Math and Global Temperature

Mary Lou Zeeman
Bowdoin College
&
Math and Climate
Research Network

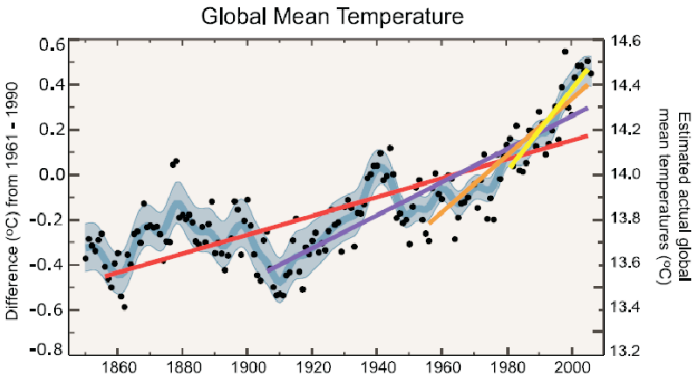
Thanks to:
many friends & colleagues

NSF MCRN Institute for Computational Sustainability ICS & Bowdoin

This talk was originally prepared by Mary Lou Zeeman for a lecture to the Coastal Studies for Girls residential high school in Freeport, Maine, in 2010. It has been modified slightly since then but remains the same in spirit. Feel free to use it. Please make sure to credit sources of information and images.

Evidence of a Changing Climate

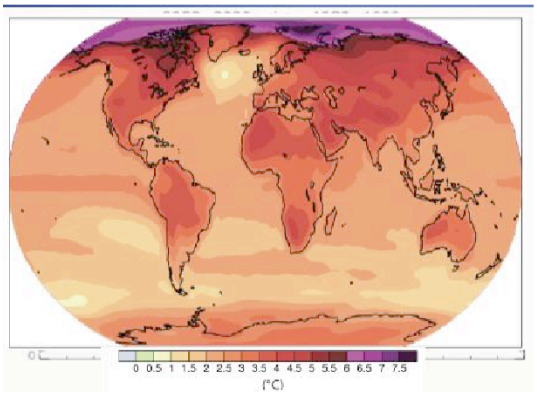
Intergovernmental Panel on Climate Change (IPCC):
Warming is "unequivocal"



Dots: yearly average
Curve: decadal average
Blue: uncertainty interval

We're going to focus on global mean temperature
(mean over planet and over a year)

Why do we try to predict?



Warming is NOT uniform

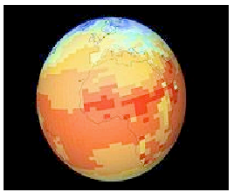
IPCC

Need to make adaptation policy decisions to protect areas of vulnerability

Why use Mathematical Models?

1

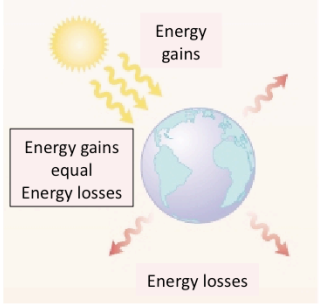
many



C. Jones

We've only got one real experiment…

Simplest Energy Balance Model



CIMSS/ U Wisc

Simple concept that underlies a family of models

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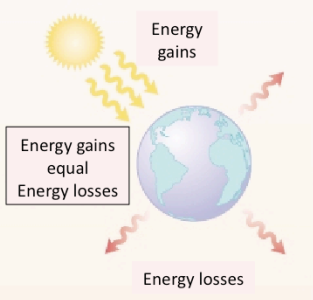
Simplest Energy Balance Model

PLAN: Use this model to understand the role of greenhouse gases in regulating our temperature



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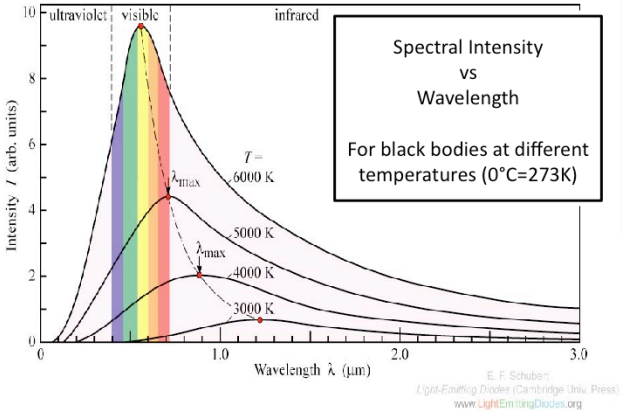
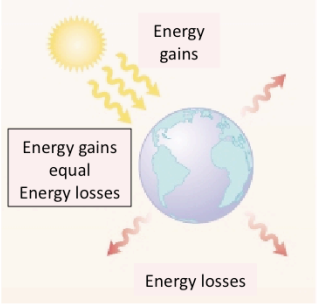
Understanding Radiated Energy

Black Body Radiation - Planck





Sun: ~6000K, emits mainly in ultraviolet (UV) and visible spectrum (shortwave)
Earth: ~300K, emits mainly in infrared (IR) spectrum (longwave)

Total energy flux over all wavelengths for temp T is $\sigma T^4 = (5.67 \times 10^{-8}) T^4$ -- Stefan-Boltzmann

A "Black Body" absorbs everything at all wavelengths
It radiates energy according to it's temperature (discovered by Planck)
Shorter wavelength has higher energy
Earth is an order of magnitude cooler than any of the graphs shown
So Earth radiates energy at much longer wavelengths than sun

Degrees K and C are the same size
Flux is energy/unit area

Not a coincidence that sun emits in our visible spectrum.
A consequence of evolution that our vision has evolved to optimize use of available spectrum

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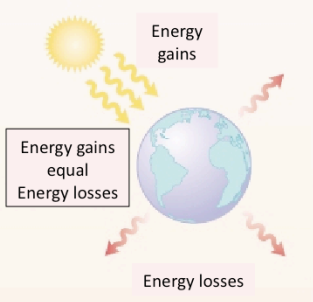
Earth's Average Temperature

Question: What is Earth's average global annual temperature T? What do you think?



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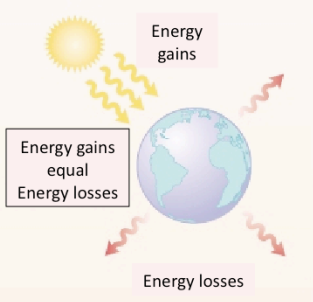
Answer: 15°C (59°F)



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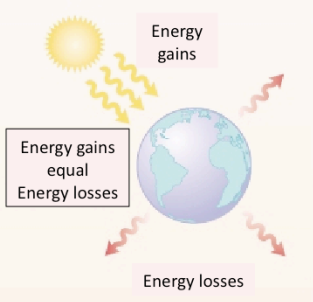
Question: Why? What controls it?



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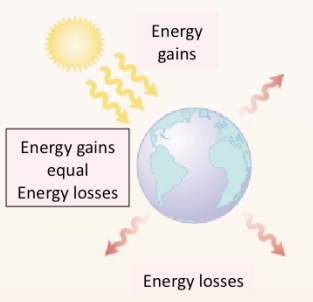
Answer: Lots of things!



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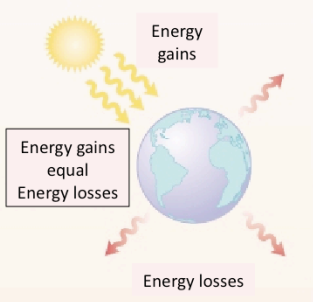

Question: Why? What controls it?

Answer: Lots of things!

Let's use math to test this hypothesis: Earth's temperature is a consequence of

1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

So we pretend the atmosphere is transparent to radiation: "no atmosphere"

Nice example of using mathematical models for hypothesis testing
ATMOSPHERE TRANSPARENT to radiation means no greenhouse gases

Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

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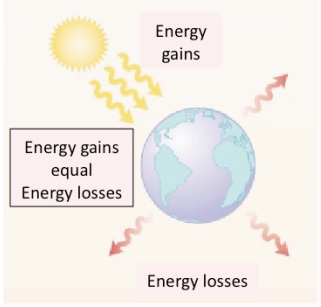

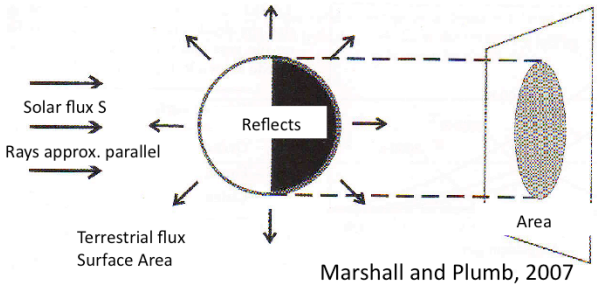

Marshall and Plumb, 2007

Let's relate diagram carefully to each hypothesis

Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

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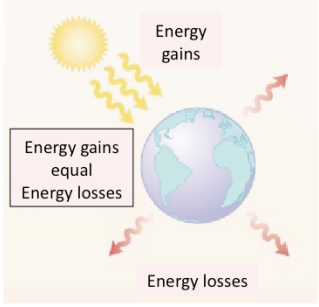
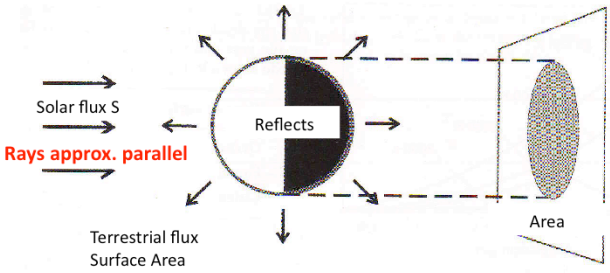
Question: Why are the rays from the sun approx. parallel? Which hypothesis?





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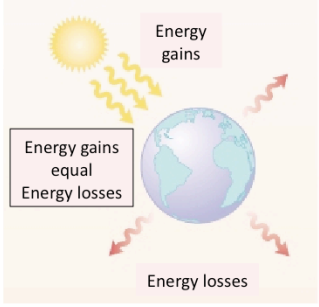
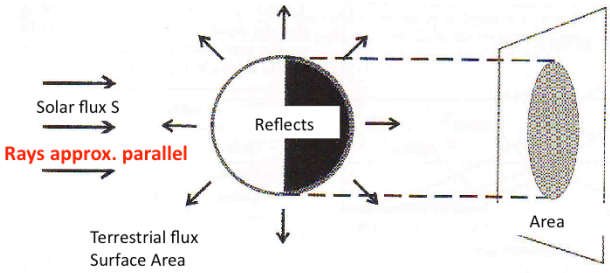


Question: Why are the rays from the sun approx. parallel? Which hypothesis?

Answer: Sun is so far away (hypothesis 2)



Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

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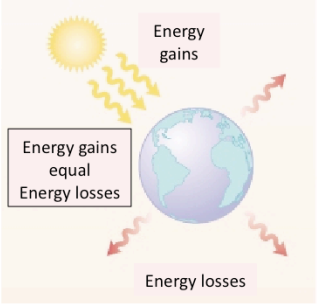
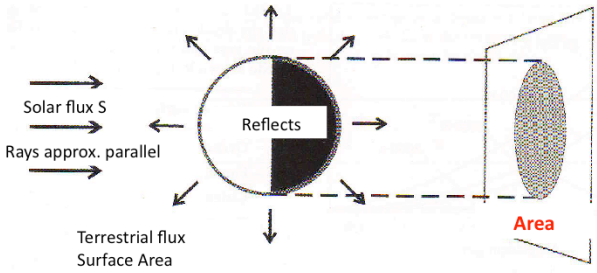
Question: What's the cross sectional area of the Earth (radius r)? Which hypothesis?





Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

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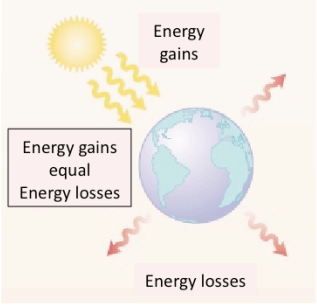
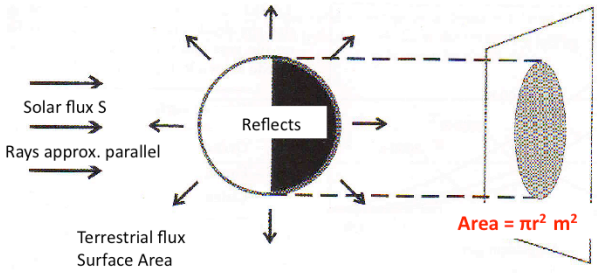
Question: What's the cross sectional area of the Earth (radius r)? Which hypothesis?

Answer: Area = πr^2 m^2 (hypothesis 3)





Energy Balance, No Atmosphere

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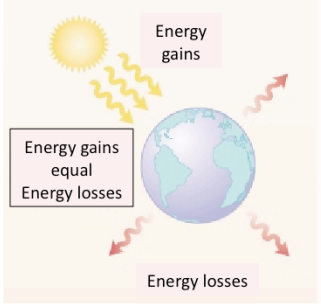
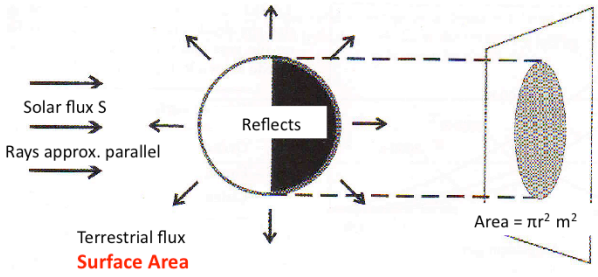
Question: What's the surface area of the Earth (radius r)? Which hypothesis?





Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of
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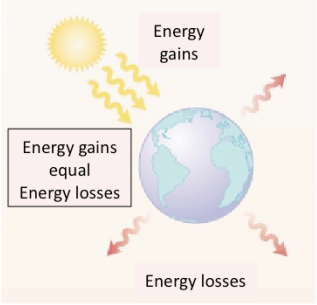
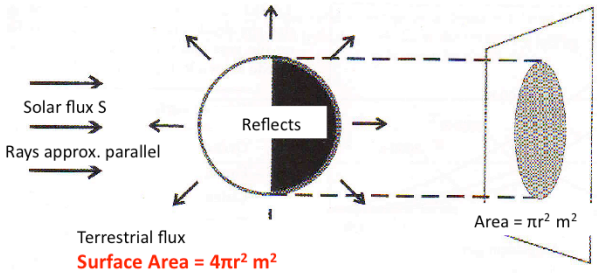
Question: What's the surface area of the Earth (radius r)? Which hypothesis?

Answer: Surface Area = $4\pi r^2$ (hypothesis 3)





Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

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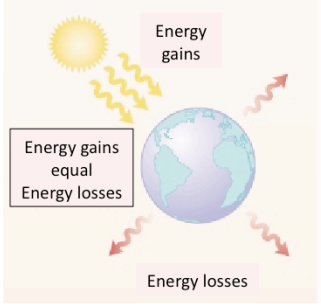
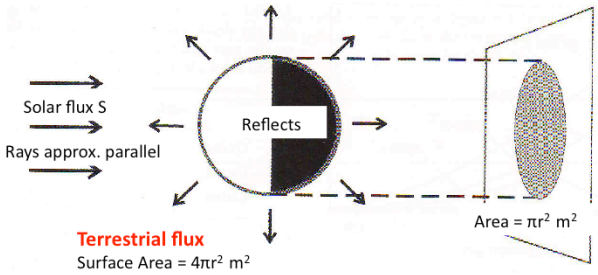
Question: What's the terrestrial flux? Which hypothesis?

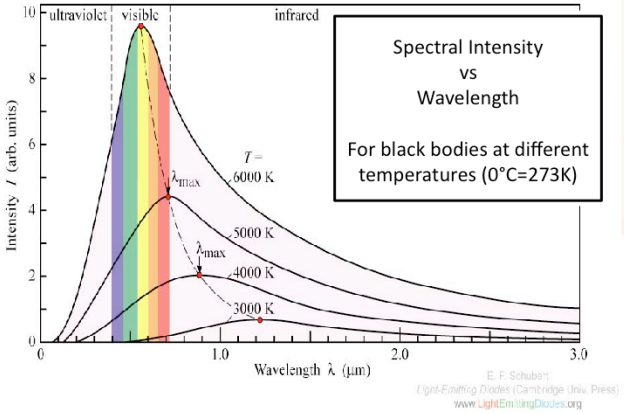


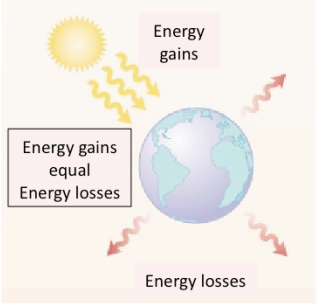


This one is not so obvious – do you remember the formula?

Figuring Out Terrestrial Flux

Black Body Radiation - Planck





Sun: ~6000K, emits mainly in ultraviolet (UV) and visible spectrum (shortwave)
Earth: ~300K, emits mainly in infrared (IR) spectrum (longwave)

Total energy flux over all wavelengths for temp T is $\sigma T^4 = (5.67 \times 10^{-8}) T^4$ -- Stefan-Boltzmann

We are going to use this to find an equation for T

.

Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of
1. Blackbody radiation
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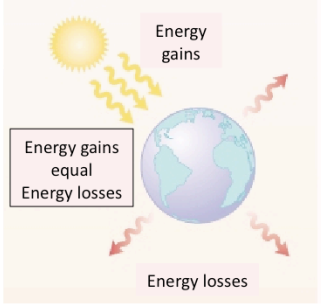
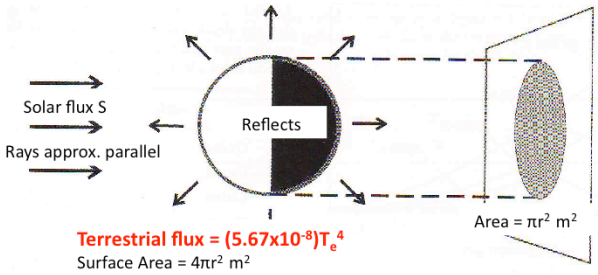
Question: What's the terrestrial flux? Which hypothesis?

Answer: Terrestrial flux = $(5.67 \times 10^{-8})T^4$ Wm^{-2} (hyp. 1)





Terrestrial flux = $(5.67 \times 10^{-8})T_e^4$
Surface Area = $4\pi r^2$ m^2

T is global annual average temperature of the earth

We already know it's 15 degrees because we've measured it, but we're going to build model equations to see what temp. these hypotheses predict.

Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of
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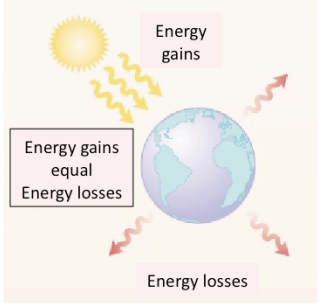
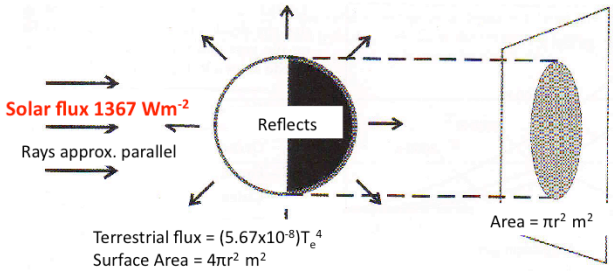
Question: What's the Solar flux? Which hypothesis?

Answer: Solar flux S = 1367 Wm^{-2} (hypothesis 1,2)





This is figured out using sigmaT^4 for the sun, and the distance from the sun. Intensity drops like inverse square of distance.

Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

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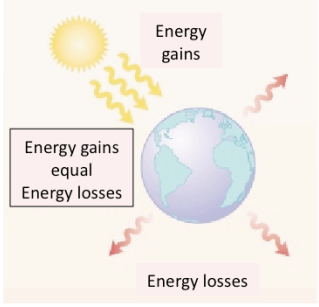
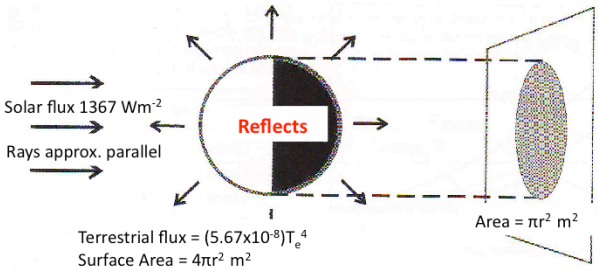
Question: What fraction of solar radiation does the earth reflect? What hypothesis?





What do you guess? On average over the whole planet?

Energy Balance, No Atmosphere

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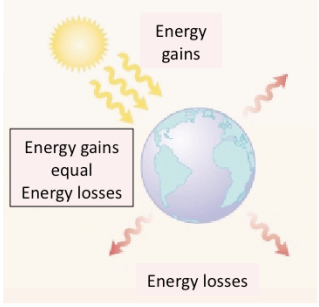
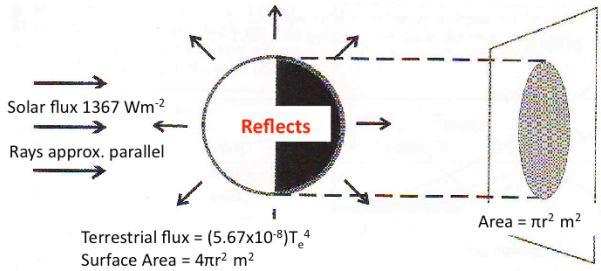


Question: What fraction of solar radiation does the earth reflect? What hypothesis?

Examples:
- Fresh snow reflects 85%
- Sandy desert reflects 40%
- Grasslands reflect 18%
- Cities reflect 16%
- Ocean reflects 6%



Albedo means the fraction that gets reflected

Energy Balance, No Atmosphere

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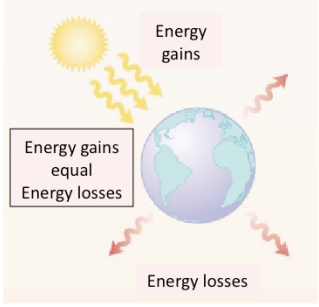

Question: What fraction of solar radiation does the earth reflect? What hypothesis?

Answer: Planetary average albedo 0.3 (hyp. 4)

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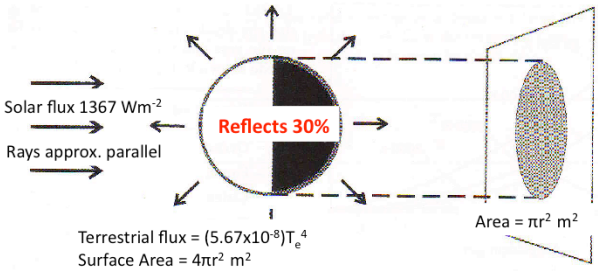

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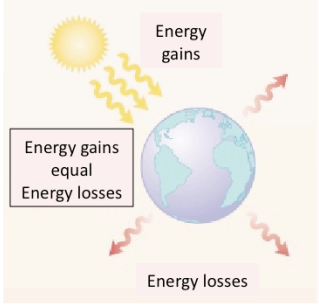
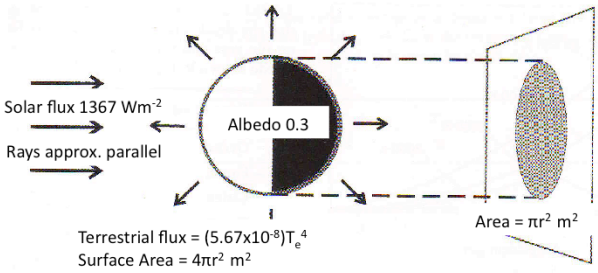


OK! Now we understand the picture,
Let's do energy balance…



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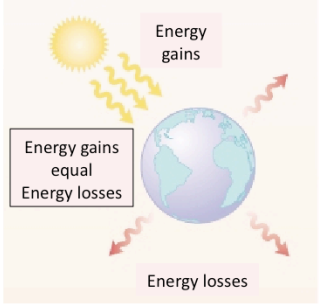
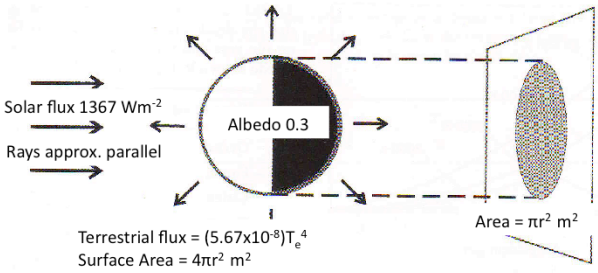


Energy Gain

Solar flux * cross-sect area = ???



This is how much solar flux the earth intercepts, in total.
Flux is energy/unit area
So energy = flux * area

Energy Balance, No Atmosphere

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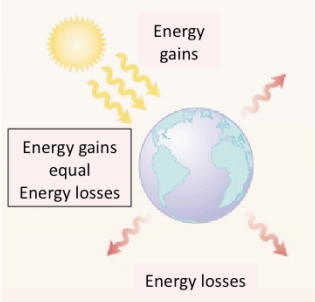
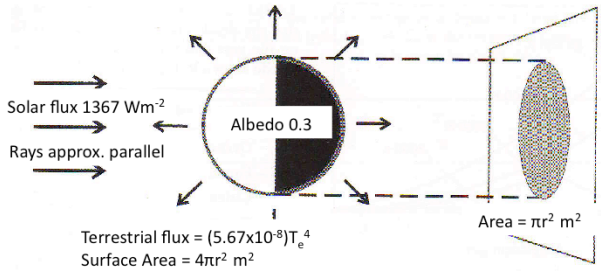


Energy Gain

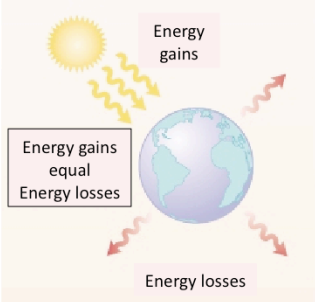
Solar flux * cross-sect area = 1367 Wm^{-2} * πr^2 m^2 = $1367\pi r^2$ W



Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

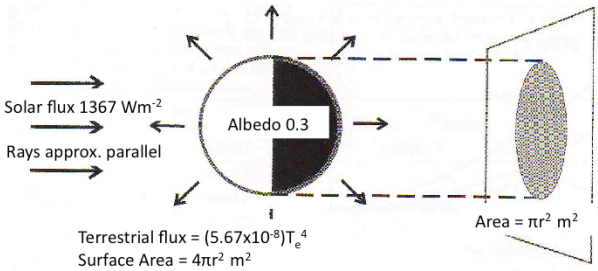


Energy Gain

Solar flux * cross-sect area = $1367\pi r^2$ W

Energy loss by Terrestrial Radiation

Terrestrial flux * surface area = ???



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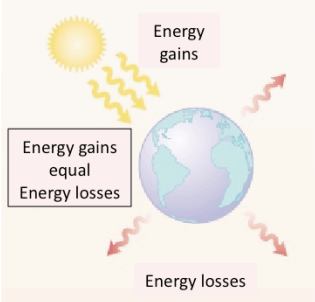
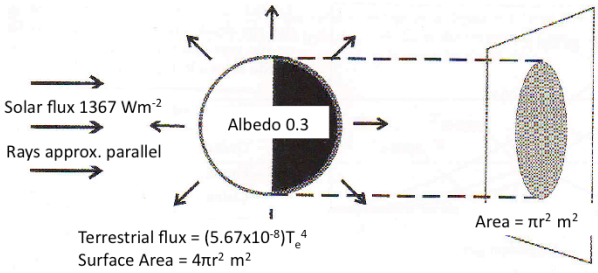


Energy Gain

Solar flux * cross-sect area = $1367\pi r^2$ W

Energy loss by Terrestrial Radiation

Terrestrial flux * surface area = $(5.67 \times 10^{-8})T^4\ 4\pi r^2$ W



Terrestrial flux given by Stefan-Bolzman law of radiation from a black body at temperature T

Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of
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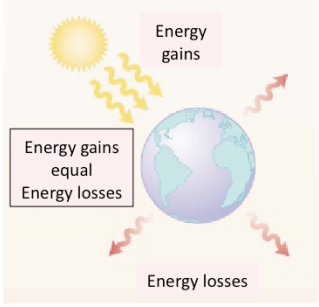
4. Albedo (reflectivity)



Energy Gain

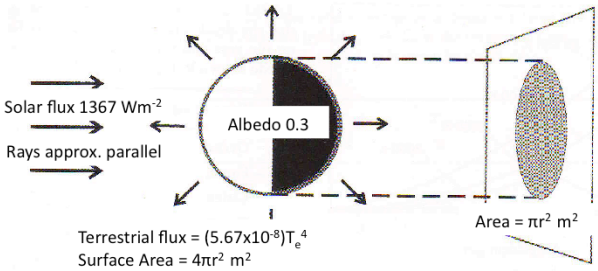
Solar flux * cross-sect area = $1367\pi r^2$ W

Energy loss by Terrestrial Radiation

Terrestrial flux * surface area = $(5.67 \times 10^{-8})T^4\ 4\pi r^2$ W

Energy loss by Albedo

Reflected solar radiation = ???



Albedo (reflectivity) * solar radiation arriving

Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

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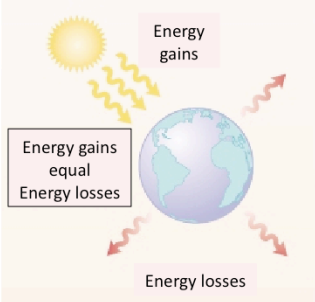


Energy Gain

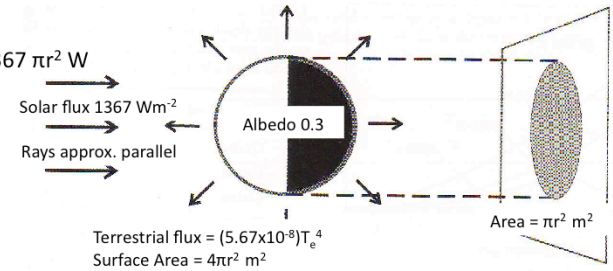
Solar flux * cross-sect area = $1367\pi r^2$ W

Energy loss by Terrestrial Radiation

Terrestrial flux * surface area = $(5.67 \times 10^{-8})T^4\ 4\pi r^2$ W

Energy loss by Albedo

Reflected solar radiation = $0.3 \times 1367\ \pi r^2$ W



Energy Balance, No Atmosphere

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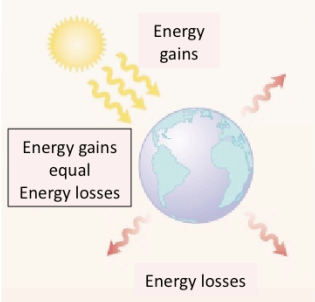


Energy Gain

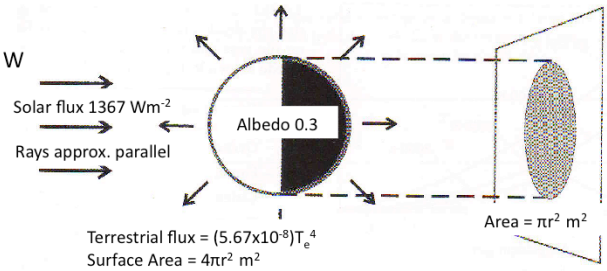
Solar flux * cross-sect area = $1367\pi r^2$ W

Energy loss by Terrestrial Radiation

Terrestrial flux * surface area = $(5.67 \times 10^{-8})T^4\ 4\pi r^2$ W

Energy loss by Albedo

Reflected solar radiation = $410\pi r^2$ W



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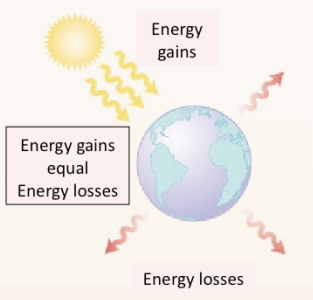

Energy Gain

Solar flux * cross-sect area = $1367\pi r^2$ W

Energy loss by Terrestrial Radiation

Terrestrial flux * surface area = $(5.67 \times 10^{-8})T^4\ 4\pi r^2$ W

Energy loss by Albedo

Reflected solar radiation = $410\pi r^2$ W

Balance

$1367\pi r^2 = 410\pi r^2 + (5.67 \times 10^{-8})T^4\ 4\pi r^2$

Cancel pi r^2 and solve for T

Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

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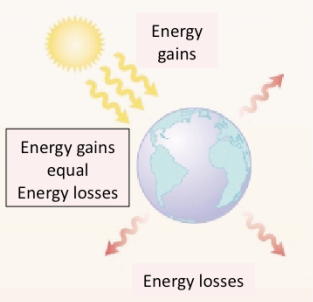

Energy Gain

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Energy loss by Terrestrial Radiation

Terrestrial flux * surface area = $(5.67 \times 10^{-8})T^4\ 4\pi r^2$ W

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Balance

$1367\pi r^2 = 410\pi r^2 + (5.67 \times 10^{-8})T^4\ 4\pi r^2$

$1367 = 410 + 4(5.67 \times 10^{-8})T^4$

Energy Balance, No Atmosphere

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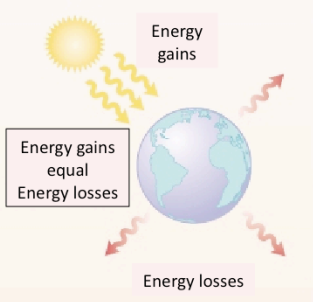

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Energy loss by Albedo

Reflected solar radiation = $410\pi r^2$ W

Balance

$1367\pi r^2 = 410\pi r^2 + (5.67 \times 10^{-8})T^4\ 4\pi r^2$

$1367 = 410 + 4(5.67 \times 10^{-8})T^4$

$(5.67 \times 10^{-8})T^4 = 239$

Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

1. Blackbody radiation
2. Distance from Sun
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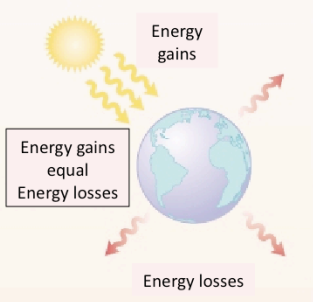

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Balance

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$1367 = 410 + 4(5.67 \times 10^{-8})T^4$

$(5.67 \times 10^{-8})T^4 = 239$

$T = 239^{(1/4)}(5.67 \times 10^{-8})^{(-1/4)}$

Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

1. Blackbody radiation
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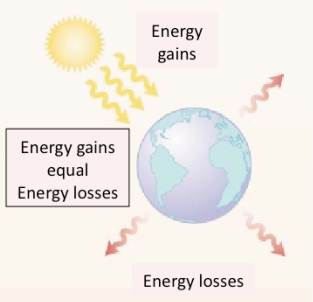

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$T = 3.93 \times 64.8$

Energy Balance, No Atmosphere

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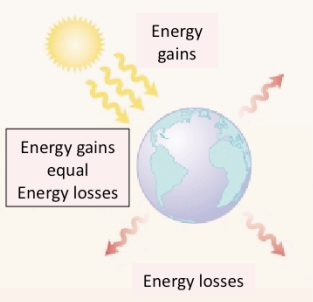

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$(5.67 \times 10^{-8})T^4 = 239$

$T = 239^{(1/4)}(5.67 \times 10^{-8})^{(-1/4)}$

$T = 3.93 \times 64.8$

So…….

$T = 3.93 \times 64.8 = 255K$

Energy Balance, No Atmosphere

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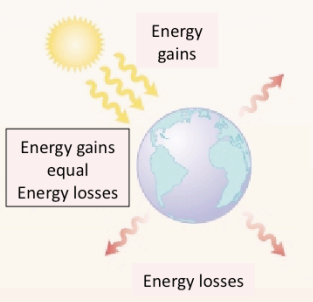

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So.....

$T = 3.93 \times 64.8 = 255K =$???°C

(Remember 273K = 0°C)

Energy Balance, No Atmosphere

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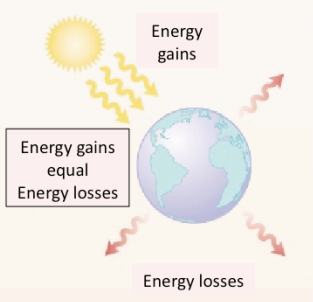

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Does that sound about right?

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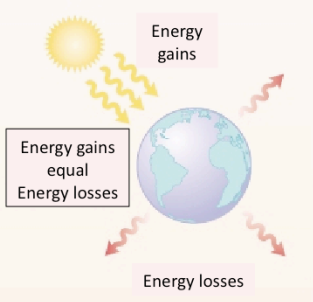

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Too Cold – Snowball Earth!

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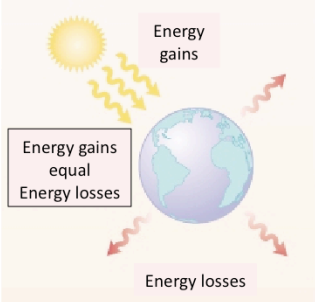

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Recall: our current global average surface temp is 15°C

Energy Balance, No Atmosphere

Hypothesis: Earth's temperature is a consequence of

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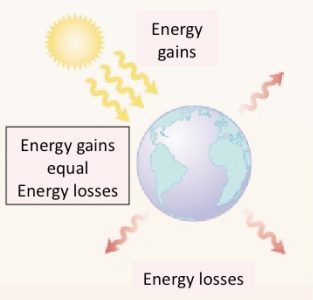
3. Size
4. Albedo (reflectivity)

Conclusions:

With these hypotheses, surface temp is 33°C too low.
We need to include atmosphere to correct this.

255K is a good estimate for the top of the atmosphere,
Which does satisfy the hypotheses more closely.



So.....

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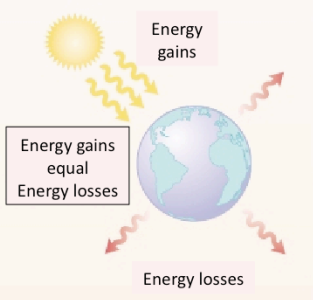


Conclusions:

With these hypotheses, surface temp is 33°C too low.
We need to include atmosphere to correct this.

255K is a good estimate for the top of the atmosphere, Which does satisfy the hypotheses more closely.

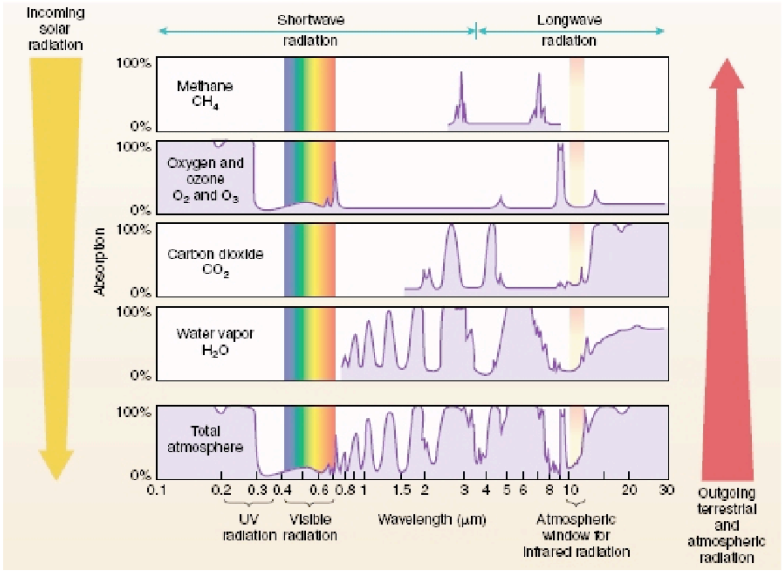
Need new hypotheses about absorption of solar and terrestrial radiation by the atmosphere.

So.....

$T = 3.93 \times 64.8 = 255K = -18°C$

Does that sound about right?
Too Cold – Snowball Earth!
Recall: our current global average surface temp is 15°C

Greenhouse Gases (GHGs)
Atmospheric Absorption Spectrum



CIMSS/ U Wisc

Atmosphere is not transparent.

Figure shows absorption spectrum due to major greenhouse gas constituents of the atmosphere

Shortwave range: solar radiation coming in – atmosphere is mainly transparent to that

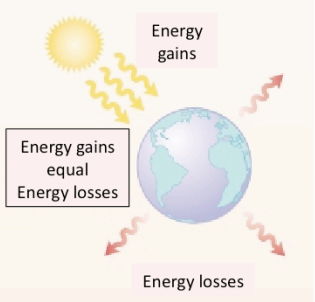
Longware (IR) range: terrestrial radiation leaving – atmosphere is partially opaque to IR

Energy Balance, GHGs in Atmosphere

Hypotheses: Earth's temperature is a consequence of

1. Radiation, distance from sun, albedo
2. Atmosphere transparent to solar radiation
3. Atmosphere absorbs some terrestrial radiation

How do these assumptions affect our energy balance computation?



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Energy Gain - unchanged by new assumptions

Solar flux * cross-sect area = $1367\pi r^2$

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Energy loss by Albedo – unchanged

Reflected solar radiation = $410\pi r^2$ W

Energy Balance, GHGs in Atmosphere

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Energy loss by Terrestrial Radiation - reduced! Let's use parameter α to reduce it.

α(Terrestrial flux * surface area) = $\alpha(5.67 \times 10^{-8})T^4\ 4\pi r^2$

Energy Balance, GHGs in Atmosphere

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New Balance

$1367\pi r^2 = 410\pi r^2 + \alpha(5.67 \times 10^{-8})T^4\ 4\pi r^2$

$$T = \frac{255}{\alpha^{1/4}}$$

Energy Balance, GHGs in Atmosphere

Hypotheses: Earth's temperature is a consequence of

1. Radiation, distance from sun, albedo
2. Atmosphere transparent to solar radiation
3. Atmosphere absorbs some terrestrial radiation

New Energy Balance: $T = \frac{255}{\alpha^{1/4}}$

Case α=1: Corresponds to no greenhouse gases

Atmosphere is transparent to terr. radiation, &

T = 255 K = -18°C, as before

Snowball Earth! We need *some* GHGs to avoid this.

As greenhouse gases increase: α does what??? Increase or decrease?

Energy Balance, GHGs in Atmosphere

Hypotheses: Earth's temperature is a consequence of
1. Radiation, distance from sun, albedo
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As greenhouse gases increase: α decreases
So predicted temperature T does what **???** Increase or decrease?

Energy Balance, GHGs in Atmosphere

Hypotheses: Earth's temperature is a consequence of

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T = 255 K = -18°C, as before

Snowball Earth! We need *some* GHGs to avoid this.

As greenhouse gases increase: α decreases

So predicted temperature T increases

Energy Balance, GHGs in Atmosphere

Hypotheses: Earth's temperature is a consequence of
1. Radiation, distance from sun, albedo
2. Atmosphere transparent to solar radiation
3. Atmosphere absorbs some terrestrial radiation

New Energy Balance: $T = \frac{255}{\alpha^{1/4}}$

Case α=1: Corresponds to no greenhouse gases
Atmosphere is transparent to terr. radiation, &
T = 255 K = -18°C, as before
Snowball Earth! We need *some* GHGs to avoid this.

As greenhouse gases increase: α decreases
So predicted temperature T increases

Conclusion
As greenhouse gases keep increasing, temperature keeps increasing…

Energy Balance, GHGs in Atmosphere

Hypotheses: Earth's temperature is a consequence of
1. Radiation, distance from sun, albedo
2. Atmosphere transparent to solar radiation
3. Atmosphere absorbs some terrestrial radiation

New Energy Balance: $T = \frac{255}{\alpha^{1/4}}$

Case α=1: Corresponds to no greenhouse gases
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Want to learn more???...

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To Learn More…

www.mathclimate.org

Mathematics and Climate Research Network – Education pages

www.mpe2013.org

Mathematics of Planet Earth 2013 – Education pages

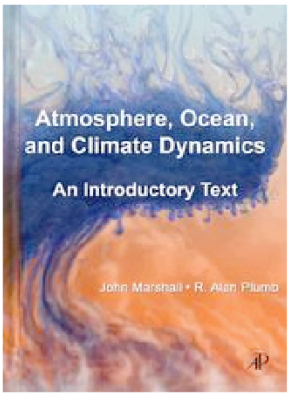
www.mathaware.org

Math Awareness Month 2013 – Mathematics and Sustainability

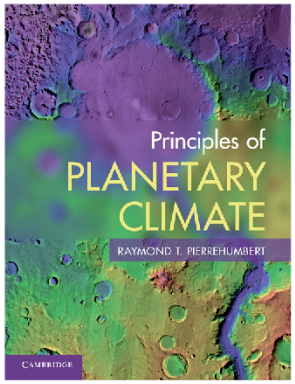
www.dimacs.rutgers.edu/MPE

DIMACS Mathematics and Sustainability Curriculum Modules

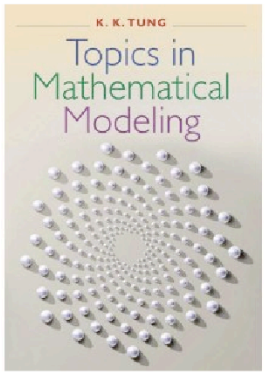
And More…



John Marshall and Alan Plumb (MIT)



Ray Pierrehumbert



Ka-Kit Tung

Forthcoming text on Mathematics and Climate, Hans Kaper and Hans Engler. SIAM

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Math and Global Temperature

THANK YOU!

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