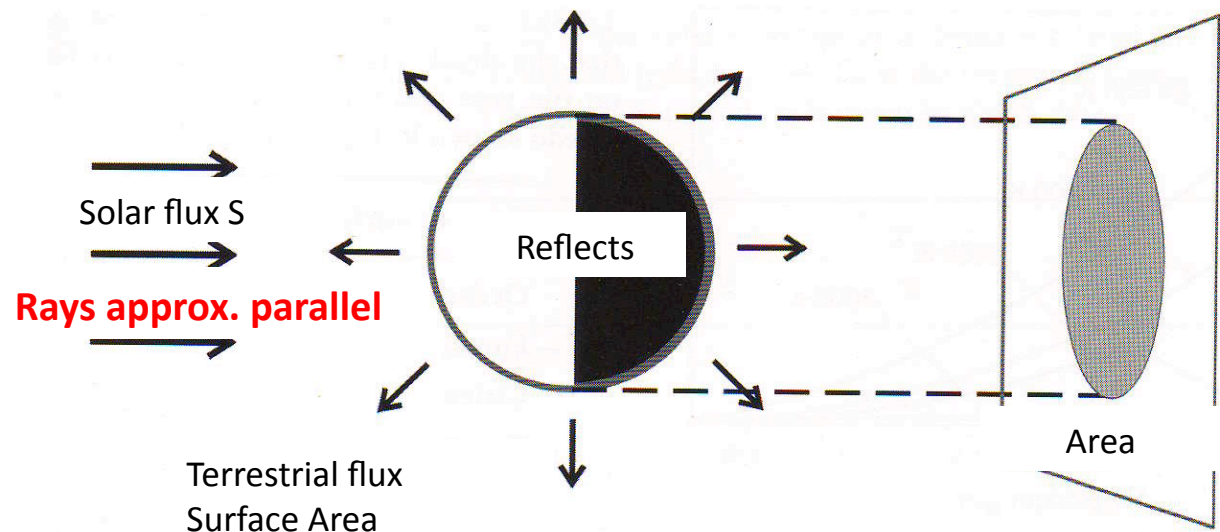
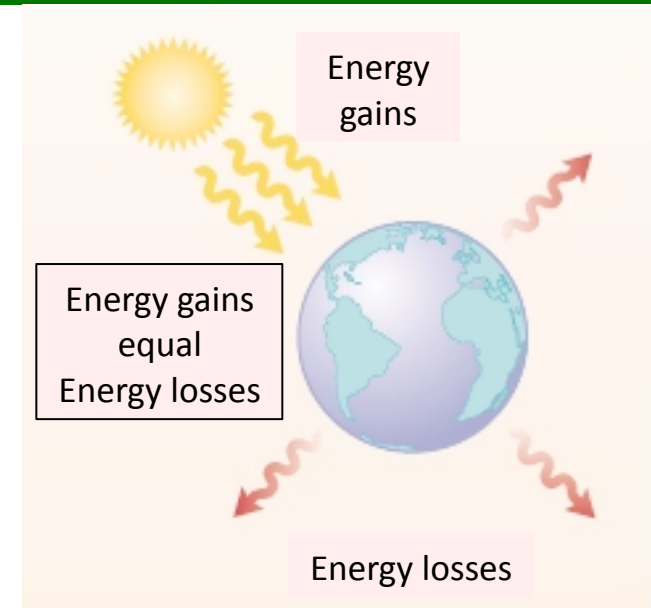


Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

1. Blackbody radiation
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Question: Why are the rays from the sun approx. parallel? Which hypothesis?





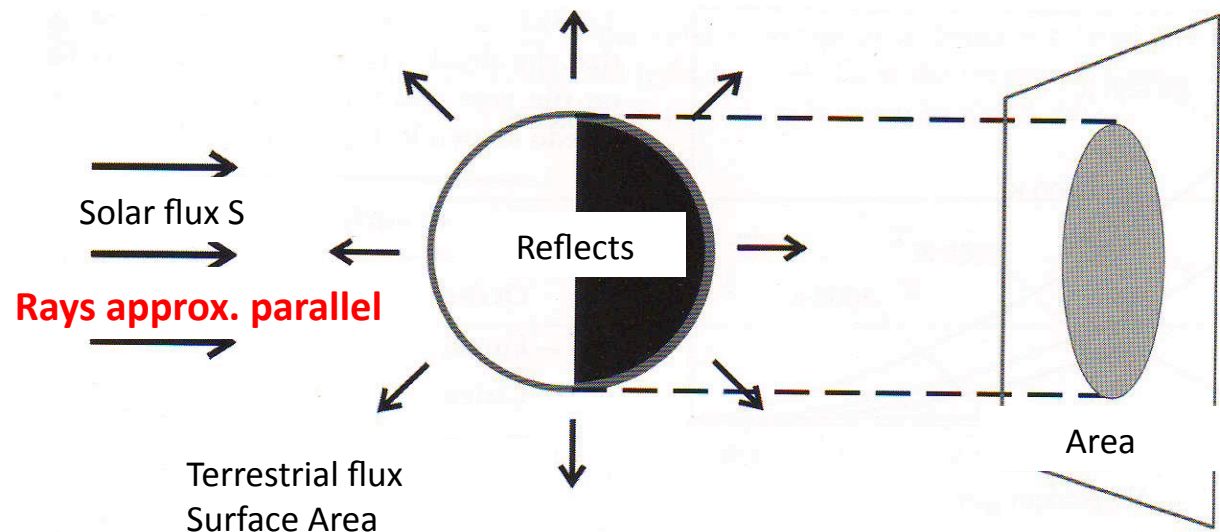
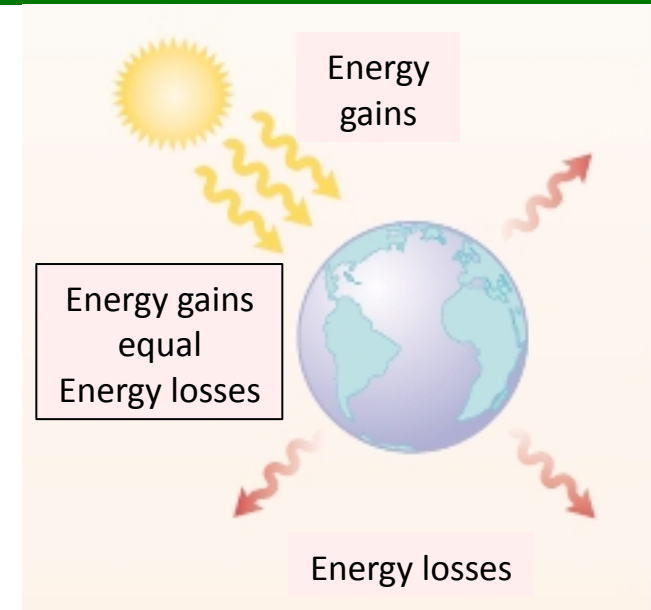
Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

1. Blackbody radiation
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Question: Why are the rays from the sun approx. parallel? Which hypothesis?

Answer: Sun is so far away (hypothesis 2)



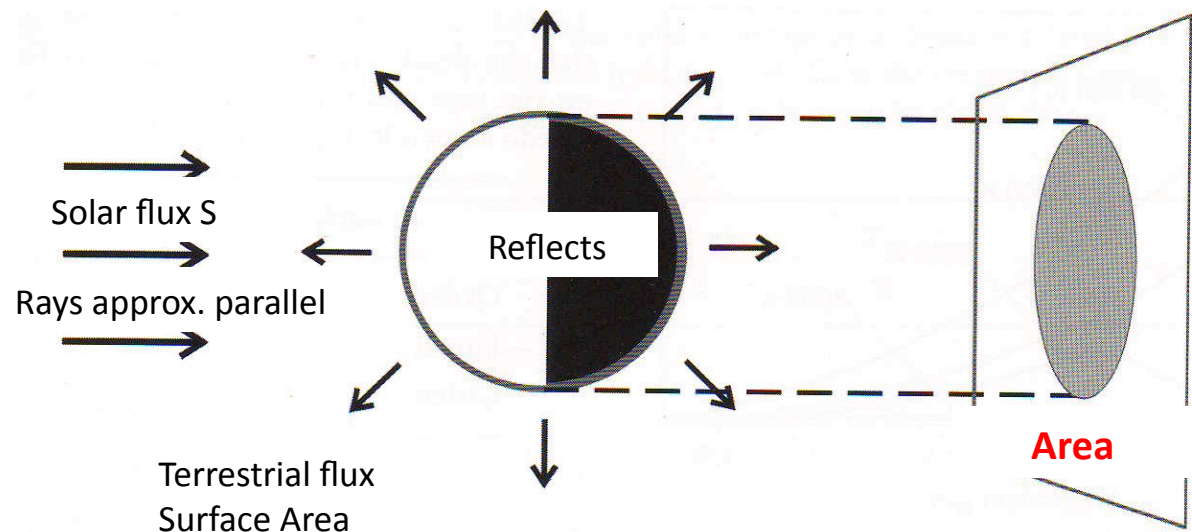
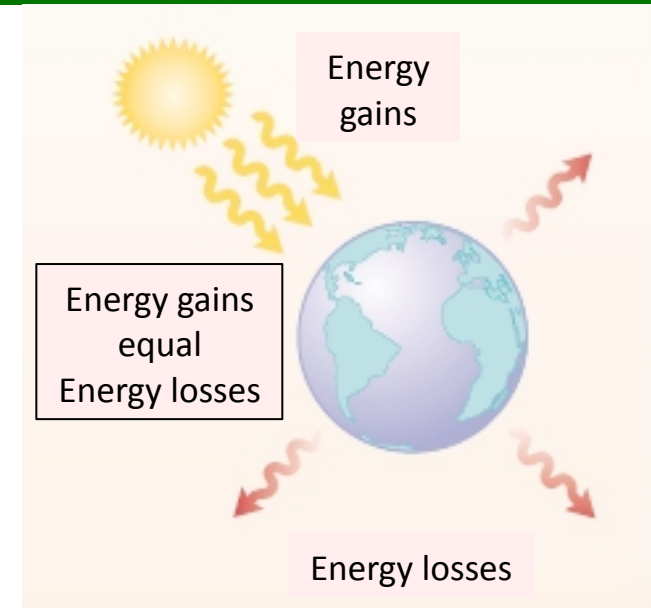


Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

1. Blackbody radiation
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Question: What's the cross sectional area of the Earth (radius r)? Which hypothesis?





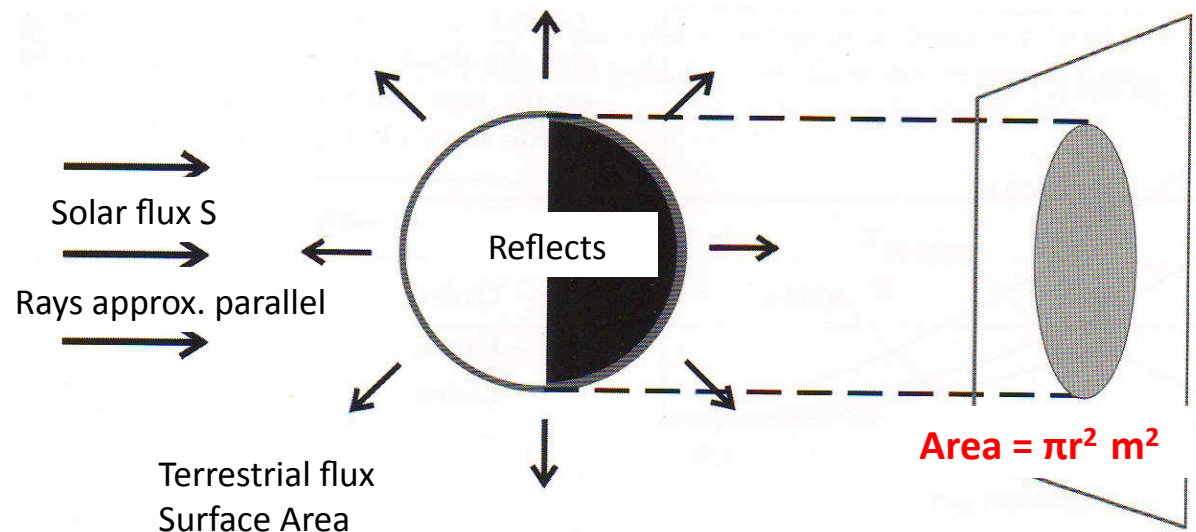
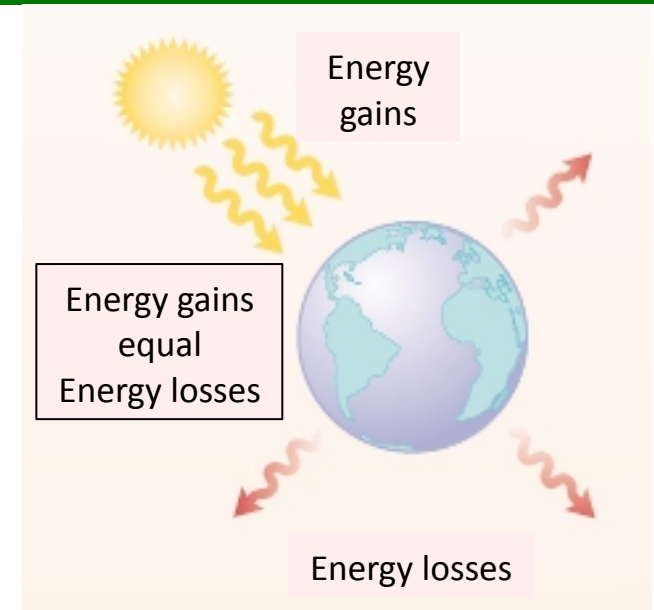
Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

1. Blackbody radiation
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Question: What's the cross sectional area of the Earth (radius r)? Which hypothesis?

Answer: Area = πr^2 m² (hypothesis 3)



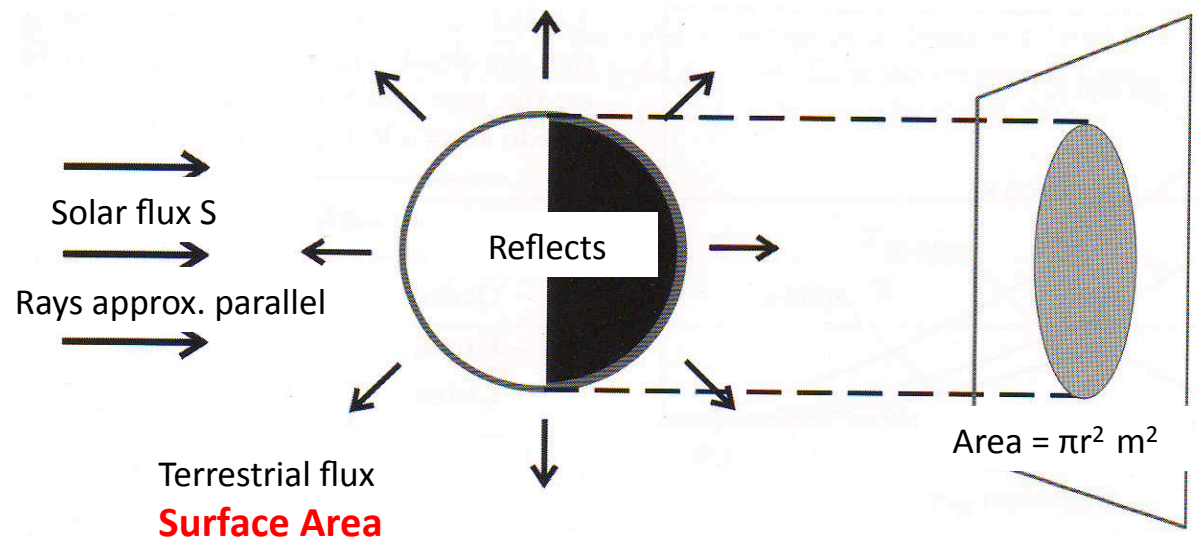
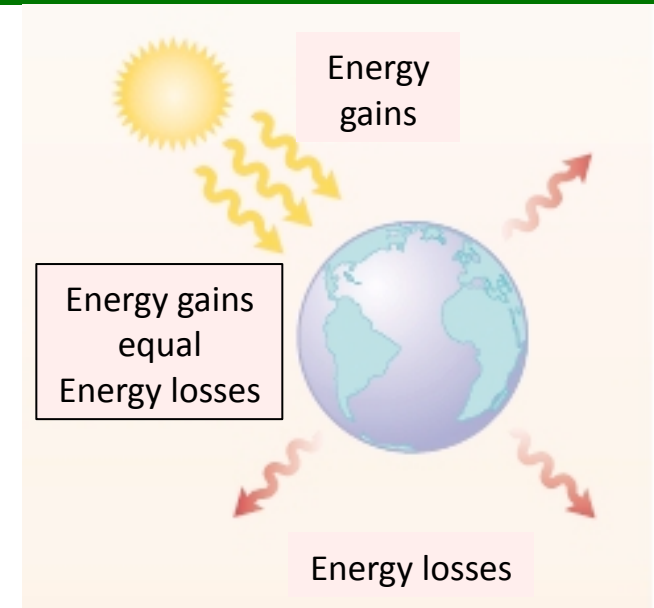


Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

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Question: What's the surface area of the Earth (radius r)? Which hypothesis?





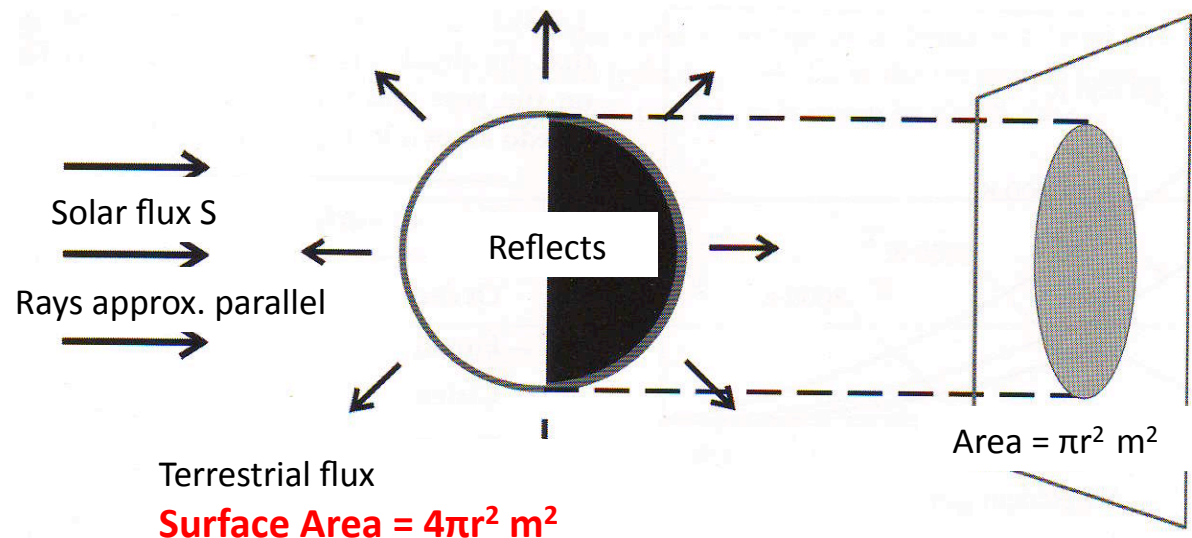
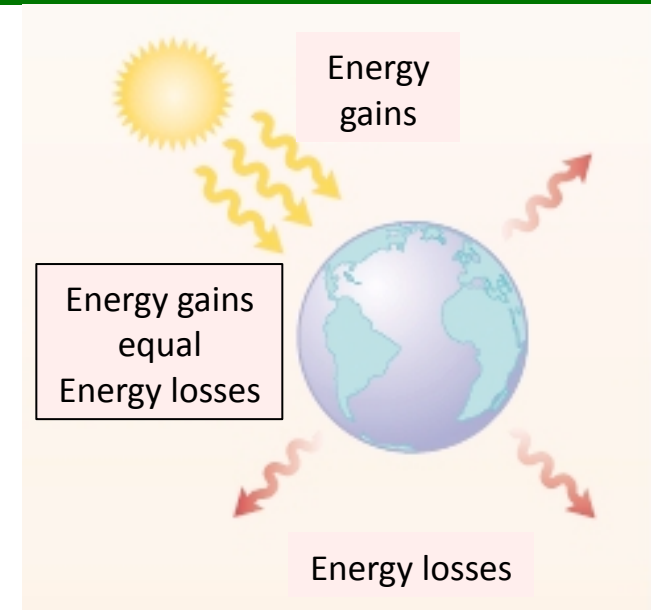
Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

1. Blackbody radiation
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3. Size
4. Albedo (reflectivity)

Question: What's the surface area of the Earth (radius r)? Which hypothesis?

Answer: Surface Area = $4\pi r^2$ (hypothesis 3)



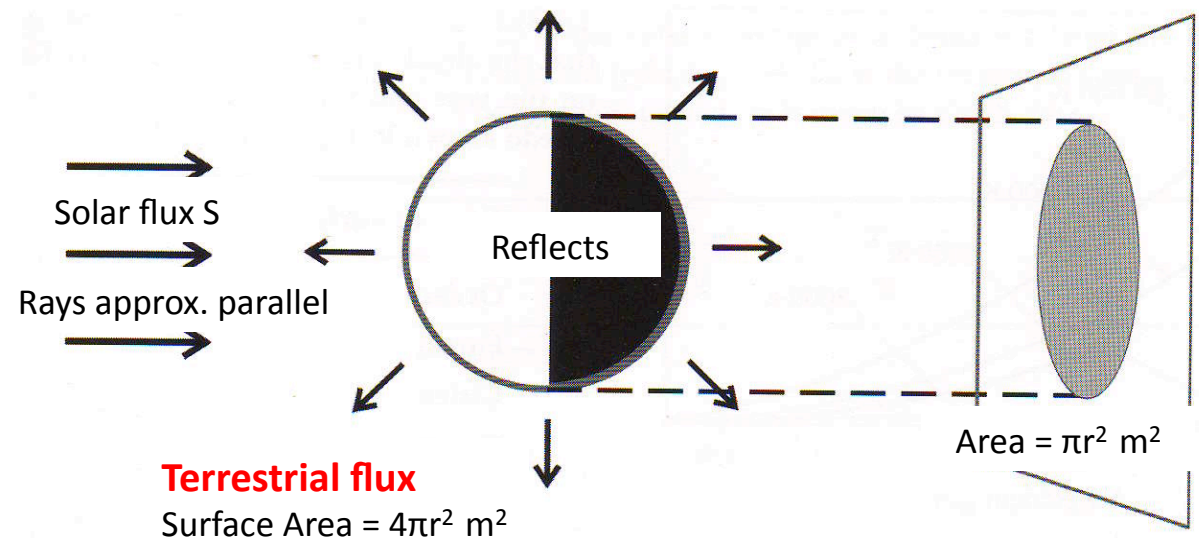
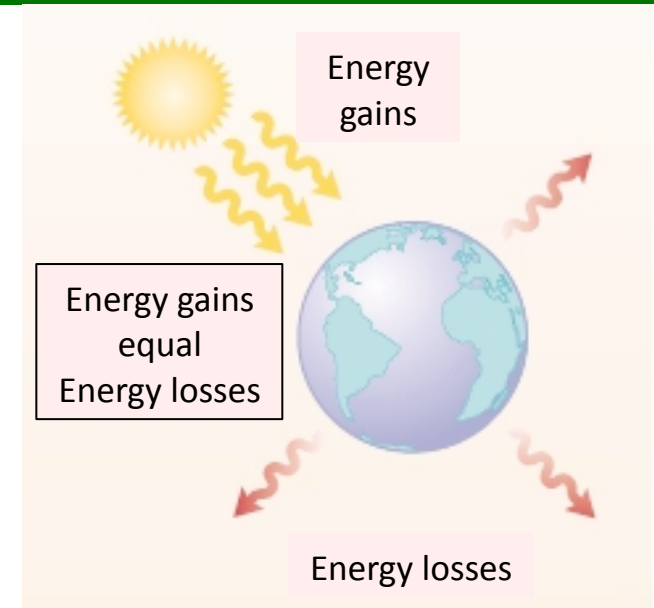


Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

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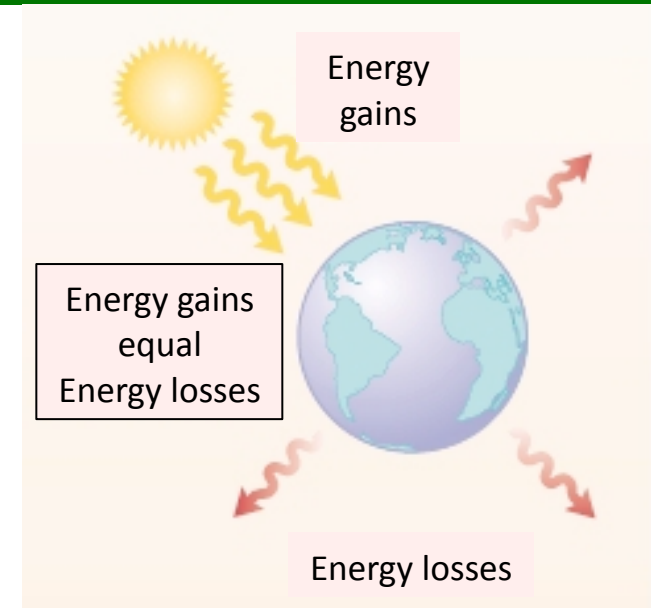
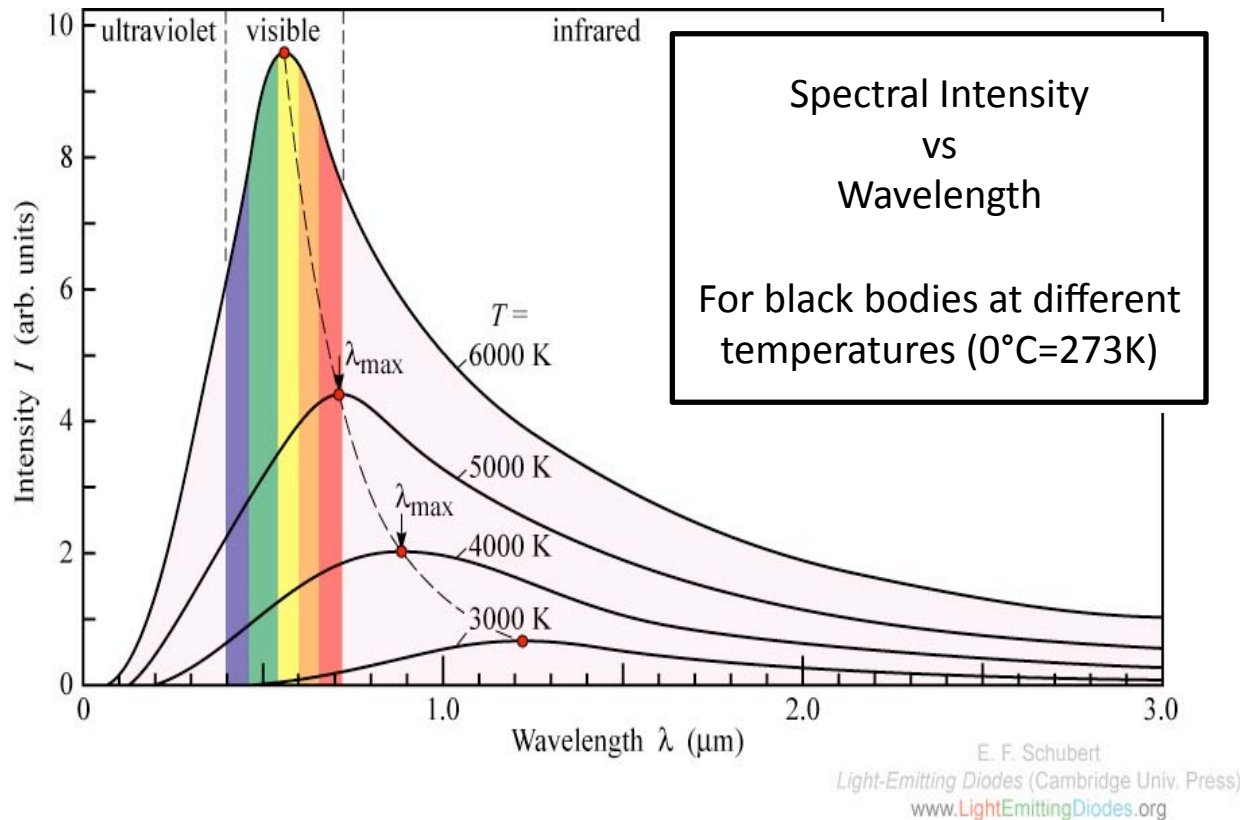
Question: What's the terrestrial flux? Which hypothesis?





Figuring Out Terrestrial Flux

Black Body Radiation - Planck



Sun: $\sim 6000\text{K}$, emits mainly in ultraviolet (UV) and visible spectrum (shortwave)

Earth: $\sim 300\text{K}$, emits mainly in infrared (IR) spectrum (longwave)

Total energy flux over all wavelengths for temp T is $\sigma T^4 = (5.67 \times 10^{-8}) T^4$ -- Stefan-Boltzmann



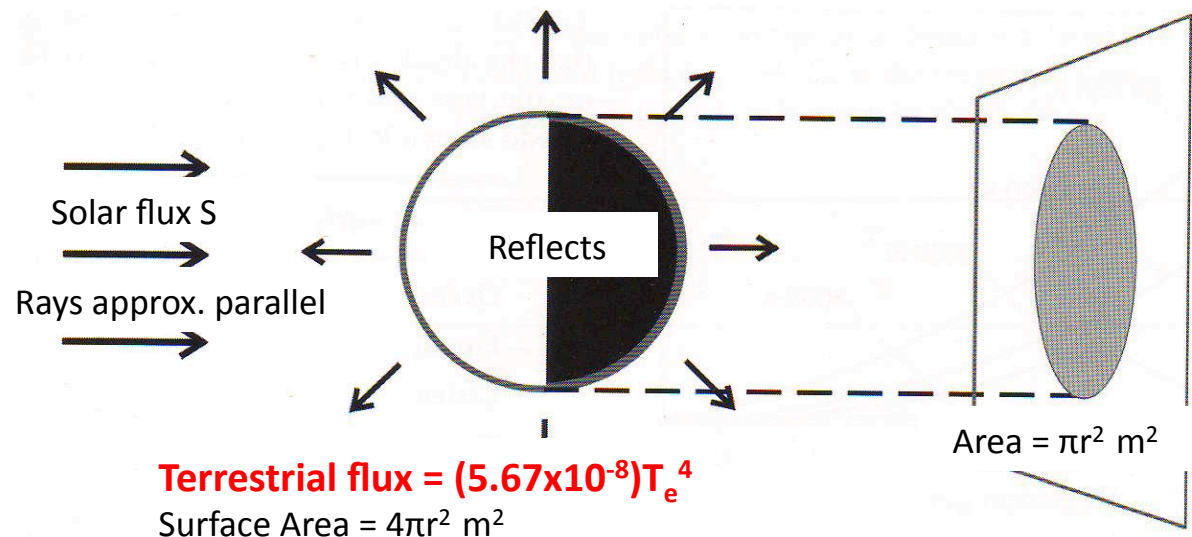
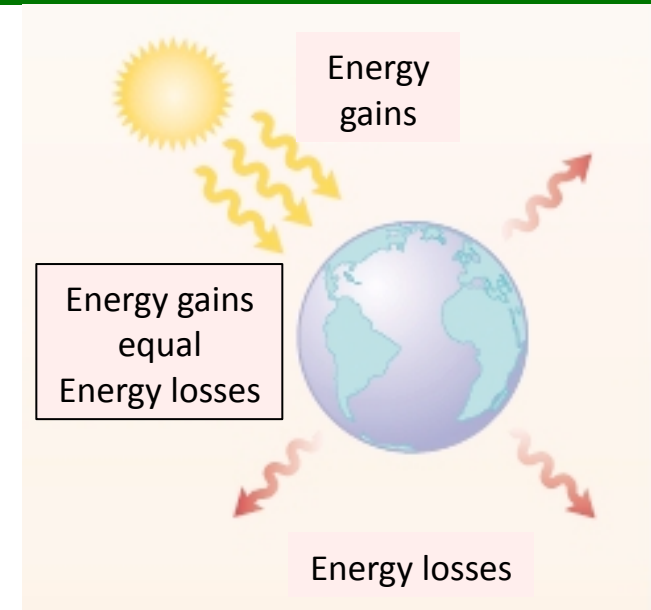
Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

1. Blackbody radiation
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4. Albedo (reflectivity)

Question: What's the terrestrial flux? Which hypothesis?

Answer: Terrestrial flux = $(5.67 \times 10^{-8}) T^4 \text{ Wm}^{-2}$ (hyp. 1)





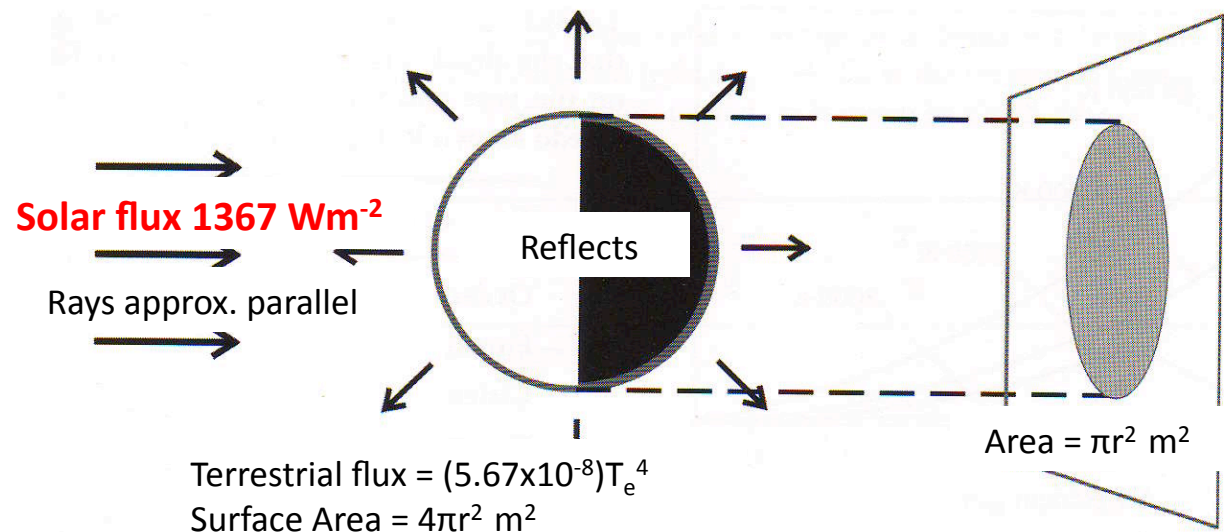
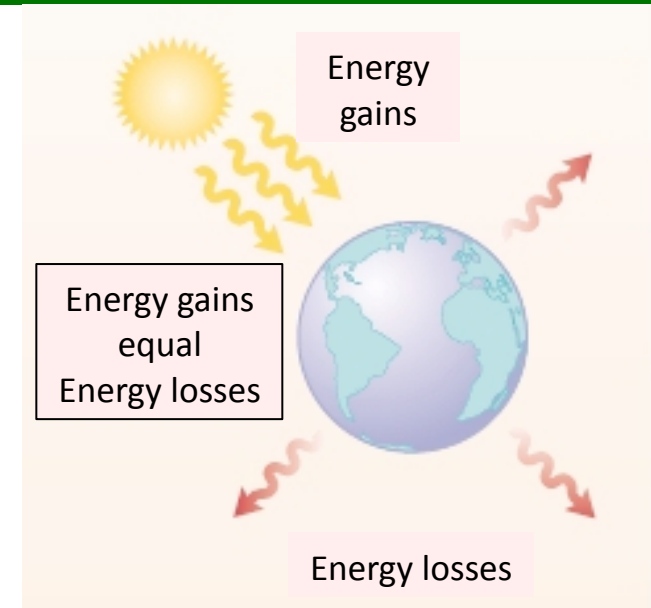
Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

1. Blackbody radiation
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4. Albedo (reflectivity)

Question: What's the Solar flux? Which hypothesis?

Answer: Solar flux $S = 1367 \text{ Wm}^{-2}$ (hypothesis 1,2)



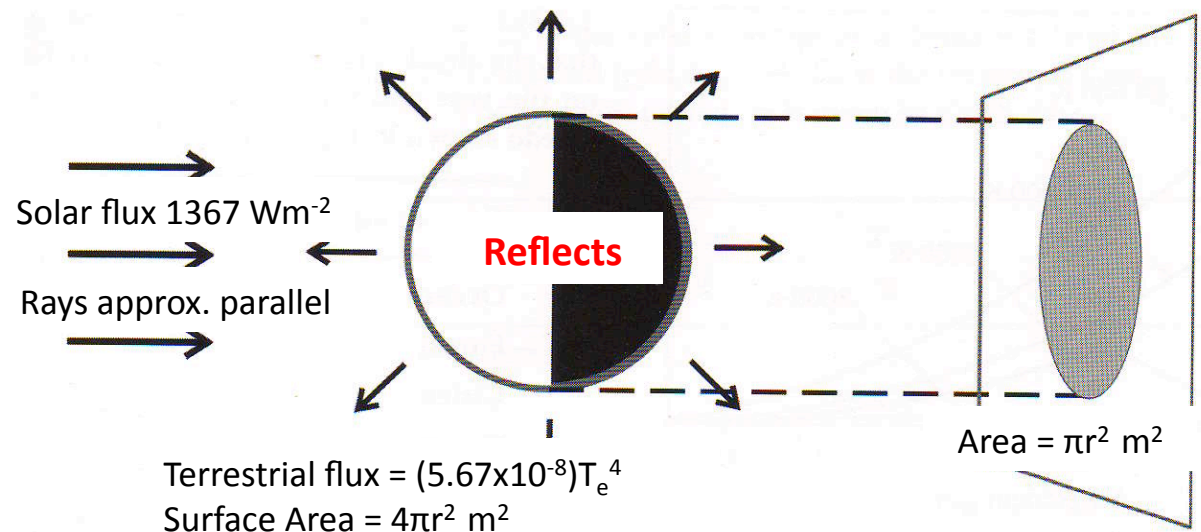
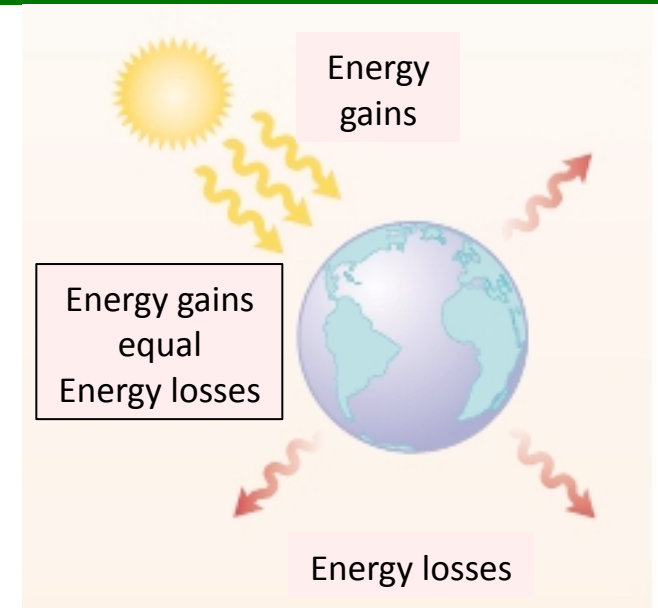


Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

1. Blackbody radiation
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Question: What fraction of solar radiation does the earth reflect? What hypothesis?





Energy Balance, No Atmosphere

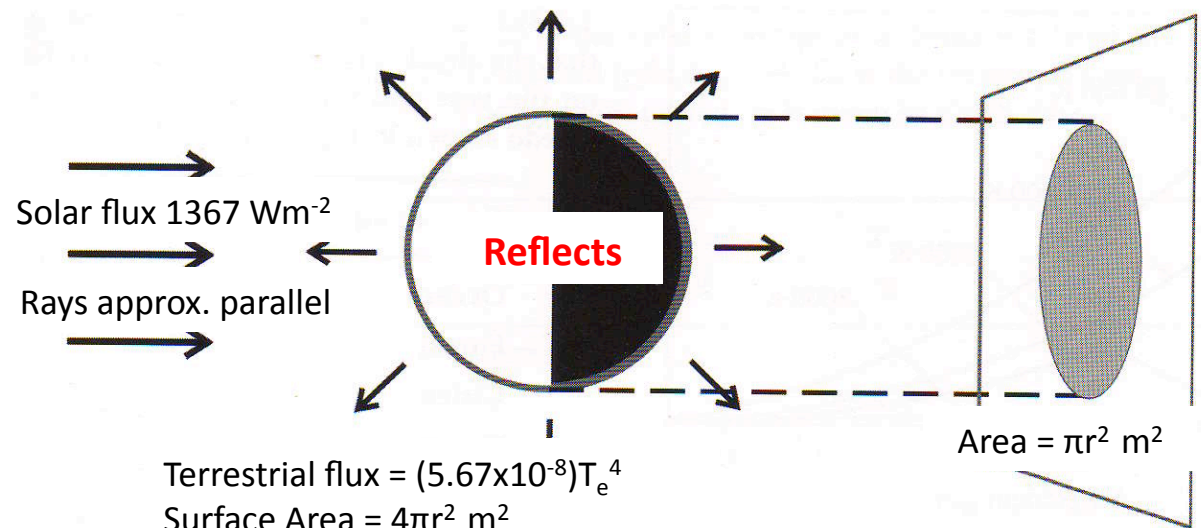
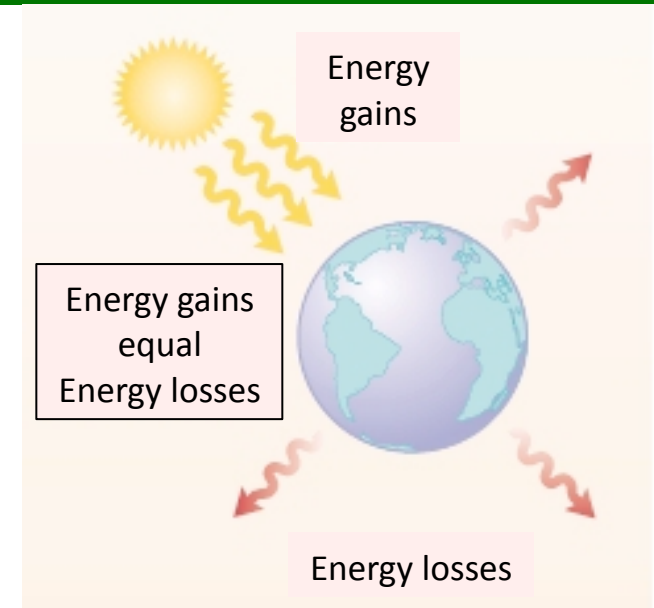
Hypothesis: Earth's temperature is a consequence of

1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

Question: What fraction of solar radiation does the earth reflect? What hypothesis?

Examples:

- Fresh snow reflects 85%
- Sandy desert reflects 40%
- Grasslands reflect 18%
- Cities reflect 16%
- Ocean reflects 6%





Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

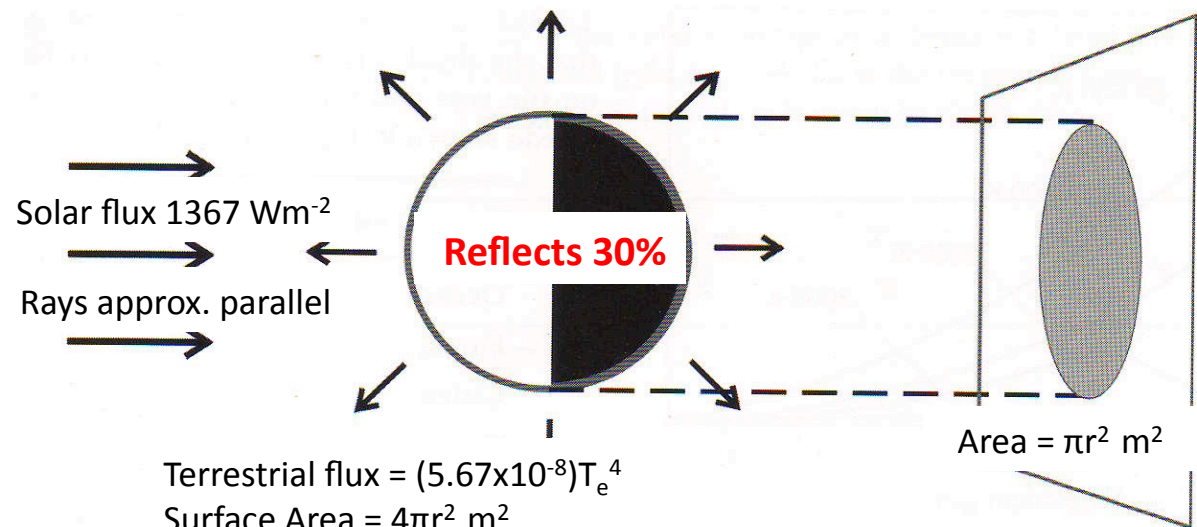
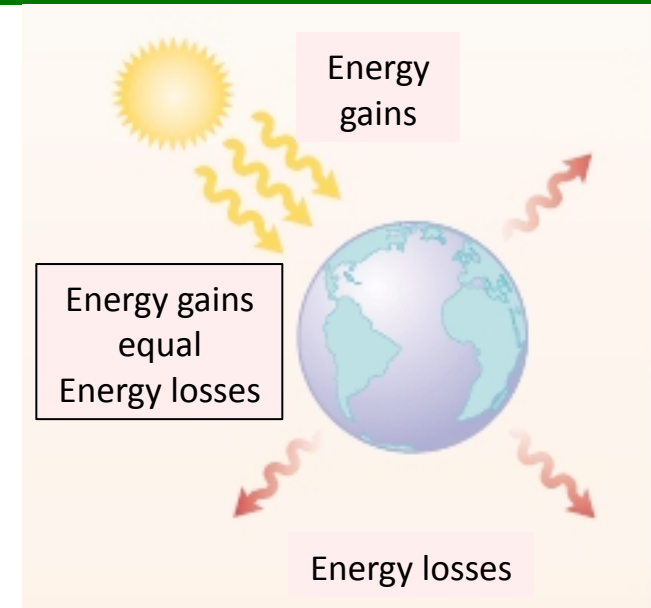
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

Question: What fraction of solar radiation does the earth reflect? What hypothesis?

Answer: Planetary average albedo 0.3 (hyp. 4)

Examples:

- Fresh snow reflects 85%
- Sandy desert reflects 40%
- Grasslands reflect 18%
- Cities reflect 16%
- Ocean reflects 6%



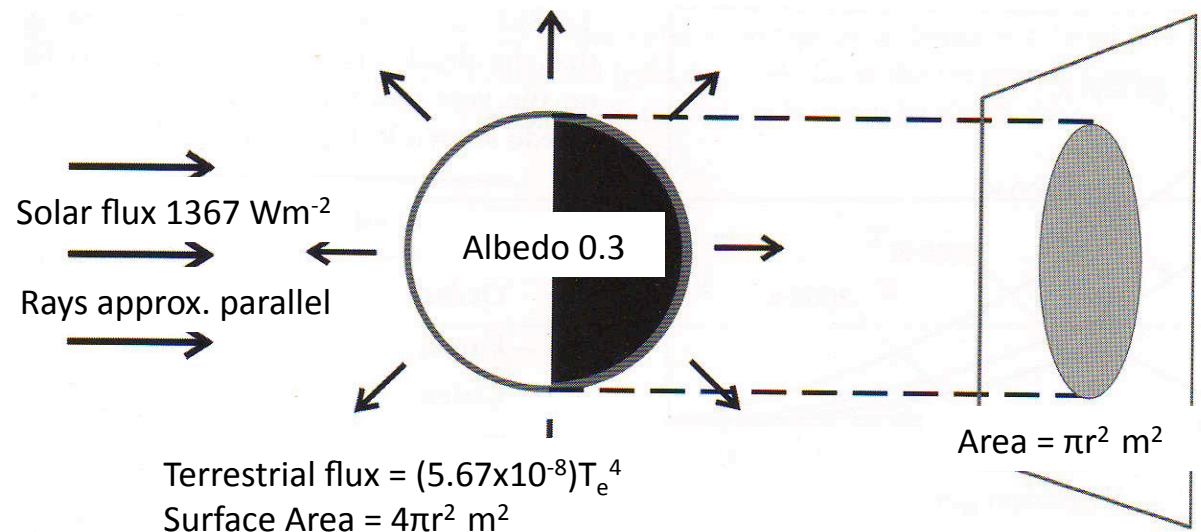
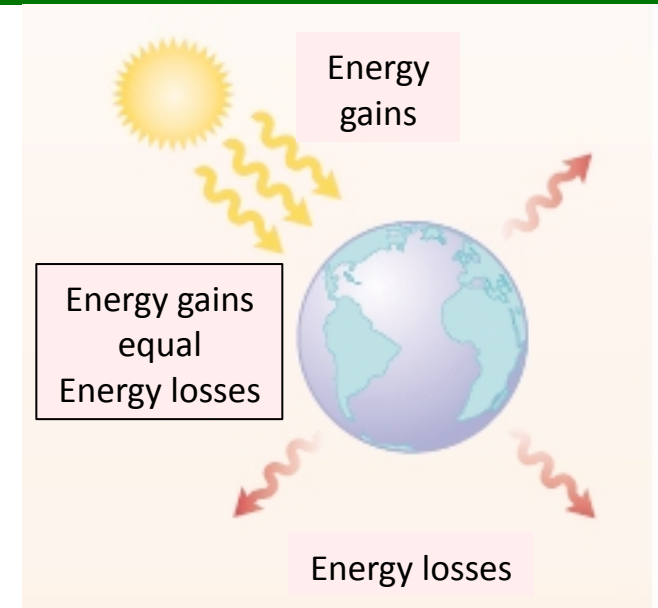


Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

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OK! Now we understand the picture,
Let's do energy balance...





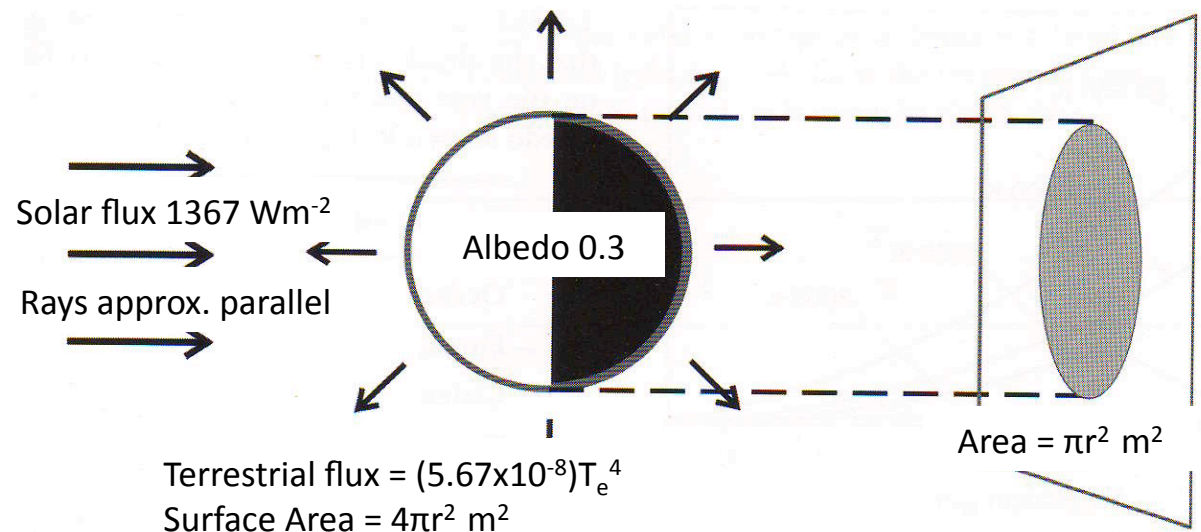
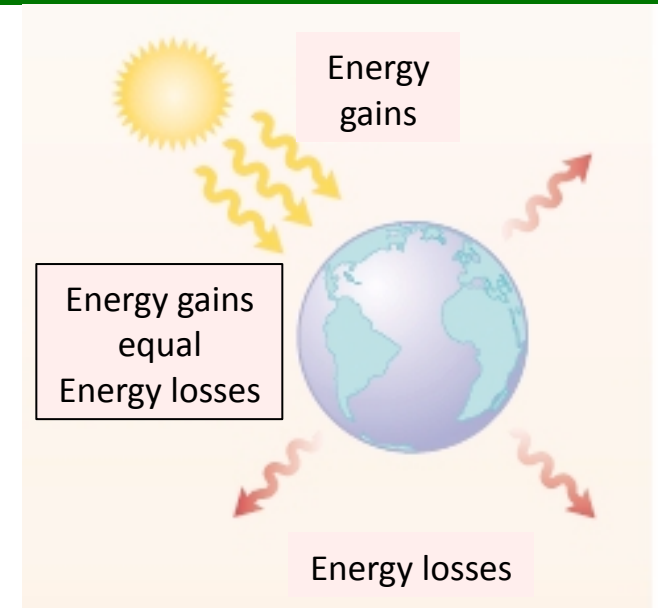
Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

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Energy Gain

Solar flux * cross-sect area = ???





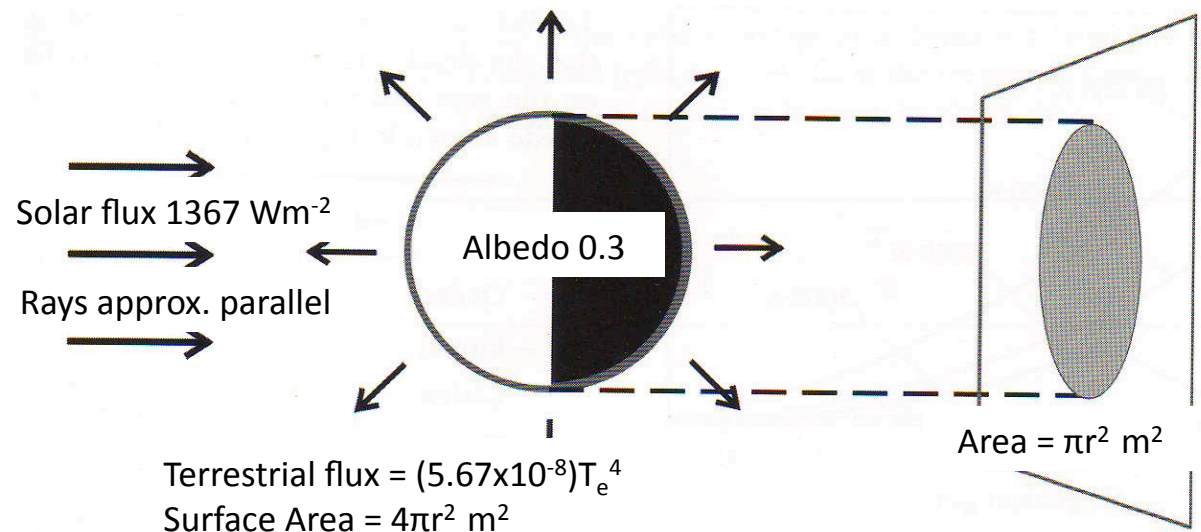
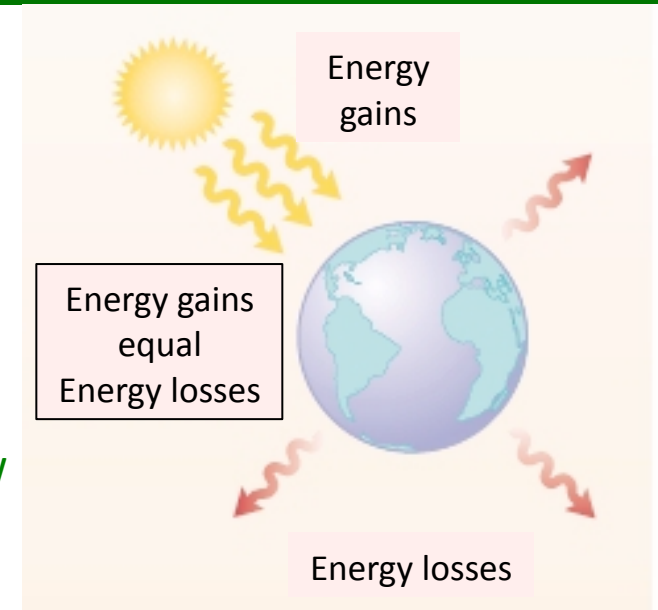
Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

1. Blackbody radiation
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Energy Gain

Solar flux * cross-sect area = $1367 \text{ Wm}^{-2} * \pi r^2 \text{ m}^2 = 1367\pi r^2 \text{ W}$





Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

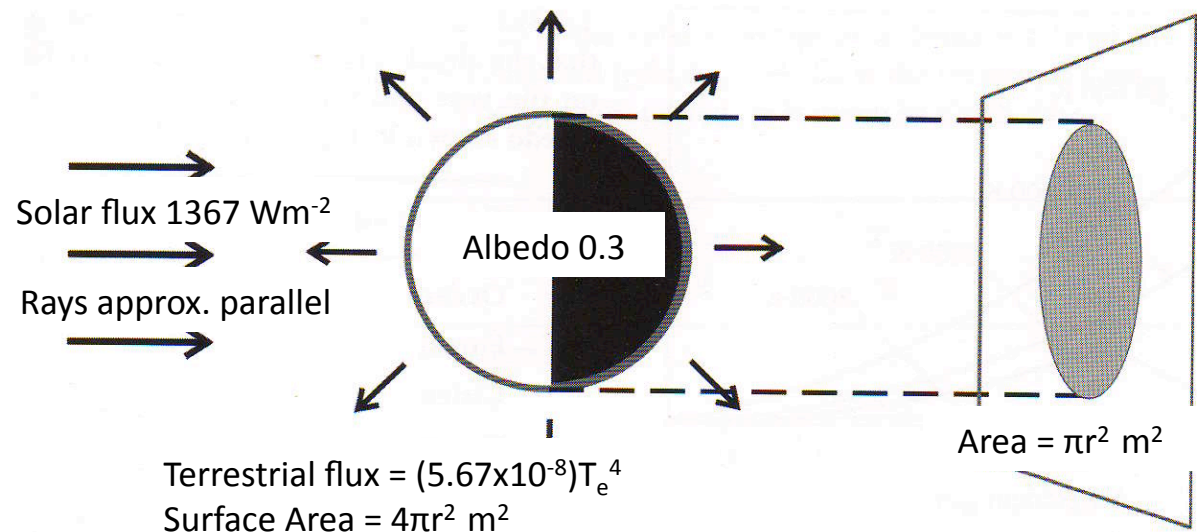
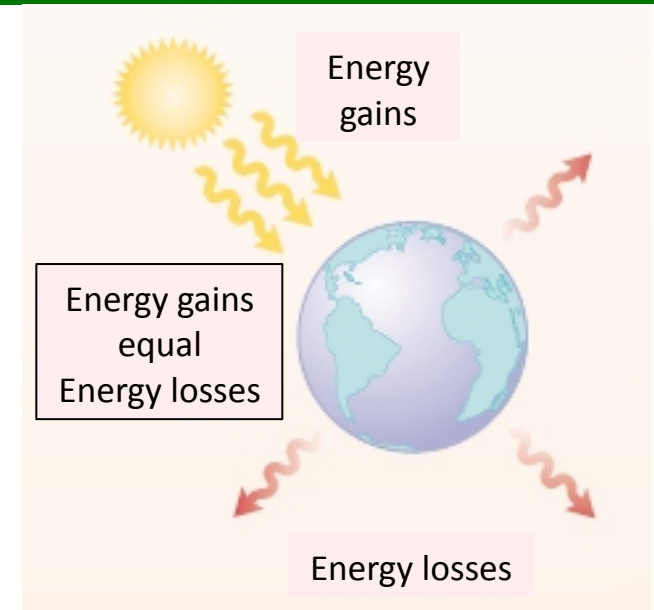
1. Blackbody radiation
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3. Size
4. Albedo (reflectivity)

Energy Gain

Solar flux * cross-sect area = $1367\pi r^2$ W

Energy loss by Terrestrial Radiation

Terrestrial flux * surface area = ???





Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

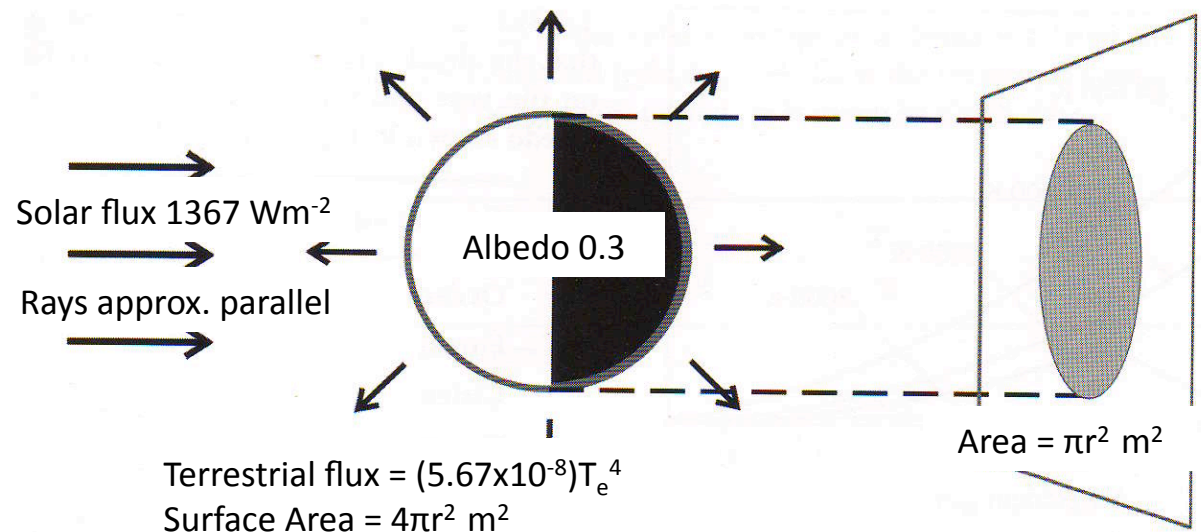
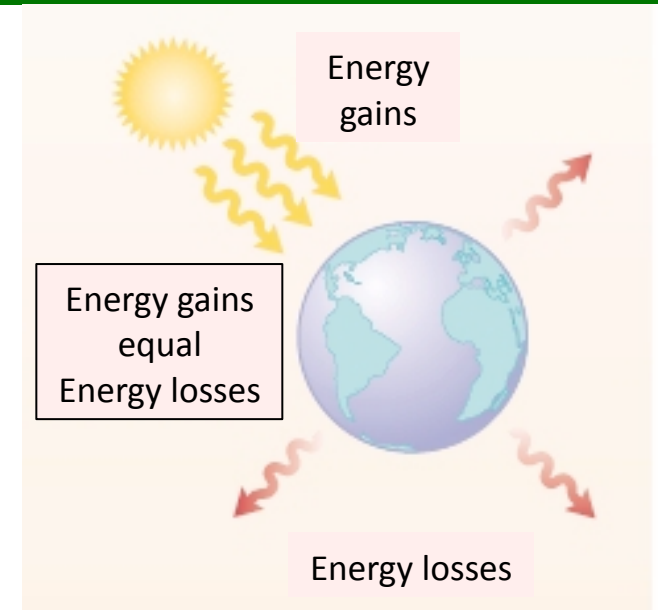
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

Energy Gain

Solar flux * cross-sect area = $1367\pi r^2$ W

Energy loss by Terrestrial Radiation

Terrestrial flux * surface area = $(5.67 \times 10^{-8})T_e^4 4\pi r^2$ W





Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

1. Blackbody radiation
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Energy Gain

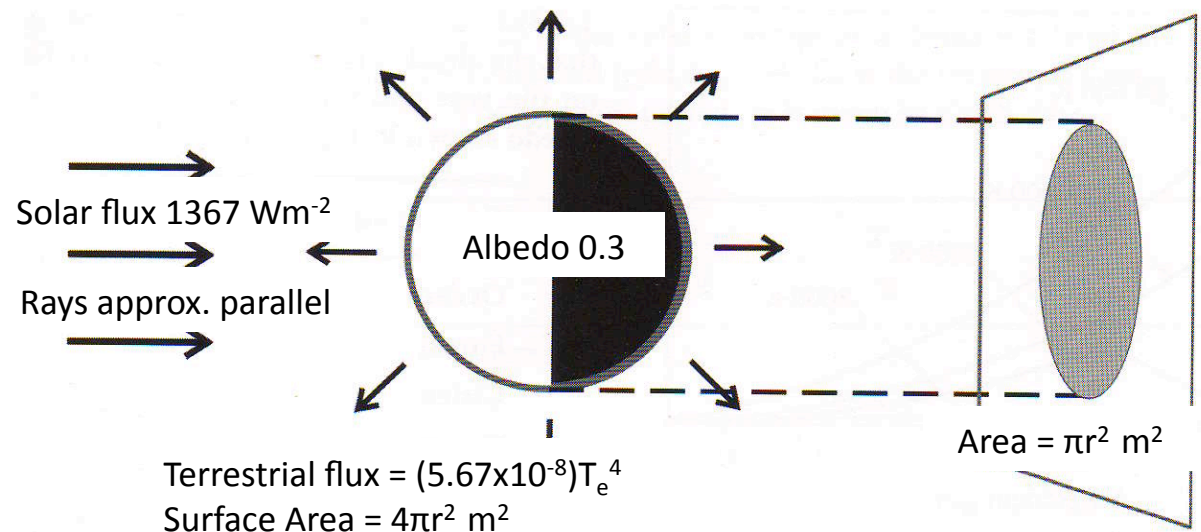
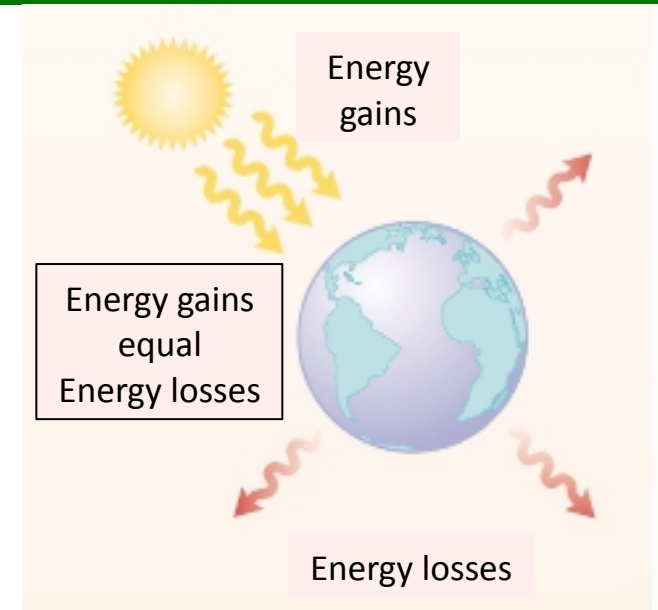
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Energy loss by Terrestrial Radiation

Terrestrial flux * surface area = $(5.67 \times 10^{-8})T_e^4 4\pi r^2$ W

Energy loss by Albedo

Reflected solar radiation = ???





Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

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4. Albedo (reflectivity)

Energy Gain

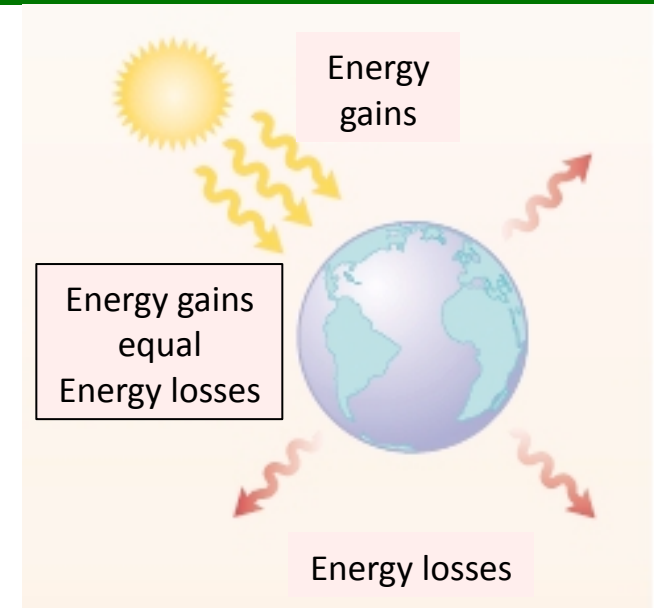
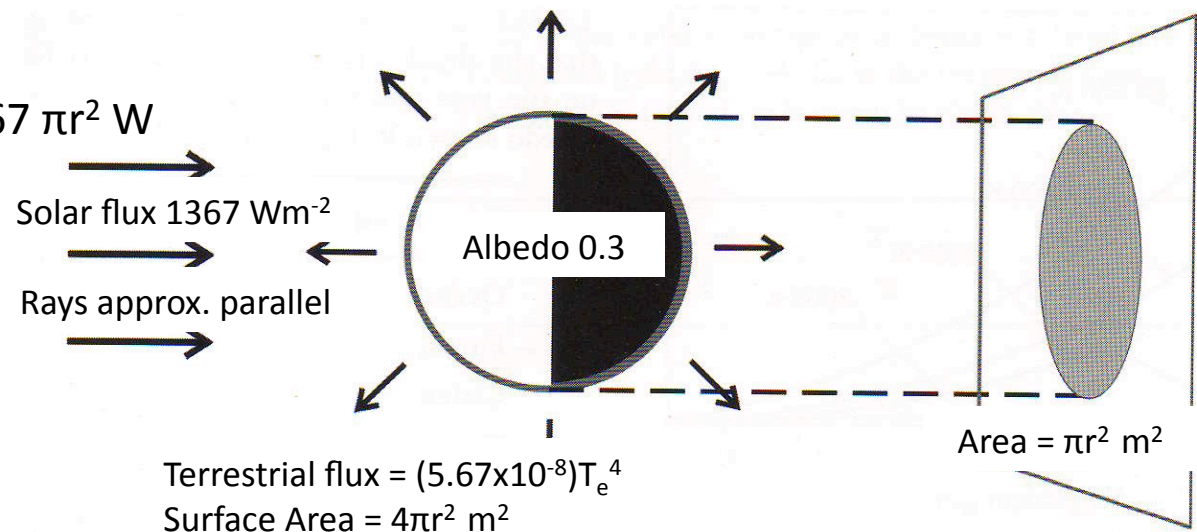
Solar flux * cross-sect area = $1367\pi r^2$ W

Energy loss by Terrestrial Radiation

Terrestrial flux * surface area = $(5.67 \times 10^{-8})T_e^4 4\pi r^2$ W

Energy loss by Albedo

Reflected solar radiation = $0.3 \times 1367 \pi r^2$ W





Energy Balance, No Atmosphere

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Energy Gain

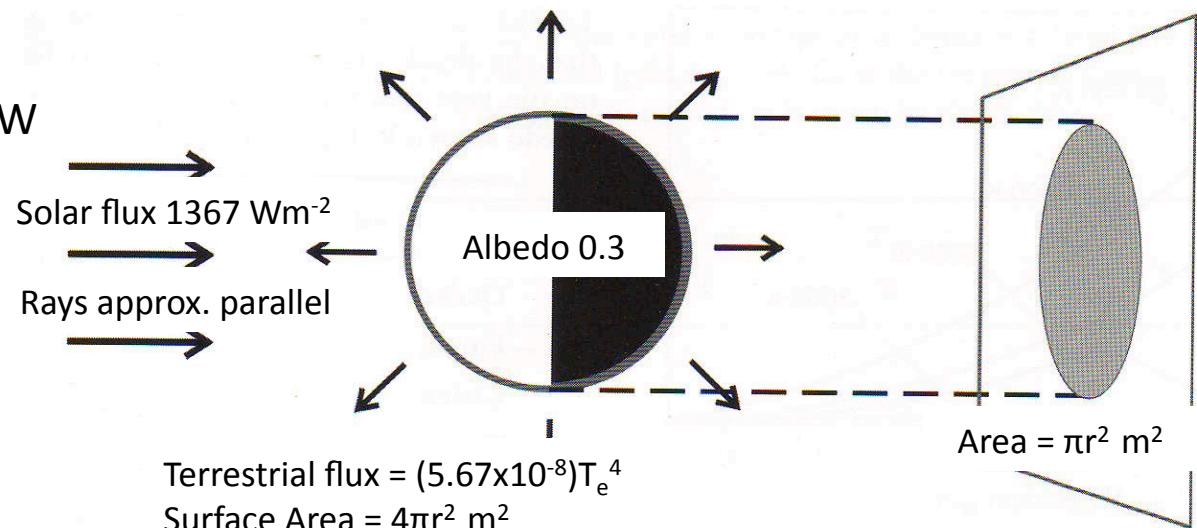
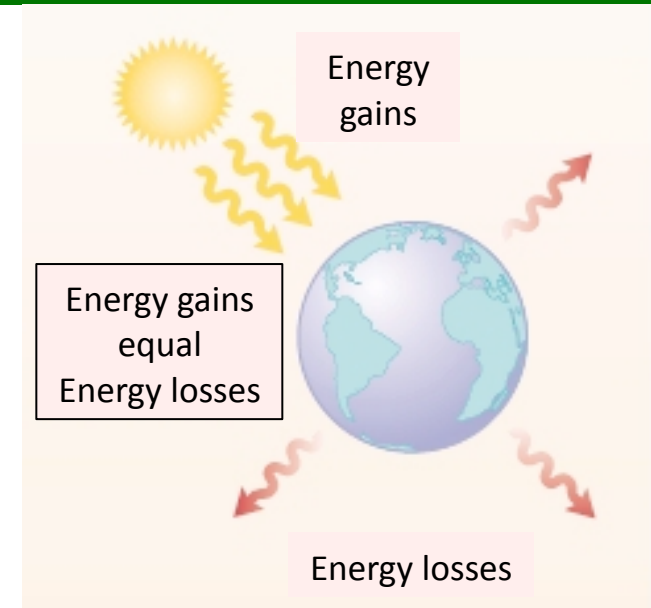
Solar flux * cross-sect area = $1367\pi r^2$ W

Energy loss by Terrestrial Radiation

Terrestrial flux * surface area = $(5.67 \times 10^{-8})T_e^4 4\pi r^2$ W

Energy loss by Albedo

Reflected solar radiation = $410\pi r^2$ W





Energy Balance, No Atmosphere

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3. Size
4. Albedo (reflectivity)

Energy Gain

Solar flux * cross-sect area = $1367\pi r^2$ W

Energy loss by Terrestrial Radiation

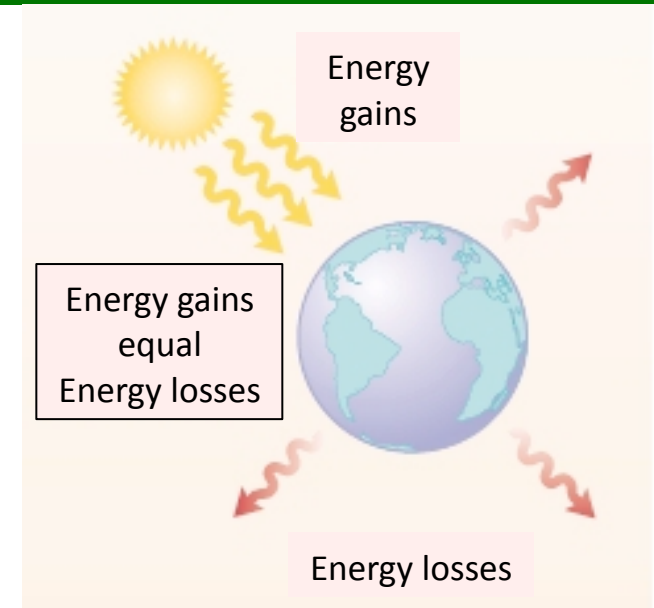
Terrestrial flux * surface area = $(5.67 \times 10^{-8})T^4 4\pi r^2$ W

Energy loss by Albedo

Reflected solar radiation = $410\pi r^2$ W

Balance

$$1367\pi r^2 = 410\pi r^2 + (5.67 \times 10^{-8})T^4 4\pi r^2$$





Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

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3. Size
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Energy Gain

Solar flux * cross-sect area = $1367\pi r^2$ W

Energy loss by Terrestrial Radiation

Terrestrial flux * surface area = $(5.67 \times 10^{-8})T^4 4\pi r^2$ W

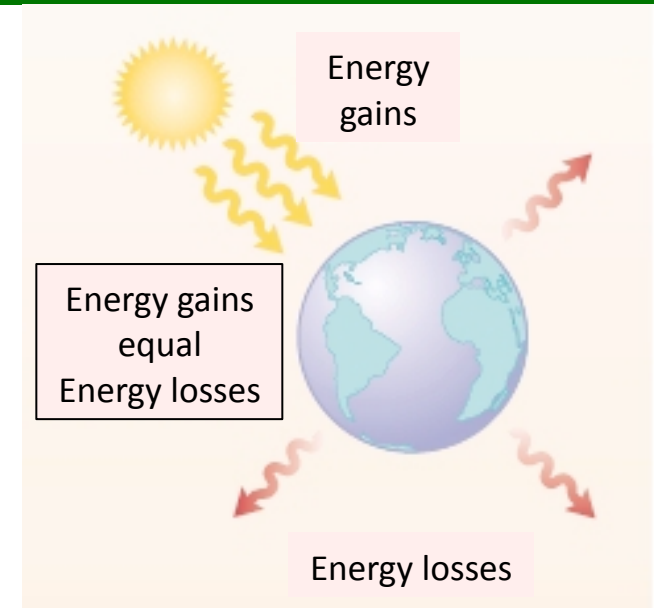
Energy loss by Albedo

Reflected solar radiation = $410\pi r^2$ W

Balance

$$1367\pi r^2 = 410\pi r^2 + (5.67 \times 10^{-8})T^4 4\pi r^2$$

$$1367 = 410 + 4(5.67 \times 10^{-8})T^4$$





Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

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Energy Gain

Solar flux * cross-sect area = $1367\pi r^2$ W

Energy loss by Terrestrial Radiation

Terrestrial flux * surface area = $(5.67 \times 10^{-8})T^4 4\pi r^2$ W

Energy loss by Albedo

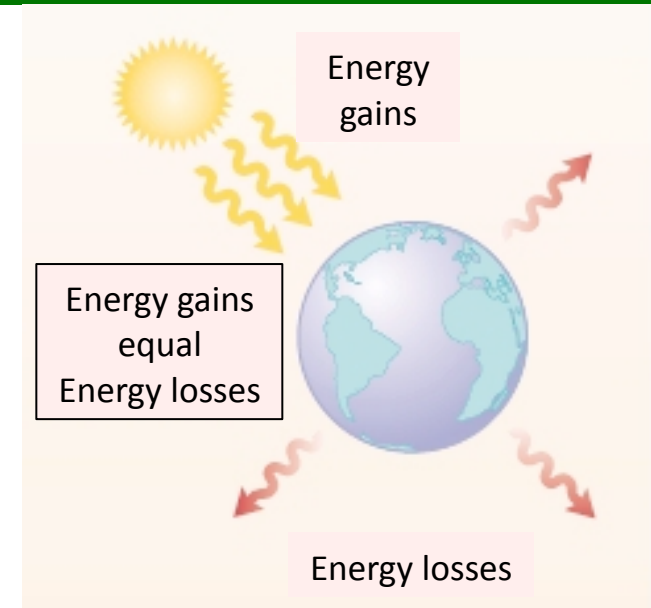
Reflected solar radiation = $410\pi r^2$ W

Balance

$$1367\pi r^2 = 410\pi r^2 + (5.67 \times 10^{-8})T^4 4\pi r^2$$

$$1367 = 410 + 4(5.67 \times 10^{-8})T^4$$

$$(5.67 \times 10^{-8})T^4 = 239$$





Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

Energy Gain

Solar flux * cross-sect area = $1367\pi r^2$ W

Energy loss by Terrestrial Radiation

Terrestrial flux * surface area = $(5.67 \times 10^{-8})T^4 4\pi r^2$ W

Energy loss by Albedo

Reflected solar radiation = $410\pi r^2$ W

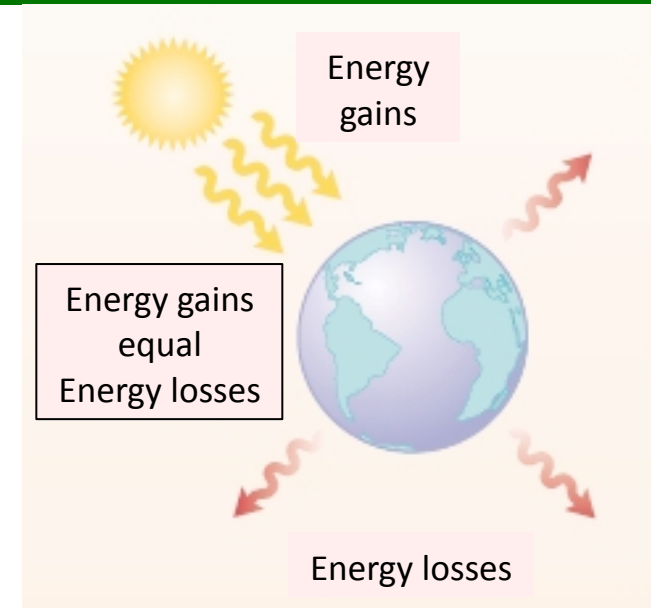
Balance

$$1367\pi r^2 = 410\pi r^2 + (5.67 \times 10^{-8})T^4 4\pi r^2$$

$$1367 = 410 + 4(5.67 \times 10^{-8})T^4$$

$$(5.67 \times 10^{-8})T^4 = 239$$

$$T = 239^{(1/4)}(5.67 \times 10^{-8})^{(-1/4)}$$





Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

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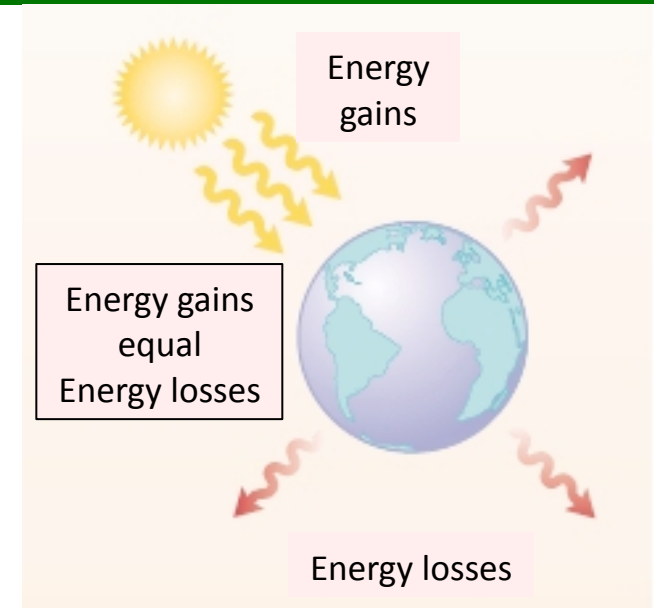
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$$T = 3.93 \times 64.8$$





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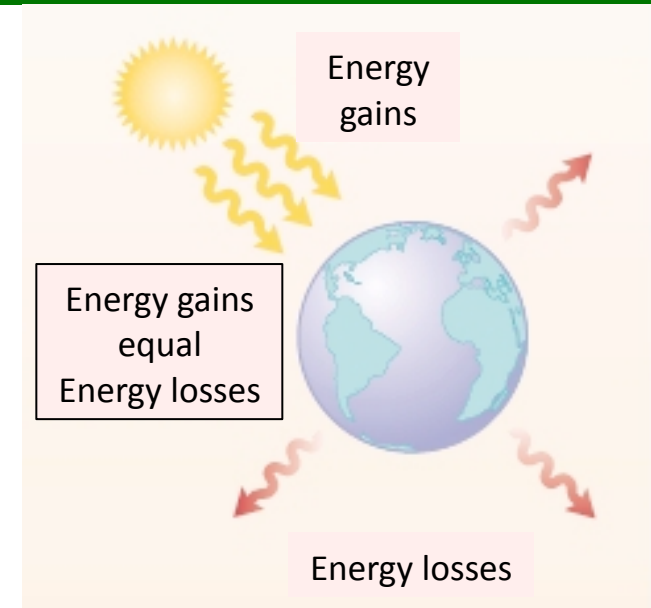
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$$T = 3.93 \times 64.8$$



So.....

$$T = 3.93 \times 64.8 = 255\text{K}$$



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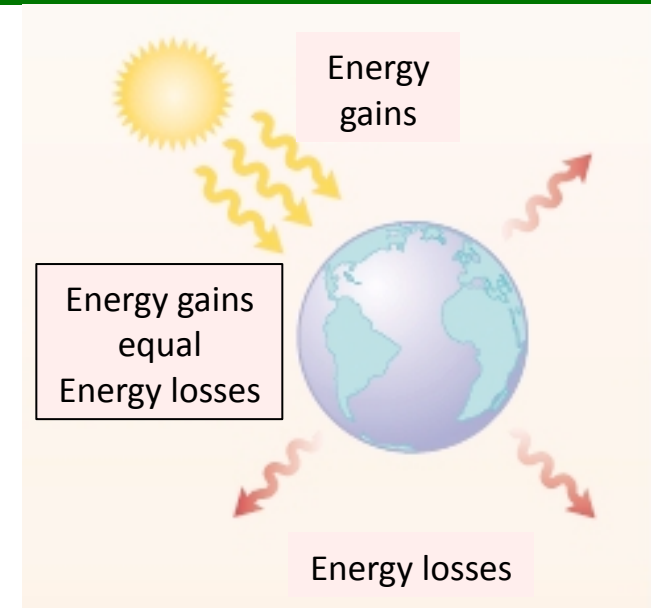
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So.....

$$T = 3.93 \times 64.8 = 255\text{K} = \text{???}^\circ\text{C}$$

(Remember $273\text{K} = 0^\circ\text{C}$)



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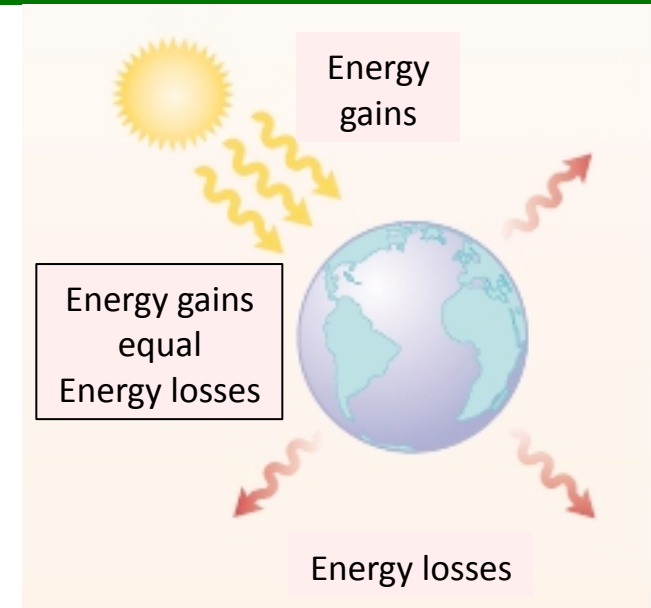
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So.....

$$T = 3.93 \times 64.8 = 255\text{K} = -18^\circ\text{C}$$

Does that sound about right?



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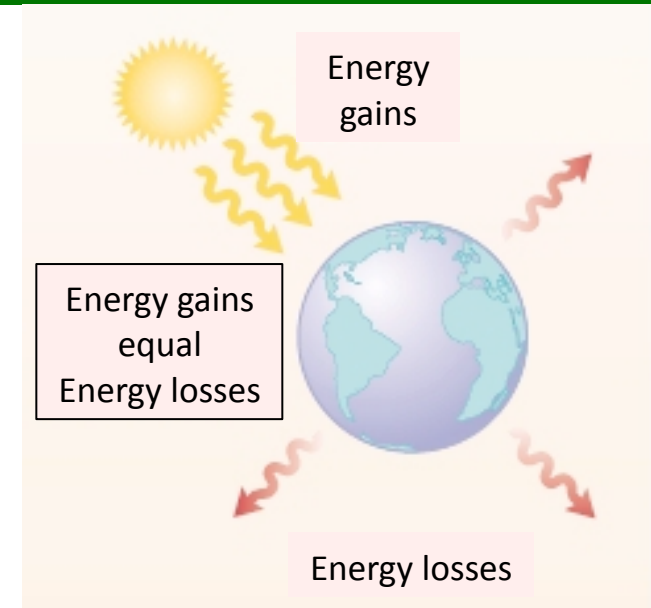
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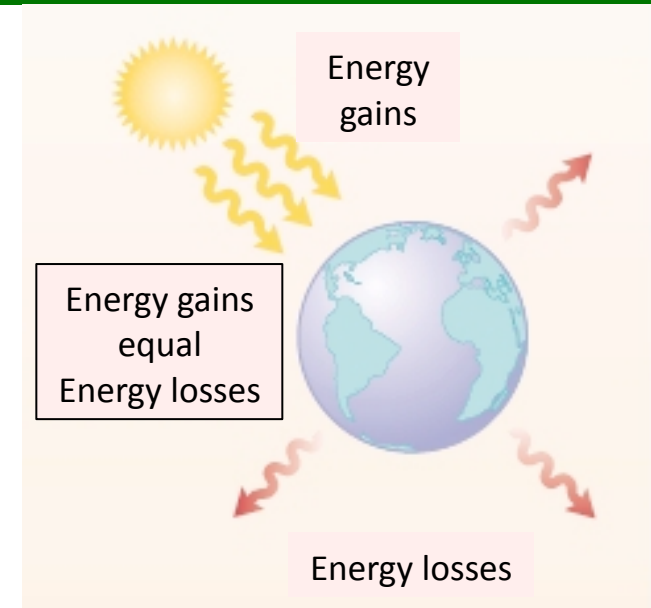
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Recall: our current global average surface temp is 15°C



Energy Balance, No Atmosphere

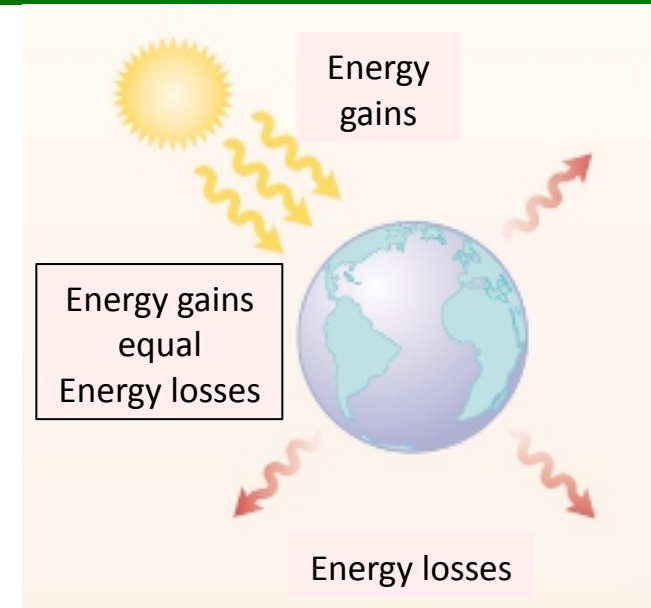
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Conclusions:

With these hypotheses, surface temp is 33°C too low.
We need to include atmosphere to correct this.

255K is a good estimate for the top of the atmosphere,
Which does satisfy the hypotheses more closely.



So.....

$$T = 3.93 \times 64.8 = 255\text{K} = -18^\circ\text{C}$$

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Energy Balance, No Atmosphere

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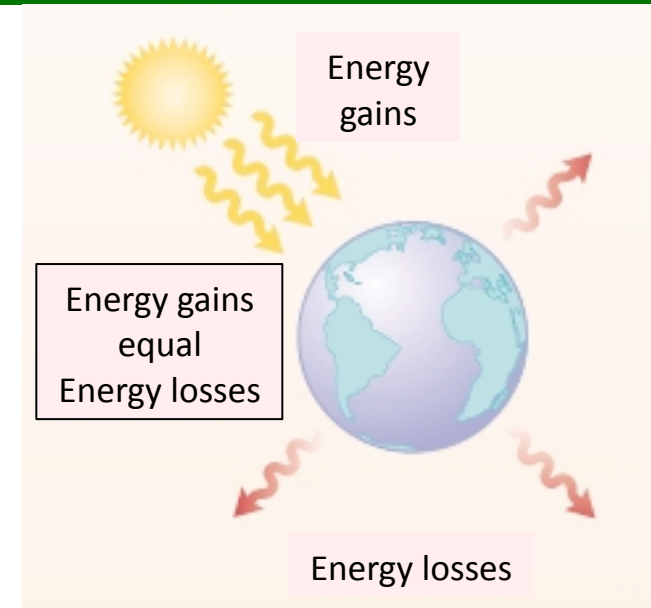
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Need new hypotheses about absorption of solar and
terrestrial radiation by the atmosphere.



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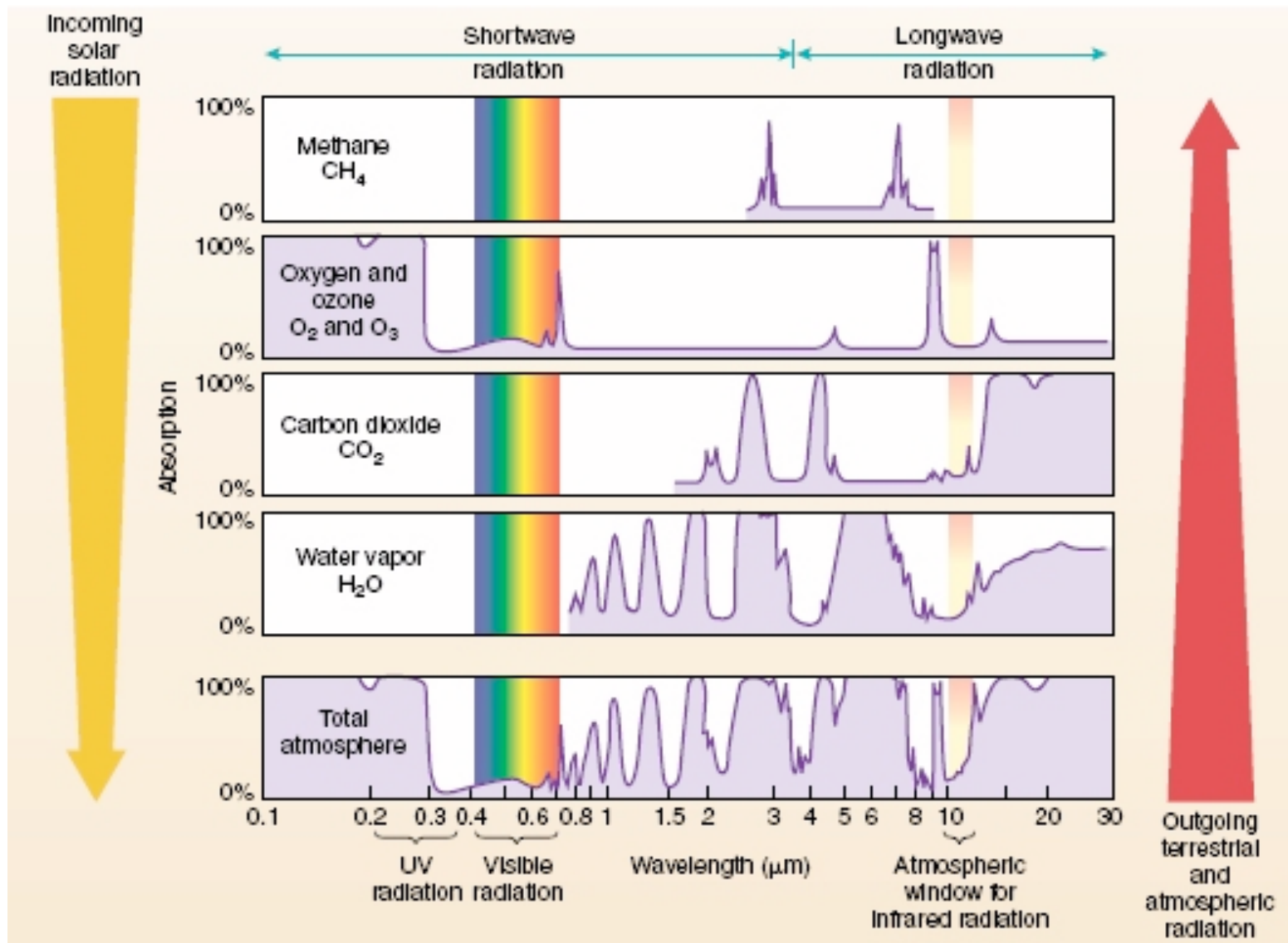
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Greenhouse Gases (GHGs)

Atmospheric Absorption Spectrum



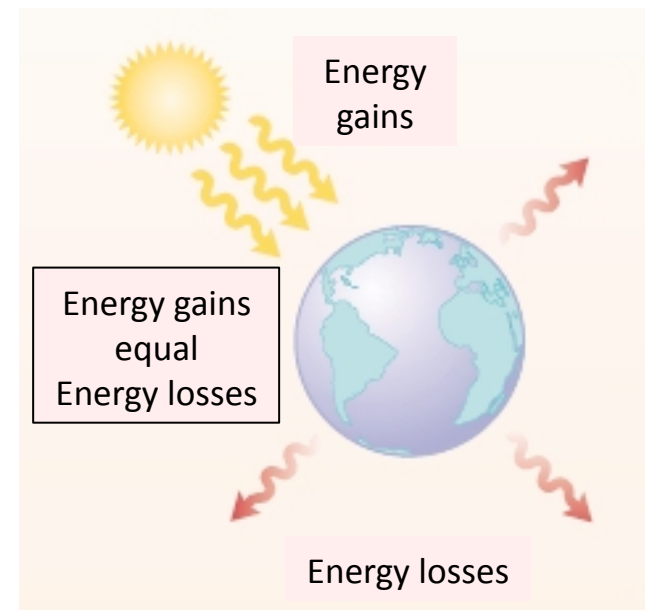


Energy Balance, GHGs in Atmosphere

Hypotheses: Earth's temperature is a consequence of

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3. Atmosphere absorbs some terrestrial radiation

How do these assumptions affect our energy balance computation?





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$$\alpha(\text{Terrestrial flux} * \text{surface area}) = \alpha(5.67 \times 10^{-8}) T^4 4\pi r^2$$



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$$1367\pi r^2 = 410\pi r^2 + \alpha(5.67 \times 10^{-8}) T^4 4\pi r^2$$

$$T = \frac{255}{\alpha^{1/4}}$$



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Case $\alpha=1$: Corresponds to no greenhouse gases

Atmosphere is transparent to terr. radiation, &

$$T = 255 \text{ K} = -18^\circ\text{C}, \text{ as before}$$

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As greenhouse gases increase: α does what??? Increase or decrease?



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So predicted temperature T does what ??? Increase or decrease?



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Want to learn
more???

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To Learn More...



www.mathclimate.org

Mathematics and Climate Research Network – Education pages

www.mpe2013.org

Mathematics of Planet Earth 2013 – Education pages

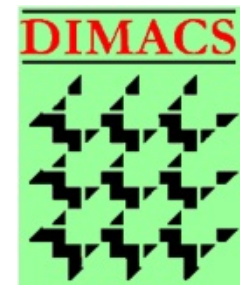


www.mathaware.org

Math Awareness Month 2013 – Mathematics and Sustainability

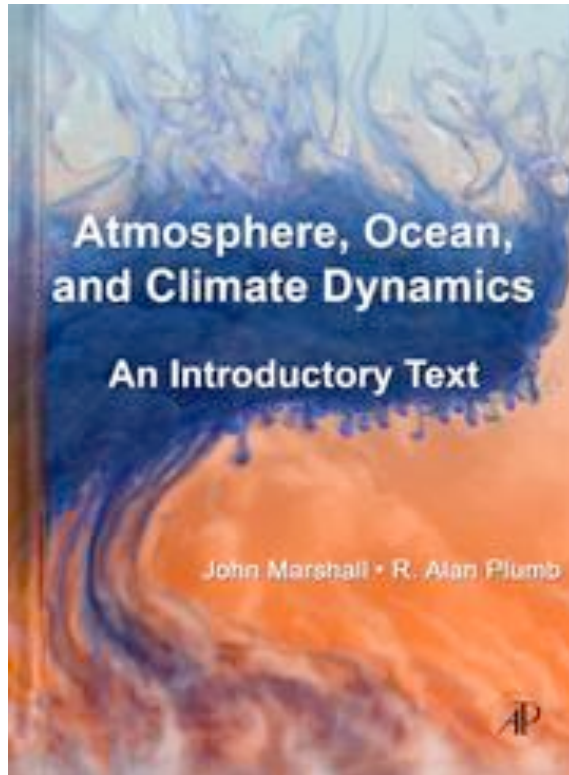
www.dimacs.rutgers.edu/MPE

DIMACS Mathematics and Sustainability Curriculum Modules

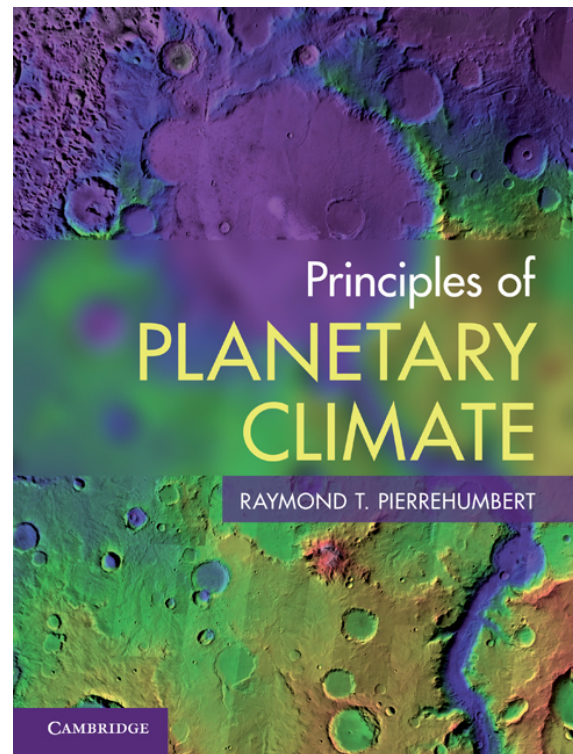




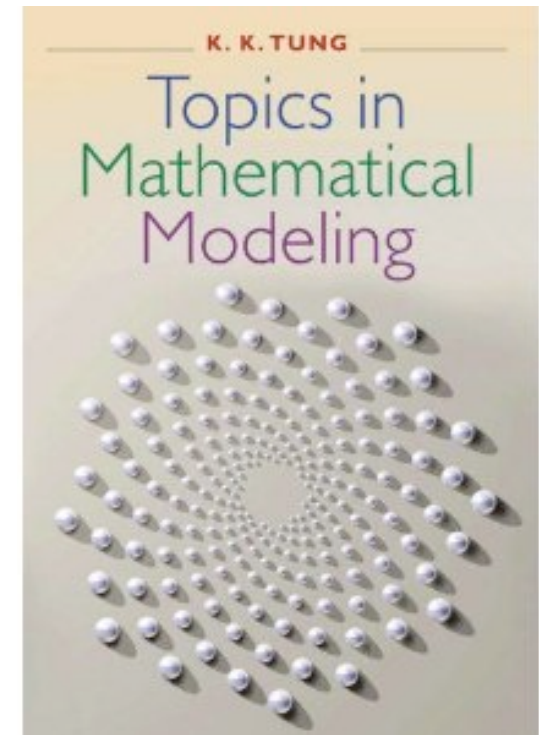
And More...



John Marshall and
Alan Plumb (MIT)



Ray Pierrehumbert



Ka-Kit Tung

Forthcoming text on Mathematics and Climate, Hans Kaper and Hans Engler. SIAM



Math and Global Temperature

THANK
YOU!



Thanks to:
**many friends
& colleagues**



&

Bowdoin