# Percentage Play in Tennis 

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#### Abstract

Tennis scoring makes tennis different from most games in that scoring at the point level is not cumulative and it is thus possible for a player scoring fewer points than her or his opponent to win a match. In this paper, a probabilistic model for a tennis match is constructed in which the probabilities associated with winning points are used to analyze the chances of winning matches. Charts are used to illustrate the consequences of the model and some numerical outcomes are highlighted to indicate how raw match data might be used to indicate how a player might improve her or his game. The notion of a "big" shot is examined within the model and the possibility of a "big" shot turning a losing game into a winning game is addressed.


## Introduction

One of the intriguing supplements to television commentary for tennis matches these days is the computer analysis from software packages that compile the results from the charting of matches. Charting is not new, but the facility that the computer provides in creating, tabulating, and cross-referencing data from the charts has made more information easily accessible. The computer can use the charts to provide ready information on the progress of a match or data for the analysis of a player's strengths and weaknesses over several or many matches. Furthermore, software that can provide such data is readily available. A fundamental question that follows is "What does a player need to know to put the data to best use?"

If the commentators who do tennis on television or the writers who do analysis in the tennis magazines are representative, some grave misunderstandings exist as to what the mathematical consequences of the percentages associated with stroke production are. The big question may not be "How well does a player play the big points?"; it is probably "How well does a player have to play to get to play a big point?". (Vic Braden would probably add "...more than once a summer?"!) In Tennis for the Future, Braden emphasizes that the laws of physics are a given; a player can either fight them or use them to his or her benefit. The laws of probability are also a given. A better understanding of tennis from a mathematical point of view can give a player a clearer picture of how good she or he needs to be relative to the strength of the opponent in order to get to the point that strategy has a reasonable chance of making a difference.

A mathematics/tennis intuition test:

1. Suppose that a player expects to win, on the average, one point for each three the opponent gets on serve. How often during the course of a match should the player expect to be able to break serve?
A. one game in three?
B. one game in six?
C. one game in twenty?
2. Suppose that a player expects to win four points out of five on serve. How frequently can the player expect to win six consecutive service games?
A. four times out of five?
B. two times out of three?
C. three times out of five?

If you took the test and answered "one game in twenty" to the first question and "none of these, they're all too high" to the second; your tennis intuition is in line with the mathematical realities. If you missed one of the questions, however, you can probably benefit from learning how to make a mathematical model of a tennis match. After all, a player's view of how a tennis match will go is likely biased by how the player would like it to go. What's more, understanding what a model predicts can put a player or coach in a position to answer questions that practice can help.

## The Model

The basic element of tennis scoring is the point, but tennis matches are won by the player who wins two out of three (or three out of five) sets --- a scoring tally three steps removed from the point. Players can practice making shots which can win points. In order, however, to plan on winning matches, a player must understand the cumulative impact that winning points is likely to have on winning matches. The initial assumption for our model is that there exists a source for knowledge of the expectation of winning any one point. This is a good assumption mathematically because it begins with the basic element of scoring and, from it, conclusions about the likely outcomes of games, and therefore sets and matches are available. It is a good assumption practically because data can be gathered whenever players have played each other, and these data can be used to suggest plausible probabilities. What's more, data can be collected that will afford analysis of parts of the game which can be directed at individual strokes or special tactics.

Suppose that a model is to be made to prepare for an upcoming match between two players. To begin, assign to each player the probability of winning a point against the other; that is, represent the probability of the first player winning a point against the second and vice versa, with non-negative numbers that sum to 1 . The second assumption for the model is that a player is as likely to win any one point in a game as any other. Mathematically this makes the model computationally tractable. Practically it eliminates factors such as momentum, changes in circumstance, or "mental toughness." The assumption allows the model to act as if there are no corrupting intangibles involved and that the match will be played under similar conditions as before, and thus to predict what is likely to happen based on previous experience. (Ironically, eliminating intangibles from the model can provide a baseline against which the existence of intangibles such as "playing the big points well" can be studied scientifically.)

## The Calculations

The model begins with two probabilities that sum to 1 and the assumption that the probability for winning a point does not depend on the outcome of the previous point. Represent these two probabilities by $u$ and $f$, $u$ for the likelihood that the underdog will win a point and $f$ for the likelihood that the favorite will win a point. In traditional scoring, there are four ways to win a game: at love(0), 15, 30, or after deuce. Thus the probability of the underdog winning a game can be calculated by computing the separate probabilities of winning each type of game and then summing these probabilities. For a love game, one player must win all four points, so for the underdog this gives a probability of $u^{4}$. For a game at 15 , the probability of the underdog winning is $4 u^{4} f$ (win the fifth point and three of the other four; that is, win the first three and the fifth points, or win the first two and last two points, or win the first point and last three points, or win the last four points). For the underdog to win a game at 30, the player must win the sixth point and three of the first five for a total probability of $10 u^{4} f^{2}$. The probability of the underdog winning a deuce game is $\left(20 u^{5} f^{3}\right) /(1-2 u f)$. In Van Alen (no-ad) scoring, if the score is tied at three points for each player, instead of playing until one player achieves a two-point advantage, a single point
is played, with the receiver choosing court, to decide the winner of the game. If Van Alen scoring is used, the probability of the underdog winning a no-ad game replaces that of a deuce game and is $20 u^{4} f^{3}$. Thus the probability of winning a game can be computed given the probability of winning a point. Chart 1 shows comparative values that come from the formulas.

Chart 1

| Probability underdog will win point | Probability underdog will win game <br> (standard scoring) | Probability underdog will game <br> (Van Alen scoring) |
| :--- | :--- | :--- |
| $.05(1$ point in 20) | .00009232 | .000193 |
| .1 (1 point in 10) | .0014478 | .002728 |
| $.15(3$ points in 20) | .0071371 | .012103 |
| $.2(1$ point in 5) | .0217788 | .033344 |
| $.25(1$ point in 4$)$ | .050781 | .070557 |
| $.3(3$ points in 10) | .099211 | .126036 |
| $.33(1$ point in 3$)$ | .139081 | .168214 |
| $.35(7$ points in 20) | .1703553 | .1999846 |
| $.375(3$ points in 8$)$ | .21462 | .243021 |
| $.4(4$ points in 10) | .264271 | .289792 |
| $.45(9$ points in 20) | .37685 | .391712 |
| .46 | .4 | .413 |
| .47 | .4254 | .4346 |
| .48 | .45 | .456 |
| .49 | .475 | .478 |
| .5 | .5 | .5 |

Entries in the table may be interpreted relative to some match situations. What might give the illusion of an underdog giving someone a good match? How about winning every other point, that is, getting to 30-40 consistently? This means winning two points for each three the opponent wins, corresponding to a probability of .4. This forecasts winning about $1 / 4$ of the games, or about four out of sixteen. That's losing 6-2, 6-2! How about the probability associated with losing deuce games on the first deuce? (Consistently getting to deuce would justify the illusion of being competitive.) A losing one-deuce game means winning three points while losing five and corresponds to a probability of .375. In this case, on the average, the underdog can expect to win about one game out of five; that's a 6-1 set for each 6-2 set! There is a lesson here : Being competitive in individual games does not translate into any realistic hope of winning sets, much less matches. (Nevertheless, comparing the far right-hand column to the middle column, given the choice, an underdog should prefer Van Alen scoring to standard scoring.) To re-emphasize the consequences of the data in Chart 1, Chart 2 contains data that relate probabilities of winning a point to probabilities of winning a set or a match.

Chart 2

| Probability of winning <br> point | Probability of winning <br> set | Probability of winning <br> match (2 out of 3) | Probability of winning <br> match (3 out of 5) |
| :--- | :--- | :--- | :--- |
| .05 | $1.3 \times 10^{-22}$ | tiny | really tiny |
| .1 | $1.928 \times 10^{-21}$ | $1.1 \times 10^{-29}$ | tiny |
| .15 | $2.73 \times 10^{-11}$ | $2.246 \times 10^{-21}$ | $2.05 \times 10^{-31}$ |
| .2 | .00000021534 | $1.39 \times 10^{-15}$ | $10^{-22}$ |
| .25 | .0000033 | $3.2646 \times 10^{-11}$ | $3.6 \times 10^{-16}$ |
| .3 | .0001161363 | .000000843 | $4.71 \times 10^{-11}$ |
| .33 | .001161363 | .00000404316 | .0000000156 |
| .35 | .003605583 | .00003890694 | .0000004662 |
| .375 | .01255762 | .0004691209 | .00001943146 |
| .4 | .03652245 | .003904234 | .00046087 |
| .45 | .1846393 | .08968572 | .04680059 |

These numbers tell a sad story. Unless a player can win almost half the points, the player is going to lose the match. To get to the point of occasional upset (just under one two-out-of-three match out of every twelve played), a player needs to expect to win nine points for every eleven of the opponent.

## Big Shot Strategies

Fortunately for those who might not typically go on court expecting to win more points than the day's opponent, there is relief from the numbers. The model employed so far does not take into account the conditional probabilities associated with the "big" shot, a shot that, if a player gets to hit it, increases the chances for winning the point increase considerably. To make a model that will assess the impact of such a shot, three qualities need to be known: the "effectiveness" the shot is likely to have, the likelihood the player will get to hit the shot, and the likelihood the player will win the point if she or he doesn't get to hit the shot. All of these factors can be quantified from match charts. A reasonable way to model effectiveness is to count points in which the stroke being studied is hit and note the outcomes. An effectiveness probability can then be computed by dividing the number of points during which the shot was hit into the number of points won during which the shot was hit. To model the likelihood that a shot will be hit, divide the number of points observed into the number of points in which the shot being studied was hit. Finally, to model the effectiveness (or lack of effectiveness) when the shot is not hit, divide the total number of points in which the shot being studied was not hit into the number of points won in which the shot was not hit. (This idea can be expanded to include combinations of shots.)

Let A represent the probability that the shot being studied will be hit, E represent the effectiveness probability of the shot, and $E$ 'the effectiveness probability when the shot is not hit. The probability of winning a point is $(A E)+((1-A) E)$. Since, from previous considerations, it is clear that a player must win more than half of the points to win consistently, the focus belongs on the relationships among $A, E$, and $E^{\prime}$ in the formula and a search for values for which the formula yields a value of at least .5. Assuming that $E>E^{\prime}$ makes sense ${ }^{1}$ because if the shot

[^0]being analyzed is not more efficient than the player's game without it, then it hardly qualifies as a "big" shot. Mathematically, this has the useful side effect of guaranteeing that increased opportunity or efficiency means increased expectation of winning points. This enables computation of values for $A, E$, and $E^{\prime}$ so that raising the value of $A$ or $E$ while maintaining the value of $E^{\prime}$ results in a higher expectation of winning a point. Hence the basis for the analysis can be focused on computing values for $A, E$, and $E^{\prime}$ where the expectation of winning a point is exactly .5 with the assurance that any higher values of $A$ and $E$ will increase the expectation of winning. Chart 3 lists values for $A, E$, and $E^{\prime}$ in which $E>E^{\prime}$ and the resulting probability of winning a point.

## Chart 3

| Expectation of hitting <br> shot | Effectiveness with shot | Effectiveness without <br> shot | Probability of winning <br> point |
| :--- | :--- | :--- | :--- |
| .2 | 1 | .375 | .5 |
| .39 | .7 | .375 | .5 |
| .455 | .65 | .375 | .5 |
|  |  |  |  |
| .17 | 1 | .4 | .5 |
| .33 | .7 | .4 | .5 |
| .4 | .65 | .4 | .5 |
|  | 1 | .45 |  |
| .09 | .7 | .45 | .5 |
| .2 | .65 | .45 | .5 |
| .25 |  | .5 |  |
|  | 1 | .49 |  |
| .02 | .7 | .49 | .5 |
| .05 | .65 | .49 | .5 |
| .06 | .55 | .49 | .5 |
| .15 |  |  |  |

The values in the third column were chosen to conform to the game situations discussed earlier. For . 375 and .4, the opportunity levels associated with the efficiencies that can turn the game in favor of the big shot are prohibitively high. On the other hand, at the level starting with .45, an absolute putaway shot (effectiveness 1) need occur in less than one point out of ten in order to turn the probabilities. In addition, shots lethal enough to participate in winning two points out of three need be hit only one point in four. At the levels of real competitiveness, even a small change in effectiveness is associated with a realistic opportunity level. The numbers again speak clearly : A game that is competitive at the point level, but not at the match level, can be made competitive with a "big" shot ${ }^{2}$. A game that is competitive at the game level can be made competitive with only marginal improvement of some shot.

The challenge here tactically is to determine which shots in a player's repertoire are "big" shots, what the player's efficiency level is without them, and how to optimize the opportunity to hit them. Opportunity is important. Suppose, for instance, that a player has a great overhead. Unless the player's opponent is willing to cooperate and

[^1]hit some lobs no one will ever find out. Suppose that a player has a great cross-court forehand from mid-court. Unless the player's opponent is willing to hit a short ball on the correct side of the court no one will ever know. If a player's "big" shot is a groundstroke, the player had better learn patience and hit deep ground strokes so that she or he will be able to wait for an opportunity to hit the "big" shot. On the other hand, if a player's "big" shot is a volley, the player gets to control the opportunity to hit the shot since she or he controls when to come to net.

Then, of course, there is serve, the one shot whose opportunity to play is completely under a player's control and that affords a natural approach shot behind which to volley. A player gets to, within the rules, choose where to stand and how to toss the ball; the player's opponent cannot dictate what stroke the player will use nor to which part of the service box the ball will be hit. The server exercises complete control over one of the three factors in the formula, namely how often she or he gets the opportunity to play the chosen stroke. The model can be easily refined so as to account for double faults by using the percentage of second serves in, $A^{\prime}$, with the factors defined before to give the expected probability of winning a point as

$$
(\mathrm{AE})+\left((1-\mathrm{A}) \mathrm{A}^{\prime} \mathrm{E}\right) .
$$

In this context, A , the likelihood the server will get to hit the shot, is the percentage of first serves in and E is the probability of winning the point if the server's first serve goes in.

To make sure that the data this formula can give is applied plausibly, the analysis must be different than that made earlier. Whereas before any point in the set was modeled, and thus showed what the set as a whole looked like, now only points on serve are modeled and a player gets to serve only every other game. One way to account for this is to analyze the likelihoods associated with playing six games, since by the time six service games are played, the set is over or it is time to play a tiebreaker. If a player can make some assessment as to her or his ability to break serve, such information can be used to assess the overall chances of winning.

Chart 4 is made by starting with the probability of winning a game on serve. The pertinent formulas are, with $p$ representing the probability of winning a game, $p^{6}$ for winning six in a row, 6(1-p) $p^{5}$ for winning exactly five of six, and $15(1-p)^{2} p^{4}$ for winning exactly four of six.
CHART 4

| Probability of winning <br> game | Probability of winning <br> six of six | Probability of winning <br> five of six | Probability of winning <br> four of six |
| :--- | :--- | :--- | :--- |
| .55 | .02768 | .16 | .44 |
| .58 | .038 | .2 | .5 |
| .74 | .164 | .51 | .81 |
| .89 | .497 | .86553 | .98 |

The analysis creates a position from which to study serve as a "big" shot. Game probabilities of more than . 58 mean a player should expect to win four of six more than half the time, more than .74 means a player should expect to win five of six more than half the time, and more than .89 means a player should expect to win six of six more than half the time. Going back to the work done in linking point expectation to game expectation, the critical point probabilities are, respectively, .535 for winning $58 \%$ of the games, .605 for winning $74 \%$ of the games, and .7 for winning $89 \%$ of the games. Chart 5 gives service data and the associated point probabilities that come from the model. The chart is divided into four parts. The first treats in-percentages and efficiency levels at the break-even mark. The other three give point probabilities associated with the game probabilities from the previous tables.

## CHART 5

| Probability first serve in (A) | Effectiveness of first serve (E) | Probability second serve in (A) | Effectiveness of second serve (E) | Probability point won |
| :---: | :---: | :---: | :---: | :---: |
| . 35 | . 95 | . 7 | . 4 | . 514 |
| . 4 | . 85 | . 7 | . 4 | . 508 |
| . 45 | . 8 | . 7 | . 4 | . 514 |
| . 5 | . 75 | . 7 | . 4 | . 515 |
| . 55 | . 7 | . 7 | . 4 | . 511 |
| . 25 | . 95 | . 7 | . 5 | . 5 |
| . 3 | . 85 | . 7 | . 5 | . 5 |
| . 35 | . 8 | . 7 | . 5 | . 5705 |
| . 4 | . 75 | . 7 | . 5 | . 51 |
| . 45 | . 7 | . 7 | . 5 | . 5075 |
| . 5 | . 65 | . 7 | . 5 | . 5 |
| . 3 | . 95 | . 8 | . 4 | . 509 |
| . 35 | . 85 | . 8 | . 4 | . 505 |
| . 4 | . 8 | . 8 | . 4 | . 512 |
| . 45 | . 75 | . 8 | . 4 | . 513 |
| . 5 | . 7 | . 8 | . 4 | . 51 |
| . 2 | . 9 | . 8 | . 5 | . 5 |
| . 25 | . 8 | . 8 | . 5 | . 5 |
| . 3 | . 75 | . 8 | . 5 | . 505 |
| . 35 | . 7 | . 8 | . 5 | . 505 |
| . 4 | . 65 | . 8 | . 5 | . 5 |

Before completing the chart, note what the numbers say about the first serve as an equalizing "big" shot. Suppose that point expectation is examined by assuming that a player hits her or his second serve stroke as a first serve as well. A second serve that goes in $70 \%$ of the time and is $40 \%$ efficient (. 7 and .4 in the chart) gives a point probability of .364 , for .7 and .5 , a probability of .455 , for .8 and .4 , a probability of .384 , and for .8 and .5 , a probability of .48. All of these are losing games. Even the most miserable of these, however (. 7 and .4) can be turned into a winning game if the player has a first serve that she or he can get in two times out of five and win behind $85 \%$ of the time it goes in. The numbers also indicate that double faults are not nearly as disastrous as they are often considered to be. Compare the probability associated with .7 and .5 with that from .8 and .4 by taking one more double fault for each ten second serves and trade it for one more point out of each ten second serves in. . 455 is much greater than .384 !

## CHART 5 (continued)

| Probability $1^{\text {st }}$ <br> serve in (A) | Effectiveness 1 $^{\text {st }}$ <br> serve (e) | Probability 2 <br> nd <br> serve in (A) | Effectiveness 2 <br> nd <br> serve (e) | Probability point <br> won |
| :--- | :--- | :--- | :--- | :--- |
| .45 | .85 | .7 | .4 | .5365 |
| .5 | .8 | .7 | .4 | .54 |


| . 3 | . 95 | . 7 | . 5 | . 53 |
| :---: | :---: | :---: | :---: | :---: |
| . 35 | . 9 | . 7 | . 5 | . 5425 |
| . 4 | . 8 | . 7 | . 5 | . 53 |
| . 35 | . 95 | . 8 | . 4 | . 54 |
| . 4 | . 85 | . 8 | . 4 | . 532 |
| . 45 | . 8 | . 8 | . 4 | . 536 |
| . 25 | . 95 | . 8 | . 5 | . 5375 |
| . 3 | . 85 | . 8 | . 5 | . 535 |
| . 4 | . 75 | . 8 | . 5 | . 54 |
| . 2 | . 85 | . 9 | . 5 | . 53 |
| . 4 | . 65 | . 9 | . 5 | . 53 |
| . 5 | . 95 | . 7 | . 4 | . 615 |
| . 6 | . 85 | . 7 | . 4 | . 622 |
| . 5 | . 85 | . 7 | . 5 | . 6 |
| . 6 | . 8 | . 7 | . 5 | . 62 |
| . 45 | . 95 | . 8 | . 4 | . 6035 |
| . 5 | . 9 | . 8 | . 4 | . 61 |
| . 4 | . 95 | . 8 | . 5 | . 62 |
| . 5 | . 85 | . 8 | . 5 | . 625 |
| . 35 | . 9 | . 9 | . 5 | . 6075 |
| . 5 | . 8 | . 9 | . 5 | . 625 |
| . 65 | . 95 | . 7 | . 4 | . 715 |
| . 6 | . 95 | . 7 | . 5 | . 71 |
| . 65 | . 9 | . 7 | . 5 | . 7075 |
| . 65 | . 95 | . 8 | . 4 | . 697 |
| . 7 | . 9 | . 8 | . 4 | . 726 |
| . 55 | . 95 | . 8 | . 5 | . 7025 |
| . 6 | . 9 | . 8 | . 5 | . 7 |
| . 5 | . 95 | . 9 | . 5 | . 7 |
| . 55 | . 9 | . 9 | . 5 | . 6975 |

The data from the second part of the chart give a baseline from which to evaluate a player's service. The data given is associated with high efficiency levels since the premise is that serve is a "big" shot. For second serve levels that are close to being winning games (.7,.5; .8,.5; and .9,.5) the first serve percentages associated with winning five of six half the time or six of six half the time are attainable. The numbers speak clearly once again. From the first part of the chart: A player can win games with a strong first serve. But from the second part of the chart: If a player wants to win sets, her or his second serve should break even by itself and first serve must win points when it goes in. ${ }^{3}$

[^2]
## Analyzing Serve

In practically any televised match in which a big server begins to struggle, the commentator in charge of professional expertise is almost certain to say something on the order of "Isner should take something off and just get his first serve in." The service model we have developed enables an appraisal of whether this is a good idea in the long run.

The likelihood of winning a point on serve is $(A E)+\left((1-A) A^{\prime} E\right)$. $E$ is larger than $E^{\prime}$, or else the second serve stroke is more effective than the first serve stroke and anyone should realize that she or he should be hitting it all the time. Similarly, $A^{\prime}$ should be larger than $A$, since if a player could get her or his first serve stroke in more often than the second serve stroke, the player should use it all the time. Thus the conditions $A<A^{\prime}$ and $E>E^{\prime}$ can be added to the analysis.

To assess the impact of hitting the second serve stroke as a first serve, adjust the formula by using $A^{\prime}$ and $E^{\prime}$ in the roles of $A$ and $E$. This gives a probability of winning a point of $(A E)+((1-A) A E)$. Mathematically, testing the pertinence of the commentator's observation reduces to answering the question, "Is (AE) +( (1-A)A'E) bigger than (AE) $+((1-A) A E)$ )?". From match data, values for $A, E, A^{\prime}$, and $E^{\prime}$ can be computed, and thus the efficacy of the strategy can be assessed. By taking values for $A^{\prime}, E^{\prime}$, and $E$, and solving
$(A E)+\left((1-A) A^{\prime} E\right)=(A E)+\left((1-A) A^{\prime} E\right)$ for $A$, we create the data for Chart 6. Chart 6 tabulates values for $A^{\prime}$ and $E^{\prime}$ and threshold values for $A$ and $E$ for which any larger efficiency or percentage give a higher expected return for hitting the first serve stroke, then the second serve stroke (if necessary) than for hitting the second serve stroke as a first serve.

[^3] and maintained his ground game at the level he had previously played it pushed him to the top.

## CHART 6

| Second serve percentage (A) | Second serve efficiency (E) | For first serve efficiency of | First serve \% must be at least |
| :---: | :---: | :---: | :---: |
| . 7 | . 4 | 1 | . 12 |
|  |  | . 84 | . 15 |
|  |  | . 73 | . 18 |
|  |  | . 67 | . 22 |
|  |  | . 5 | . 38 |
|  |  |  |  |
| . 7 | . 5 | 1 | . 22 |
|  |  | . 82 | . 32 |
|  |  | . 75 | . 38 |
|  |  | . 7 | . 45 |
|  |  |  |  |
| . 8 | . 4 | 1 | . 22 |
|  |  | . 82 | . 13 |
|  |  | . 72 | . 16 |
|  |  | . 6 | . 23 |
|  |  | . 5 | . 36 |
|  |  |  |  |
| . 8 | . 6 | 1 | . 18 |
|  |  | . 92 | . 22 |
|  |  | . 82 | . 28 |
|  |  | . 75 | . 35 |
|  |  | . 71 | . 41 |
|  |  |  |  |
| . 9 | . 4 | 1 | . 06 |
|  |  | . 76 | . 09 |
|  |  | . 59 | . 16 |
|  |  | . 5 | . 25 |
|  |  |  |  |
| . 9 | . 6 | 1 | . 12 |
|  |  | . 9 | . 15 |
|  |  | . 83 | . 18 |
|  |  | . 75 | . 25 |
|  |  | . 7 | . 35 |

The numbers in Chart 6 contain information that runs counter to the traditional wisdom. If a player has a second serve that goes in $90 \%$ of the time and is $40 \%$ effective, the player only has to get a serve that wins three of four points in one time in ten to improve her or his results. What's more, the chart contains break-even values; increase either the efficiency or the percentage in of the first serve values and the likelihood of winning a point increases. Thus, even though hitting a $90 \%$ in, $60 \%$ effective stroke twice will give the same expectation for winning a point as a $15 \%$ in, $90 \%$ effective serve backed by a $90 \%$ in, $60 \%$ effective second serve, increasing to $30 \%$
in, $90 \%$ effective further increases the expectation of winning the point. If one considers the above scenario, that is, increasing the percentage in on a $90 \%$ effective first serve being backed by the $90-60$ second serve hypothesized translates into winning $83 \%$ of the service games. And, if one could get half her or his first serves in and maintain the $90 \%$ effectiveness, the number soars to an astounding $98 \%$ of the games. (These numbers were computed using the numbers in Chart 1, which relates point probabilities to game probabilities.) If further encouragement were needed, the "percentage in" factor can be accurately gauged on the practice court, although the effectiveness factor is likely dependent on the opponent.

The numbers speak quite clearly about service. If a player can get her or his second serve in, it makes little sense to use the second serve stroke as a first serve. If a player can't get a second serve in, the player shouldn't be worrying about tactics, she or he should be hitting buckets of serves.

## AFTERTHOUGHTS

The folk wisdom associated with a subject quite often turns out to be based in fact. The experiment discussed in the previous pages of looking at tennis through its numbers gives a window through which to examine some of the perceived truisms of the game.

Why is there such a big emphasis on playing the "big points" well? Our analysis indicates that, for a match to be close, it is extremely likely that the number of points won by each player will be close. Perhaps the "big points" are magnified in the players' minds because they are actually the margin of victory. Put those points anywhere else in the match and the outcome is not likely to change. For example, consider a deuce game that Player 1 wins. Let $P_{i}$ stand for Player 1 winning point number i and $\mathrm{O}_{\mathrm{i}}$ stand for Player 1's opponent winning point number i . Schematically, a game might look like
$\mathrm{P}_{1} \mathrm{P}_{2} \mathrm{P}_{3} \mathrm{O}_{4} \mathrm{O}_{5} \mathrm{O}_{6} \mathrm{P}_{7} \mathrm{P}_{8}$.

Now make your own game. You may fill in the first six spaces with three P's and three O's any way you like, but $P_{7}$ and $P_{8}$ must fill in the last two. Now exchange $P_{8}$ with any of the $O^{\prime} s$ (that is, put the "big point" outcome in a non-"big point" place) and look at the resulting game. Player 1 won the game without going to deuce. Serve changes and the players go to a different set of performance parameters without Player 1 having to face a deuce. The argument might be made that, in the context of the match, this may have made it an even bigger point. To argue that "big point" performance is a factor, one would need to show that if the outcome on the perceived "big points" were turned around, then the opponent wouldn't have had the edge in points, and that if the "big points" were placed elsewhere in the match under similar parameters, then the outcome would have been different.

The "experts" often harp about the intangibles affecting outcomes. Things like "mental toughness" and "killer instinct" often come up in discussions of winning the "big points." Appropriately enough, the model that has been built in this paper can allow testing for intangibles. If actual set scores (or scores in "big" shot games, possibly games on serve if serve is a "big" shot) routinely outperform what the observed point probabilities predict, then perhaps we could explain the phenomenon through intangibles. In the matches I have charted in which players have outperformed their numbers (and the great majority of matches go exactly as the numbers say they should), a breakdown into service games has removed the anomaly. I also suspect that a "big" shot analysis would be telling in practically any scenario in which a player outperforms her or his numbers. It would also be interesting to see if a player's reputed strengths were those that were statistically indicated.

Bill Tilden is supposed to have said "Never change a winning game. Always change a losing game." He apparently sensed what the analysis in this paper indicates. The analyses from Chart 1 and Chart 2 show that winning points wins matches and the "big" shot analysis shows that the addition of a weapon can change an otherwise losing game into a winning game.

This discussion is not intended to be exhaustive, but rather to indicate that tools to decide what parts of traditional wisdom are actually wisdom and what parts are simply wishful thinking are available. It appears to be reasonable to understand the tangibles of a player's game before putting its fate in the hands of intangibles.

## Reference

1. Vic Braden and Bill Bruns, Vic Braden's Tennis for the Future, Little, Brown (Boston), 1977.
G. EDGAR PARKER received his BS degree in Mathematics with a minor in Philosophy and Religion from Guilford College in 1969 and his PhD in mathematics from Emory University in 1977. In a teaching career that now spans 40 years, Ed has taught in high school, junior college, an open-admissions university, and selective universities, and remains committed to Inquiry-Based Learning. Although the bulk of his research has been done on differential equations, sports (in particular baseball) are his great recreational passion, and he is often led to think about sports due to listening to or reading remarks from the media or coaches as they analyze games he has seen. Ed began playing tennis in 1977 and it continues to be his main competitive outlet.

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[^0]:    ${ }^{1}$ The assumption that $\mathrm{E} \leq \mathrm{E}^{\prime}$ may be pertinent for other investigations however. In particular, assessing the relative payoff in improving a stroke which is currently a liability would fall under this assumption.

[^1]:    ${ }^{2}$ It is ironic that "playing the percentages" has become synonymous with keeping the ball in play.

[^2]:    ${ }^{3}$ Readers who are old enough may remember when Ivan Lendl made the jump from Top-10 player to \#1 (where he remained for some time). Commentators tended to suggest that it had something to do with mental toughness. I

[^3]:    suggest, in light of the analysis above, that the fact that he improved his first serve enough to give him some aces

