## Down 4 with a Minute to Go

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#### Abstract

According to the commentary on college basketball games, the prevailing wisdom concerning tactics associated with trying to win a game in which a team trails in the late stages, but is still close, is to "extend the game"; that is, to try to create as many more possessions as possible. Even when a team is down by only 3 points, announcers will advocate trying to score 2 points quickly rather than attempting to tie the game. In this paper, we analyze the situation in the title according to the points associated with the various outcomes from 2-point strategies and 3-point strategies, use the resulting model to suggest what the issues are in choosing a strategy for a particular team on the basis of its strengths or its opponent's weaknesses, and suggest how statistics might be gathered to inform the probabilities used in the model.


Play-by-play announcer : "The Hurrahs are down 81-77 with the ball coming out of this time-out and there are just 52 seconds left. I guess they'll shoot the three. Right, Billy?"

Expert analyst : No Jim, this is still a two-possession game. There is no need to panic; look for the easy two.

This conversation shows up time after time on both college and NBA telecasts. Apparently, if the announcing "experts" reflect the collective thinking of the coaches, this is the strategy that most coaches intend to play in this situation. In what follows plausible strategies associated with being down by four points at a time in the game when the opposition is in a position to, by exhausting the shot clock, limit a team to one additional possession are analyzed. The analysis examines the scenarios involved, and provides a theoretical model in which the strategy is dictated by the relative likelihoods of a team scoring two points or three points, or one point when trying for two or three, and the relative likelihood of the opponent scoring zero, one, or two points. The conclusion contains some questions raised by the analysis.

Suppose that the "easy two" strategy is an inferior strategy when compared to attempting a three-point shot ${ }^{1}$ and being willing to attempt a second one if the opponent makes two free throws. Then why might most everybody employ the other strategy ${ }^{2}$ ? One possible reason is that when a team is down by four in the last minute of play, it is probably going to lose in regulation. Thus, when a team in that situation does win one or get to overtime, the game, and thus the strategy being used, is probably going

[^0]to stand out in the minds of those who witness it. If most everybody uses the "easy two" strategy and such "stick-out" wins or overtime losses (a chance to win) are indeed likely to be memorable, then the rarity of such a win might reinforce the strategy rather than call it in question. To illustrate this, suppose that, in games of the type being considered, $90 \%$ of the coaches employ the "easy two" strategy and the other $10 \%$ employ a "shoot the three" strategy. Now suppose that there were 200 games where a team was down by four in the last minute and had the ball. Suppose further that the "easy two" strategy worked $5 \%$ of the time and the "shoot the three" strategy worked twice as often. There would be 9 games in which the "easy two" strategy worked; that means 18 head coaches (and probably 54 assistants) who are impressed with (or depressed by, in the case of the losing coaches) the results. Meanwhile, there would be only 2 games in which "shoot the three" worked; that would make only 4 head coaches and 12 assistants positively affected by the strategy. Empirically, the statistically better strategy attracts only 16 proponents while the inferior strategy attracts 72 . These are not good odds in breaking down a stereotype.

Also important are factors that are actually under a coach's control. For instance, a coach cannot guarantee that her/his team will make a shot; she/he can be aware of the frequency with which the team has executed the desired strategies in practices or in past games. A coach cannot guarantee that the opponent will miss a foul shot; but a coach can be aware of the frequency with which the players on the court for the opponent have made foul shots in the past. Following are the strategies being compared.

## Shoot the 3.

This strategy entails trying for a three-point field goal, and if a missed three-point field goal is rebounded by the shooting team, trying for another three-point field goal. Notice that there is also the possibility of making one, two, or four points. The shooter may be fouled and make only one or two of the free throws, a rebounder may be fouled and get only two free throws, or, in the case of an undisciplined opponent, the opponent may foul on a successful three-point shot and allow the shooting team to tie the game. The last of these outcomes is not allowed for in the model, although it adds minutely to the value of a three-point strategy.

## Shoot the "easy" two.

This strategy entails attempting to score a quick two-point field goal ${ }^{3}$. The rationale is that, since a defense is likely trying to prevent giving up three points and thus will guard the perimeter closely and is not likely to take a chance on fouling an inside shooter, that a field goal try from close range is more likely to be made than would ordinarily be the case.

[^1]Following the first possession，defensively，the strategy is typically to foul．Fouling allows the trailing team to regain possession of the ball and retain enough time ${ }^{4}$ on the clock to execute a second strategic offensive possession．If there are fewer than 35 seconds to play，when playing a two－point strategy， there is no other option（unless the opponents are incompetent ball－handlers，a condition rarely encountered at higher levels of play），since time must be left to execute an offense and the team will trail by at least two points．In playing a three－point strategy，if there is time，the option to play defense for a shot clock is more attractive in the event a three points have been scored，since even if the opponent scores two points in the possession，the possibility of tying or taking the lead in a final possession still exists．

The pertinent numbers associated with these strategies are the following：
＜T3 ：the probability of scoring three points in a possession if the commitment is made to attempt a three－point shot as described in the three－point strategy．
＜t3：the probability of scoring three points in a possession if the team is down by three points or fewer and commits to attempting a three－point shot ．

〈 D2 ：the probability of scoring two points in a possession if the team is down by more than two points and commits to attempting a two－point shot．

〈 d2 ：the probability of scoring two points in a possession if the team is down by two points or fewer and commits to attempting a two－point shot．
＜ $\mathrm{T}_{2}$ ：the probability of scoring two points in a possession if the team commits to try for a three－point field goal when down by four points．
＜ $\mathrm{T}_{1}$ ：the probability of scoring one point in a possession if the team commits to try for a three－point field goal when down by four points．
＜ $\mathrm{D} 2_{1}$ ：the probability of scoring one point in a possession if the team commits to try for a two－point field goal when down by more than two points．
＜ $\mathrm{d} 2_{1}$ ：the probability of scoring one point in a possession if the team commits to try for a two－point field goal when down by two points or fewer．
＜ $\mathrm{D} 2_{3}$ ：the probability of scoring three points in a possession if the team commits to try for a two－point field goal when down by more than two points．

〈 $\mathrm{d} 2_{3}$ ：the probability of scoring three points in a possession if the team commits to try for a two－point field goal when down by two points or fewer．

[^2]〈 X2 : the probability of the opponent scoring two points in a possession.
< X1 : the probability of the opponent scoring one point in a possession.
< XO : the probability of the opponent not scoring.
Some relationships can be inferred or deduced between and among these numbers.
$\mathrm{D} 2>\mathrm{d} 2 \geq \mathrm{T} 3>\mathrm{t} 3 \gg \mathrm{D}_{3}$.
Implicit in "easy two" is that the scoring percentage for an "easy two" should be higher than scoring a two when trailing by two, and scoring a two under "hard defense" should be no more difficult than scoring a three, even when the opponent is being careful not to foul a three-point shooter. Scoring three points when it will tie a game should be more difficult than scoring three points under the "don't foul" conditions. D2 $2_{3}$ should be negligible if the conditions for "easy two" are real.
$\mathrm{T}_{1}>\mathrm{D} 2_{1}$ and both are tiny.
Implicit in the "easy two" is that the opponent dare not foul; a team up by four will not foul a three-point shooter. Thus whatever fouls occur are predominately rebound fouls. Rebounds are more likely to occur on three-point tries since they are more likely to be missed than "easytwo" attempts, and hence there should be a greater incidence of fouls on such attempts. The reluctance of the team with the lead to let the opponent score with the clock stopped should make both numbers very small, so omitting this factor is not likely to make much difference. For the same reasons, $\mathrm{T3}_{2}$ should be tiny. $\mathrm{D} 2_{3}$ should be less than any of these since a foul on a twopoint shot occurs under controlled defensive circumstances rather than loose-ball conditions. $\mathrm{d} 2_{1}$ is dependent on how often a team will be fouled when shooting a contested two-point field goal and miss one of the foul shots.
$X 2+X 1+X 0=1$.
The opponent must score either zero, one, or two points under a "foul-them" strategy unless it rebounds a missed free throw after having made a free throw. Under such a possibility, the "easy-two" strategy must leave the trailing team in a position inferior to the one it was in before the two possessions, whereas a three-point strategy may have the team in the same position but with less time remaining. Thus, omitting the possibility of a three-point possession by the opponent, if it has an effect, skews the results in favor of an "easy-two" strategy. Given that a "foul-them" strategy is to be followed, X2, X1, and X0 can reasonably be computed from a single
parameter, $s$, the percentage of free throws the opponent is expected to make ${ }^{5}$. The computation differs according to whether the opponent shoots in a 1-and-1 or 2-shot situation.

〈 For a 1-and-1 free throw situation, $\mathrm{X} 2=s^{*} s, X 1=s^{*}(1-s)$, and $X 0=1-s$.
< For a 2-shot free-throw situation $\mathrm{X} 2=\mathrm{s}^{*} \mathrm{~s}, \mathrm{X} 1=2 * \mathrm{~s}^{*}(1-\mathrm{s})$, and $\mathrm{XO}=(1-\mathrm{s})^{*}(1-\mathrm{s})$.
When playing a three-point strategy and scoring three point in the initial possession, the opportunity exists to play defense for a shot clock. In this case, the numbers depend on offensive efficiency and X1 has a small probability relative to X 2 and X 0 .

Given these parameters, probabilities can be computed for the outcomes after two of the trailing team's possessions associated with playing the different strategies by using the following models. Regarding the third possession (second offensive possession) in the sequence, the model is made on the assumption that a team down by three points in its (possibly) last possession, regardless of strategy, will make a three-point attempt.

## Using an "easy-two" strategy :

The probability of having a two-point lead is $\mathrm{D}_{3}{ }^{*} \mathrm{XO}{ }^{*} \mathrm{~d} 2_{3}$.

The probability of having a one-point lead is $\mathrm{D} 2_{3}{ }^{*} \mathrm{X} 0^{*} \mathrm{~d} 2+\mathrm{D} 2^{*} \mathrm{X} 0^{*} \mathrm{~d}_{2}{ }_{3}+\mathrm{D} 2_{3}{ }^{*} \mathrm{X} 1^{*} \mathrm{~d} 2_{3}$.
The probability of being tied is
$\mathrm{D} 2 * \mathrm{X} 0^{*} \mathrm{~d} 2+\mathrm{D} 2_{1}{ }^{*} \mathrm{X} 0^{*} \mathrm{t} 3+\mathrm{D} 2_{3}{ }^{*} \mathrm{X} 0^{*} \mathrm{~d} 2_{1}+\mathrm{D} 2 * \mathrm{X} 1^{*} \mathrm{t} 3+\mathrm{D} 2_{3}{ }^{*} \mathrm{X} 1^{*} \mathrm{~d} 2+\mathrm{D} 2_{3}{ }^{*} \mathrm{X} 2 * \mathrm{t} 3$.

## Using a three-point strategy :

The probability of having a two-point lead is $\mathrm{T}^{*} \mathrm{XO}^{*} \mathrm{~d} 2_{3}$.

The probability of having a one-point lead, using an aggressive strategy, is
$\mathrm{T} 3 * \mathrm{XO}{ }^{*} \mathrm{~d} 2+\mathrm{T} 3_{2}{ }^{*} \mathrm{X} 0 * \mathrm{t} 3+\mathrm{T} 3 * \mathrm{X} 1 * \mathrm{t} 3$. ${ }^{6}$
The probability of having a one-point lead, using a conservative strategy, is
$T 3^{*} X 0^{*} d 2+T 3_{2}{ }^{*} X 0^{*} d 2_{3}+T 3^{*} X 1 * d 2_{3}$.
The probability of being tied, using an aggressive strategy, is
$\mathrm{T} 3 * \mathrm{XO}{ }^{*} \mathrm{~d} 2_{1}+\mathrm{T}_{2}{ }^{*} \mathrm{XO} 0^{*} \mathrm{~T}_{2}+\mathrm{T} 3_{1}{ }^{*} \mathrm{X} 0 * \mathrm{t} 3+\mathrm{T} 3 * \mathrm{X} 1^{*} \mathrm{~T} 3_{2}+\mathrm{T} 3 * \mathrm{X} 2 * \mathrm{t} 3$.
The probability of being tied, using a conservative strategy, is

[^3]$T 3^{*}{ }^{2} 0^{*} \mathrm{~d} 2_{1}+\mathrm{T} 3_{2}{ }^{*} \mathrm{X} 0^{*} \mathrm{~d} 2+\mathrm{T}_{1}{ }^{*} \mathrm{X} 0^{*} \mathrm{t} 3+\mathrm{T} 3^{*} \mathrm{X} 1^{*} \mathrm{~d} 2+\mathrm{T} 3^{*} \mathrm{X} 2^{*} \mathrm{t} 3$.

To interpret any of the calculations in the basketball context, reference the bullet list describing the pertinent numbers in the model. As an example, consider "The probability of having a two-point lead is $\mathrm{D} 2_{3}{ }^{*} \mathrm{XO}{ }^{*} \mathrm{~d} 2_{3}$." This interprets as "The trailing team scores three points in a possession in which it committed to trying for a two-point field goal when down by more than two points ( $\mathrm{D} 2_{3}$ ); the opposition scores no points in its possession (X0); the trailing team scores three points in a possession in which it commits to trying for a two-point field goal when down by two points or fewer ( $\mathrm{d}_{3}$ ). ".

The assumptions behind these calculations are that all possessions constitute independent events and that if a team can take the lead with a two-point attempt, it will commit to running an offensive possession whose first priority is a two-point attempt. The numbers for an aggressive strategy are computed under the assumption that a team with the chance to take a lead with a three-point shot or to tie with a two-point shot will commit to running an offensive possession whose first priority is to create a three-point shot. The conservative strategy chooses to try for the tie. Also, to accommodate the strategy associated with playing defense for a shot clock rather than fouling, the values for $\mathrm{X} 2, \mathrm{X} 1$, and XO associated with this strategy (that is, not consequent to deliberately fouling) may be used wherever the first factor is T 3 or $\mathrm{D} 2_{3}$.

In order to effectively compare the three-point strategies with the two-point strategy, some way to measure the relative value of being two points ahead, one point ahead, and tied with the opponent must be devised. Here the factors used are probability of winning when tied in the last $35^{7}$ or fewer seconds of a game and the opponent has the ball, when ahead by one point in the last 35 or fewer seconds of a game and the opponent has the ball; and when ahead by two points in the last 35 seconds or fewer of a game and the opponent has the ball.

Letting W2 represent the probability of winning if a team has a two-point lead with 35 or fewer seconds remaining and the opponent has the ball, W1 represent the probability of winning if a team has a onepoint lead with 35 or fewer seconds remaining and the opponent has the ball, and WE represent the probability of winning if a team is tied with 35 or fewer seconds remaining and the opponent has the ball, the models for the probabilities of winning are as follows.

Probability of winning playing an "easy-two" strategy :

$$
\begin{aligned}
& \mathrm{W} 2^{*} \mathrm{D} 2_{3}{ }^{*} \mathrm{X} 0^{*} \mathrm{~d} 2_{3}+\mathrm{W} 1^{*}\left(\mathrm{D} 2_{3}{ }^{*} \mathrm{X} 0^{*} \mathrm{~d} 2+\mathrm{D} 2^{*} \mathrm{X} 0^{*} \mathrm{~d} 2_{3}\right)+ \\
& \quad \mathrm{WE} \mathrm{E}^{*}\left(\mathrm{D} 2^{*} \mathrm{X} 0^{*} \mathrm{~d} 2+\mathrm{D} 2_{1}{ }^{*} \mathrm{X} 0^{*} \mathrm{t} 3+\mathrm{D} 2_{3}{ }^{*} \mathrm{X} 0^{*} \mathrm{~d} 2_{1}+\mathrm{D} 2^{*} \mathrm{X} 1^{*} \mathrm{t} 3+\mathrm{D} 2_{3}{ }^{*} \mathrm{X} 1^{*} \mathrm{~d} 2+\mathrm{D} 2_{3}{ }^{*} \mathrm{X} 2^{*} \mathrm{t} 3\right) .
\end{aligned}
$$

Probability of winning playing an aggressive three-point strategy :

$$
\begin{aligned}
& \mathrm{W} 2{ }^{*} \mathrm{~T} 3^{*} \mathrm{XO} 0^{*} \mathrm{~d} 2_{3}+\mathrm{W} 1^{*}\left(\mathrm{~T} 3^{*} \mathrm{X} 0^{*} \mathrm{~d} 2+\mathrm{T} 3_{2}{ }^{*} \mathrm{X} 0^{*} \mathrm{t} 3+\mathrm{T} 3^{*} \mathrm{X} 1 * \mathrm{t} 3\right)+ \\
& \mathrm{WE}{ }^{*}\left(\mathrm{~T} 3^{*} \mathrm{X} 0^{*} \mathrm{~d} 2_{1}+\mathrm{T} 3_{2}{ }^{*} \mathrm{X} 0^{*} \mathrm{t} 3_{2}+\mathrm{T} 3_{1}{ }^{*} \mathrm{X} 0^{*} \mathrm{t} 3+\mathrm{T} 3^{*} \mathrm{X} 1^{*} \mathrm{t} 3_{2}+\mathrm{T} 3^{*} \mathrm{X} 2^{*} \mathrm{t} 3\right) .
\end{aligned}
$$

[^4]Probability of winning playing a conservative three-point strategy :

$$
\begin{aligned}
& \mathrm{W} 2^{*} \mathrm{~T} 3^{*} \mathrm{XO} 0^{*} \mathrm{~d} 2_{3}+\mathrm{W} 1^{*}\left(\mathrm{~T} 3^{*} \mathrm{XO} 0^{*} \mathrm{~d} 2+\mathrm{T} 3_{2}^{*} \mathrm{X} 0^{*} \mathrm{~d} 2_{3}+\mathrm{T} 3^{*} \mathrm{X} 1^{*} \mathrm{~d} 2_{3}\right)+ \\
& \mathrm{WE}\left(\mathrm{~T} 3^{*} \mathrm{X} 0^{*} \mathrm{~d} 2_{1}+\mathrm{T} 3_{2}{ }^{*} \mathrm{X} 0^{*} \mathrm{~d} 2+\mathrm{T} 3_{1}{ }^{*} \mathrm{X} 0^{*} \mathrm{t} 3+\mathrm{T} 3^{*} \mathrm{X} 1^{*} \mathrm{~d} 2+\mathrm{T} 3^{*} \mathrm{X} 2^{*} \mathrm{t} 3\right) .
\end{aligned}
$$

That $\mathrm{W} 2>\mathrm{W} 1>\mathrm{WE}$ is a reasonable assumption; what the relative ratios actually are is not nearly as clear. Thus we allow for many possibilities to be accommodated when computing with the model.

Now it is time to test drive the model. Perhaps a good way to start is to find a "break-even" scenario. First, assume "competence" on the part of both offenses. Let the team that is leading be good free throw shooters by assigning a probability of .7 that any free throw will be made. This translates into $\mathrm{X} 2=.49, \mathrm{X} 1=.42$, and $\mathrm{X} 0=.09^{8}$ in the model. Let the team that is trailing be good on offense when trying to score two points by assigning $\mathrm{d} 2=.55^{9}$ and be near-adequate three-point shooters, with the probability of .33 (1 out of 3) for making a three-point shot in a "don't foul" situation and 25 (1 out of 4) in a normal defense situation ${ }^{10}$. Set the "referee factor" data to 1 out of 50 for each outcome except "don't foul the three-point shooter" situations, to which I have assigned 0 probability ( $\mathrm{TH}_{2}$ and $\mathrm{T}_{1}$ ), and the situation of making a three-point play in a normal defense situation. I have chosen 1 out of 20 for this datum, since it happens during regular play more frequently than the other scenarios. ${ }^{11}$ Finally, assign to the probability of scoring two points when trying for an "easy two" (D2), successively, the values .9, $.8, .7$, and .6 and assume that being tied is just as good as being ahead by assigning each of WE, W1, and W2 the value 1 . The model returns the following data:

Probability of being tied or ahead

$$
\text { with } \quad \mathrm{D} 2=.9 \quad \mathrm{D} 2=.8 \quad \mathrm{D} 2=.7 \quad \mathrm{D} 2=.6
$$

using

| "Easy two" strategy | .152156 | .136256 | .120356 | .104456 |
| :--- | :--- | :--- | :--- | :--- |
| Aggressive 3-point strategy | .096261 | .096261 | .096261 | .096261 |
| Conservative 3-point strategy | .096261 | .096261 | .096261 | .096261. |

Notice that how easy an "easy two" is is a critical issue. If an "easy two" is a virtual surety, the probability of being tied or ahead is half again as large as that associated with a three-point strategy where the shooters are marginally accurate. BUT, if "easy two" means 9\% better than an already efficient offense, a two-point strategy is only slightly better, judged on being even or ahead, with a three-point strategy where the shooters are only marginally accurate.

[^5]On the other hand, being ahead gives a better chance of winning than being tied, so perhaps ignoring the probability of winning associated with being ahead as opposed to being tied is not a very practical idea. Use WE, the conditional probability of winning given that the game is tied and the opponent of the formerly trailing team has the ball with less than a shot clock remaining or time has expired, as the basic statistic. If the formerly trailing team takes a one-point lead, depending on the time remaining, the team that was previously leading either has time to score or it doesn't. If the previously leading team scores two or three points, it wins (as it would if the game were tied); if it scores one point, overtime is played (if the game were tied, this would result in a win for the scoring team); or if it does not score, the formerly trailing team wins. Thus wins for the formerly trailing team result in all scenarios in which a tie results in a win plus those in which the other team fails to score. So W1 > WE and W1 - WE must include the value of getting to overtime should the formerly leading team score one point and the value of winning when the formerly leading team fails to score. Similarly $\mathrm{W} 2>\mathrm{W} 1$, since the formerly leading team must score two points to force overtime and three points to win, and scoring one point, a part of the gain in W1 over WE, now results in a loss for the formerly leading team. W2 - W1 must include the value of getting to overtime associated with the formerly leading team scoring two points and value for a win for the formerly trailing team resulting from the formerly leading team scoring only one point.

All of WE, W1, and W2 depend on the probability of the formerly leading team scoring in a final possession after the score has been tied or it has relinquished the lead. It is reasonable to set this probability lower than the probability of scoring in a full shot clock possession since there is the possibility of there being only a few seconds left and, in a college or high school game ${ }^{12}$, the team must travel the length of the court. Let V represent the probability of the formerly trailing team winning, given that an overtime is played, p represent the probability of the formerly leading team scoring in a final possession, and $p^{\prime}$ represent the probability of it scoring exactly one point if it scores two points or one point. Then
$W E=(1+-p)^{*} V$,
$\mathrm{W} 1=(1+-\mathrm{p})+\left(\mathrm{p}^{*} \mathrm{p} * \mathrm{~V}\right)$,
and
$\mathrm{W} 2=(1+-\mathrm{p})+\left(\mathrm{p}^{*} \mathrm{p}\right)+\left((p+-\mathrm{p})^{*} \mathrm{~V}\right)$.
Suppose that winning in overtime is a break-even proposition, that the formerly leading team is expected to score one or two points $20 \%$ of the time in its final possession, and that $5 \%$ of scoring possessions resulting in no more than two points result in a single point. Then $\mathrm{V}=.5, \mathrm{WE}=.4$, $\mathrm{W} 1=.805$, and $\mathrm{W} 2=.8525$.

Using the same data for all other factors as before, but with the weights giving proportional value for the likelihoods of winning suggested above, the model returns the following data:

Probability of winning

[^6]with $\quad \mathrm{D} 2=.9 \quad \mathrm{D} 2=.8 \quad \mathrm{D} 2=.7 \quad \mathrm{D} 2=.6$
using

| "Easy-two" strategy | .063114 | .056572 | .050030 | .043488 |
| :--- | :--- | :--- | :--- | :--- |
| Aggressive three-point strategy | .059825 | .059825 | .059825 | .059825 |
| Conservative three-point strategy | .066894 | .066894 | .066894 | .066894. |

Against a good free-throw shooting team, even with an extremely high probability of scoring the "easy two" and only marginally good three-point shooting, a conservative three-point strategy is superior to an "easy-two" strategy and an aggressive three-point strategy passes from break-even to superior between .9 and .8, and at "easy-two" levels 10\% above normal offensive production, both are $40 \%$ to $50 \%$ better.

But what about a poor free-throw shooting team? The following data is based on the team with the lead expected to make only half of its free throws. This translates, for the double bonus situation, into $\mathrm{X} 0=.25, \mathrm{X} 1=.5$, and $\mathrm{X} 2=.25 .^{13}$ Using the other input data as before, the model returns the following data:

Probability of being tied or ahead/Probability of winning,
with
D2 $=.9$
D2 $=.8$
D2 $=.7$
D2 = . 6,
using
"Easy-two" strategy .255910/.109626 .231600/.098119 .204100/.086613 .176600/.0751069

Aggressive three-point strategy .116325/.083480
Conservative three-point strategy .116325/.091895
Here, the shift from a two-point strategy being the superior strategy to a conservative three-point strategy being superior comes between $80 \%$ and $70 \%$ efficiency defining the "easy two" and for an aggressive three-point strategy between $70 \%$ and $60 \%$.

But what if, against a poor free-throw shooting team, the trailing team is actually good at shooting three-point field goals? Assign . 4 to T 3 and .33 to t3; ${ }^{14}$ that is, quantify an expectation of making 2 of 5 in a "don't foul" situation and 1 of 3 in a "normal defense" situation.

The model returns the following data:
Probability of being tied or ahead /Probability of winning,
with
D2 $=.9$
D2 $=.8$
D2 $=.7$
D2 = . 6,

[^7]using
"Easy-two" strategy .295900/.124346 .264400/.11239 .232900/.098133 .201400/.085027

Aggressive three-point strategy .165000/.117268
Conservative three-point strategy .165000/.114588 .
This chart differs from the others in that an aggressive three-point strategy is superior to a conservative three-point strategy. In all of the above scenarios, both three-point strategies are comparable to the two-point and, unless an "easy two" is really easy relative to the normal chances of scoring, superior. Also keep in mind that lowering the "normally- defended two" efficiency OR raising the "don't foul the 3-point shooter" efficiency WIDENS THE GAP.

So there are the model and some suggestive data. Rev up the computer and go to work! Create plausible data for the factors in the model. Investigate how BAD the leading team's free throw shooters have to be relative to how bad the trailing team's three-point shooters have to be in order to make the "easy-two" strategy the superior strategy. Or better yet, make charts that the coaches can use that show "break-even" values for the strategies and will allow the coach to size up, on the spot, which strategy is likely to be best on a given night.

But wouldn't it be nice to know what these input probabilities typically are and to have observational data that comment on how real things like the "easy two" are? I suggest that statistically reliable answers to the questions posed below could carry the model from "intriguing and highly suggestive" to "here's what a coach should do".

Question \# 1: For some statistically significant sample of games, what is the ratio of number of games in which a team was behind by 4 and had the ball in the final minute and won or forced overtime to the number of games in which a team was behind by 4 and had the ball in the final minute?

Prediction - This ratio will be a small number. Interesting related question: Does this ratio change appreciably among levels of play, i.e. high school boys, high school girls, college men, college women, Division I, Division III, NBA? Important side data - record the outcomes of the possessions starting with down four with the ball and less than a minute to play.

Question \#2: What do basketball coaches do when they are down by four points, have the ball and there is less than a minute left to play?

What are their strategies on this possession? What are their rationales for what they do?

Question \#3: How do the shooting percentages on two-point tries for teams behind by three or more late in a game compare to the percentages for two-point field goals in the rest of the game preceding the situation?

I would be amazed if they were appreciably different. I suspect that the "easy two" is a myth, but its existence is the only basis I can imagine for using a two-point strategy. The data offered in the charts indicates that its substance is an issue of vital importance in selecting a strategy at the end of a game.

Question \#4: Is there any difference in the likelihood of scoring when a team trails by two points as compared to being behind by one point?

Question \#5: What are the numbers, as observed in actual games, associated with the factors in the model?

If the rationale on which the easy-two strategy is based is correct, D2 should be higher than d2.
Note that neither of these is field goal percentage; each is a ratio of times a team scored exactly 2 points during a possession in which the initial shot is a 2-point try to the number of possessions in which the initial shot is a 2-point try. For D2, the possessions need to be counted in the pertinent situation, trailing by four points with less than a minute to play; for d 2 , they need to be counted when trailing by two points late in a game. There is no reason to believe that $\mathrm{T}_{1}$ and $\mathrm{D} 2_{1}$ are appreciably different since 3-point attempts are seldom fouled, and shouldn't ever be fouled in this situation. Moreover, implicit in the "easy-two" rationale is that the opposition will not risk fouling, or as the commentator's say: "let a team score while the clock is not running". Is it safe to take $\mathrm{X} 2, \mathrm{X} 1$, and XO to be the ratios for the rest of the game if you are playing defense for an entire shot clock? Or is there something real about clutch performance (offensively or defensively)? What are the typical relative sizes of $\mathrm{X} 2, \mathrm{X} 1$, and XO when a team is employing a "foul-the-opponent" strategy? These statistics for $\mathrm{X} 2, \mathrm{X} 1$, and X 0 , since they are so closely tied to the ability to shoot free throws, should be the most convenient for coaches to gather if they were to choose to use the model.

Question \#6: What are reasonable numbers for $W 2, W 1$, and $W E$ ?

Data gleaned from the model in the charts without introducing W2, W1, and WE strongly suggest that three-point strategies give comparable chances of being tied or ahead when D2 is within $10 \%$ of d 2 . But, since being ahead after three more possessions is a possible outcome in a three-point strategy whereas the best outcome, except in extremely unlikely circumstances, from a two-point strategy after three more possessions is a tie, introducing W2, W1, and WE will likely show how much more powerful a three-point strategy is, even without the assumption of proficient three-point shooting.

I'm guessing, if the questions were answered consequent to reliable data, that, scientifically at least, the issue of the superior strategy would be situationally settled. That is, unless the leading team is known to shoot free throws abysmally and such shooters are on the court to be fouled, or unless there is no one on the trailing team who can make a three-point field goal ${ }^{15}$, "extending the game" would be flying in

[^8]the face of wisdom. Until that time, as a basketball TV spectator, my experience will remain the same. Every time an "expert" tells me, "No need to panic and shoot a three.", I will yell back at the set, "You're right, they need to shoot two 3's!".

## Reference

1. Richard Miller, "Mathematical Analysis of Sport" BIS Capstone Project, James Madison University, December, 2004.
G. EDGAR PARKER received his BS degree in Mathematics with a minor in Philosophy and Religion from Guilford College in 1969 and his PhD in mathematics from Emory University in 1977. In a teaching career that now spans 39 years, Ed has taught in high school, junior college, an open-admissions university, and selective universities, and remains committed to Inquiry-Based Learning. Although the bulk of his research has been done on differential equations, sports (in particular baseball) are his great recreational passion, and he is often led to think about sports due to listening to or reading remarks from the media or coaches as they analyze games he has seen. While an undergraduate, Ed was on the basketball team his freshman year and played varsity baseball for four years. He coached junior varsity basketball and varsity baseball during his four years (1969-1973) at Bayside High School in Virginia Beach, Virginia and has actively promoted youth sports participation in all of the places he has lived.

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[^0]:    ${ }^{1}$ In case you haven't noticed the drift of the introduction, this, on the basis of the computation I have done with the model, is my position except in really exceptional situations.
    ${ }^{2}$ I would have guessed that, in the early days of the 3-point shot, that coaches would have thought of the model " $3+3=6$ (make two 3 's); $6-2$ (both free throws made) $=4$ (the margin to be made up)", but it just must not have happened.

[^1]:    ${ }^{3}$ The dominant opinion voiced among broadcasters recently has been that coaches need "to extend the game". The Gambler's Road to Ruin Theorem, which says that the longer you play a losing strategy, the more certain you are to lose, indicates that the real issue is the one addressed in this paper.

[^2]:    ${ }^{4}$ In 2004，Richard Miller investigated，statistically，the effect of time left on the shot clock on field goal percentage． He found trends in the data，but nothing statistically significant linking when a shot is taken during the shot clock＇s 35 seconds to field goal percentage．

[^3]:    ${ }^{5}$ The assumption is that the team not shooting the free throws will rebound any miss. Some teams, to ensure that its players do not make a rebound foul or nullify a successful free throw by entering the three-second lane too soon, make this a certainty by not placing any players on the three-second lane.
    ${ }^{6}$ The defense fouling the shooter on a successful 3-point shot followed by a made free throw, followed by a successful two-point attempt in the next possession would also result in a two-point lead. But this is not accounted for in the model. As before, its omission skews the model in favor of an "easy-two" strategy.

[^4]:    ${ }^{7}$ This would be 24 seconds for an NBA game.

[^5]:    ${ }^{8}$ For a 1-and-1 situation, these numbers are $\mathrm{X} 2=.49, \mathrm{X} 1=.21$, and $\mathrm{X} 0=.3$
    ${ }^{9}$ Note that this is not shooting percentage, but rather the percentage of possessions in which a team attempts a two-point shot as its first shot and scores exactly two points in the possession.
    ${ }^{10} 1$ out of 3 is chosen since this has the same expected return as shooting $50 \%$ on two-point tries and 1 out of 4 is chosen because this has the same expected return as shooting $37.5 \%$, which is less than $40 \%$, the lower threshold that coaches are generally willing to tolerate and claim they actually have an offense. Neither ratio would be associated with having a team that is "good" at shooting three-pointers; 1 out of 4 would likely be associated with bad shooting.
    ${ }^{11}$ Note that, if these assignments show bias, it is towards a two-point strategy.

[^6]:    ${ }^{12}$ NBA rules allow a team to take the ball at half court if it chooses to call time out.

[^7]:    ${ }^{13}$ For a 1-and-1 situation, these numbers are $\mathrm{X} 2=.25, \mathrm{X} 1=.25$, and $\mathrm{X} 0=.5$.
    ${ }^{14}$ These are good numbers, but not outlandishly high.

[^8]:    ${ }^{15}$ I once heard Rick Majerus say in an interview: "I've got lots of players who can shoot the three; I just don't have anybody who can make one."

