The Effects of Wind and Altitude in the 400m Sprint with Various IAAF Track Geometries

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Abstract

We investigate the effects that wind and altitude have on the 400m sprint when run on various IAAF track geometries, with the work based on the senior project written by Vanessa and supervised by Michael. We validate Quinn's ordinary differential equations model using data from the 1999 World Athletics Championships. The model is based on Newton's Law for the energy balance of a runner, and Maple is used to solve the model's equations numerically. We confirm some non-intuitive results about the effect of a constant wind blowing from a fixed direction, and we modify the model to predict wind-assisted performances on both an equal quadrant track and a track from the ancient Greek games. Comparing the tracks provides information about the effects on performances on different standard tracks. We find performance differences between running lanes, indicating possible disadvantages of running in certain lanes. We find that the effect of altitude is significant but of little consequence with respect to differences in track geometry.

1. Introduction and an Early Model

Track and field meets include many events, among them the 400m sprint. In a standard International Association of Athletics Federations (IAAF) track, there are eight lanes, and a maximum of eight runners in a race. Although each IAAF track has the same dimensions, questions have arisen as to the effect that wind and altitude have on the runners' performances, regardless of the event. Several models have been created to describe their effects on the 100m sprint, the 200m sprint, and the 4 x 100m relay. Modeling these performances proved to be relatively simple, but until 2004 no one had ever tried modeling a 400m sprint because of the difficulty of accounting for two straights and two bends. In 2004 Mike Quinn published a model for the 400m sprint, using data from the 1999 World Athletics Championships. It (Quinn, 2004) is the basis for our model. Physical intuition suggests that a constant wind blowing across a closed loop track would have a detrimental effect on a runner's time regardless of the wind direction.

The basis of most mathematical models dealing with the effects of wind on sprinting performances traces back to the work of Archibald V. Hill, a British physiologist and biophysicist, and Joseph B. Keller, Professor of Mathematics and Mechanical Engineering at Stanford University. Hill was the first to provide a model for the energy balance of a runner by using Newton's Law (Alvarez-Ramirez, 2002). His work went unpublished and forty-six years later Keller would use the ideas that Hill introduced in 1927. Keller's equation of motion of a runner was:

$$\frac{dv(t)}{dt} = f(t) - \frac{1}{\tau}v\,,$$

where *v(t)* is the runner's velocity at time t in the direction of motion, and *f*(*t*) is the runner's total propulsive force per unit mass, which drives the runner forward and overcomes both the internal and external resistive force $\frac{v}{\tau}$ per unit mass (Keller, 1973). Keller assumed that the resistance is a linear function of v and that the damping coefficient τ is a constant. The works by Hill and Keller became known as the Hill-Keller model. One consequence of the Hill-Keller model is the prediction of the existence of a lower velocity limit that can be maintained indefinitely, that is, a running pace which could be maintained over an infinite time interval, which is not possible from a physiological viewpoint.

2. Quinn's Model

In 2003 Quinn extended Keller's model, as others had before him, to determine the effects of wind and altitude in the 200m sprint. Quinn's model included the reaction time of the sprinter, as well as air resistance. The reaction time for a world-class sprinter is rarely below 0.13 s and averages about 0.15 s. Reaction times also vary by gender, with men having a slightly faster reaction time than women (Quinn, 2003). Quinn's extended model equation is:

$$dv/dt = Fe^{-\beta t} - \frac{v}{\tau} - \alpha(v - v_w)^2\,,$$

where $v = \frac{ds}{dt}$, v_w is the velocity of the wind relative to the ground and tangent to the path, and $\propto = \frac{\rho C_d A}{2M}$ where ρ is the air density, taken to be 1.184 kg/m at 25^{o} C. C_d is the coefficient of drag, taken to be 0.715 (Walpert and Kyle, 1989), A is the frontal area of the athlete, estimated to be 0.51 m^2 for men (Quinn, 2004), and M is the mass of the athlete, taken to be 76 *kg*, a typical mass for a world class 400m sprinter. Quinn replaced *f*(*t*) by $Fe^{-\beta t}$, a propulsive force per unit mass that diminishes during the race as the athlete's muscles tire.

3. The Effects of Track Geometry on Running Performance

The International Association of Athletics Federations track, otherwise known as the standard running track, is a 400m track measured along lane one, with two straights each 84.4 m long and two bends each 115.6 m long. Thus 57.8% of the 400m event is run around bends. Although each runner runs around both bends, they experience different conditions in different lanes. Lanes have different radii around bends, though the length of the straight is the same for all. Thus runners will experience different wind conditions and the maximum velocity that runners can obtain around bends varies, depending on the lane.

The maximum velocity in running around a bend is less than that obtained while running in a straight line. Greene (1985) described the effects that runners experience around bends. He pointed out that lanes are unequal because of the effect of their radii on the runners' speed, since in order to balance centrifugal acceleration, a runner must heel over into the turn, with the approximate centerline of his body making an angle θ with respect to the vertical.

The final result of Greene's analysis is that everywhere the relation of the runner's peak velocity v_0 to his velocity v in a bend of radius r is $v = v_0\sqrt{\omega}$, where ω satisfies the cubic equation $\omega^3 + r^2\omega - r^2 = 0$, $r = \frac{Rg}{{v_0}^2}$, R is the bend radius, g is the gravitational force, and v_0 is the runner's peak velocity. The cubic has a real root,

$$\omega = \left(\frac{\lambda^2}{2} + \sqrt{\frac{\lambda^4}{4} + \frac{\lambda^6}{27}}\right)^{1/3} - \left(-\frac{\lambda^2}{2} + \sqrt{\frac{\lambda^4}{4} + \frac{\lambda^6}{27}}\right)^{1/3}$$

where $\lambda = rg/v_o^2$. This may be one of the more unexpected and interesting appearances of Cardano's formula for obtaining the exact solution of a cubic polynomial equation! From Ward-Smith and Radford (2002) we obtain the following radii for the eight lanes, shown in Table 1.

Lane	**Radius (in meters)**
1	36.80
2	37.92
3	39.14
4	40.36
5	41.58
6	42.80
7	44.02
8	45.24

Table 1. Lane radii for the IAAF standard track in meters

4. Computation of the Effect of Winds

Another parameter in Quinn's model is v_w, the wind velocity relative to the ground and tangent to the path of the runner. It is different in each of the four segments of the track, and depends on the wind velocity u_w and the relative wind direction, which will vary continuously as the runners progress around the track. For each runner to run exactly 400 m, they are staggered at the starting line. The runner in lane 1 starts at the starting line, and will run 115.6 m around the first bend before entering the back straight. The runner in lane 8, however, is the farthest from the starting line, and will run 89.1 m around the first bend before entering the back straight. All runners from lanes 1 through 8 run the back straight for the entire 84.4 m before entering the second bend. Once again, each runner runs a different distance around this second bend. The runner in lane 1 will once again run 115.6 m around the second bend before entering the finishing straight. The runner in lane 8, however, will run 142.1 m around the second bend before entering the finishing straight. Finally, all runners from lanes 1 through 8 will again run the finishing straight for the remaining 84.4 m before crossing the finishing line. We can calculate the wind facing the runner in each of the eight lanes.

We determine the wind velocity v_w as a function of $s(t)$, the distance traveled around the first bend. The component of the wind blowing in the direction of the runner for a runner traveling in a straight line with a wind velocity of u_w blowing at an angle θ, is $v_w = u_w \cos(\theta)$, where θ is the angle measured counterclockwise from the finishing straight on the lower edge of the track, as illustrated in Figure 1.

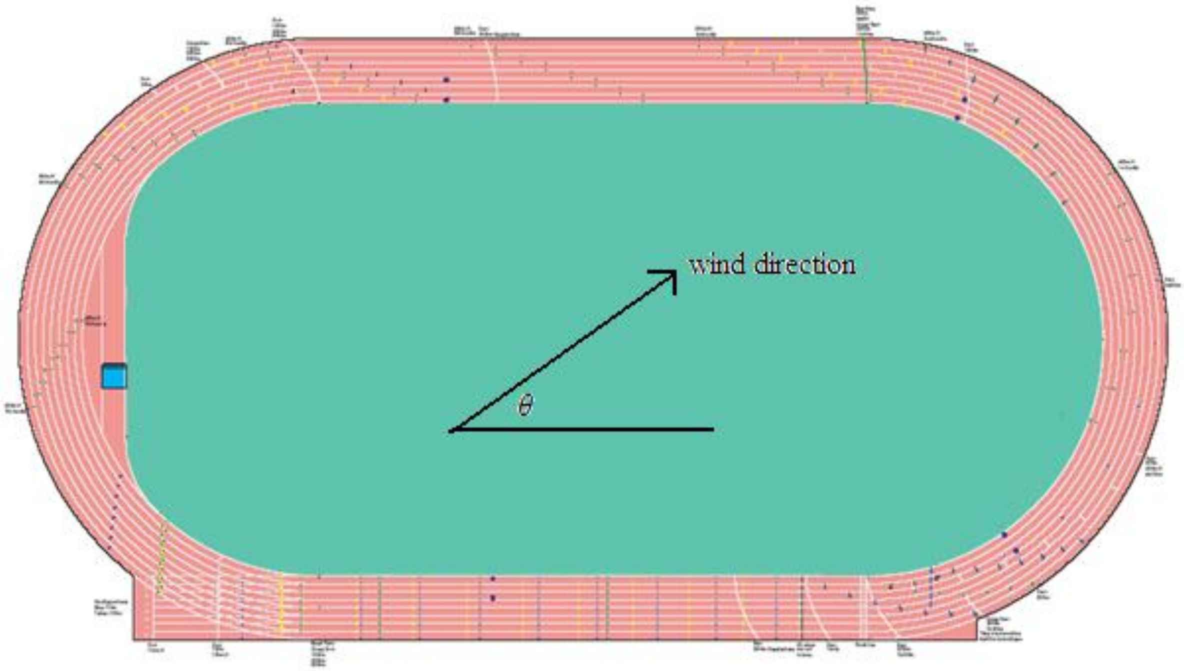

Fig. 1: The geometry of the IAAF standard track and wind direction

Trigonometry leads to expressions for v_ω in each of the four regions of the track, for lane i with corresponding radius r_i as a function of distance traveled by the runner. Table 2 lists the four formulas and the domains where they apply.

Relative Wind Velocity v_ω	Region of Application
$v_\omega = u_\omega \cos\left(\theta + \frac{231.1 - s}{r_i}\right)$	First bend: $0 \le s \le 231.2 - \pi r_i$
$v_\omega = -u_\omega \cos\theta$	Back straight: $231.2 - \pi r_i \le s \le 315.6 - \pi r_i$
$v_\omega = u_\omega \cos\left(\theta + \frac{315.6 - s}{r_i}\right)$	Second bend: $315.6 - \pi r_i \le s \le 315.6$
$v_\omega = u_\omega \cos\theta$	Finishing straight: $315.6 \le s \le 400$

Table 2: Relative wind velocity around the IAAF standard track

5. Altitude and the Propulsive Force

We now consider the propulsive force, $Fe^{-\beta t}$. Although the drag coefficient is not affected by an increase in altitude, and variations in the gravitational acceleration due to altitude are negligible (Behncke, 1994), McArdle (2007) observed that increasing altitude forces a drop in the partial pressure of oxygen, reducing the percentage of oxygen saturation in the blood. This in turn decreases the oxygen supplied to the runner's muscles by the aerobic energy system. The oxygen saturation reduction is relatively small for altitudes less than 3000 meters, but still has a detrimental effect on athletic performances (Quinn, 2004). The conclusion is that the aerobic energy system contributes significantly to the energy supply during long sprints and middle distance running. The total contribution of the aerobic energy system for the 400m event was found to be 43% (Spencer and Gastin, 2001). Since the aerobic energy system is affected by altitude, the propulsive force $Fe^{-\beta t}$ declines at a faster rate in higher altitudes than at sea level. We assume that the decay rate β depends on both the altitude H and the contribution of the aerobic energy system γ and assume that $\beta = \beta_0(1 + \gamma\sigma H)$, where β_o is the parameter value at sea level and σ is the oxygen saturation in the blood. Using estimates from McArdle (2007), we set $\beta_o = 6.00 \times 10^{-3} s^{-1}$, $\gamma = 0.35$ and $\sigma = 0.000023\ m^{-1}$.

6. Data Collected and Results from Quinn

In Quinn's 2004 model, he chose the maximum force F, the decay rate β, and the resistive force τ to fit the available 400m data from Ferro's 2001 paper. The data provided in Ferro's analysis of the 400m event were produced using twelve analog and digital video cameras. Quinn was able to fit his model to the data by setting $F = 7.91\ m/s^2$, $\beta = 6 \times 10^{-3}\ 1/s$, and $\tau = 1.45\ s$ (for men).

Using the same ordinary differential equations and the same parameters as Quinn, a program written in Maple to solve the system of differential equations numerically replicated Quinn's model results. Table 3 shows the men's 400m simulation in lane 4 from Quinn's paper and compares his model time with the data for each of the 50m intervals. We also include results from the Maple program. A reaction time of 0.15 seconds was added to determine the final finishing time, and we assumed windless conditions.

Distance (m)	**Actual Time (s)**	**Quinn's Model Time (s)**	**Alday-Frantz Time (s)**
50	5.99	6.07	6.10
100	10.95	10.99	10.99
150	15.95	15.93	15.89
200	21.07	20.98	20.95
250	26.27	26.29	26.27
300	31.51	31.75	31.76
350	37.03	37.33	37.32
400	43.03	43.03	43.03
Finish time (s)	43.18	43.18	43.18

Table 3: 400m simulation in lane 4 for world-class runners

Our model times closely approximate Quinn's results, the major difference being 0.04 seconds at 150m. The finishing time is the same for all three models.

7. Effects of Wind Direction on Overall Performance

We next consider the effects that wind has on the overall performance of a runner for different wind direction angles θ. Table 4 illustrates the effects for runners in lanes 1, 4, and 8 of a 2 m/s wind blowing from all points of the compass in 30° increments. (We use 2 m/s because it is the maximum legal speed for a world record to be recorded without the notation "wind assisted.") In addition to the predicted model times, we have included time corrections from the data obtained under windless conditions. For example, a runner in lane 4 would have an overall increase in time of 0.06 seconds corresponding to a steady 2 m/s wind blowing at an angle of 60° from the main straight.

Wind Dir. (θ°)	Lane 1 (43.15)		Lane 4 (43.03)		Lane 8 (42.89)	
	Time (s)	Correction	Time	Correction	Time	Correction
0	43.23	+0.08	43.13	+0.10	43.03	+0.14
30	43.22	+0.07	43.13	+0.10	43.04	+0.15
60	43.19	+0.04	43.09	+0.06	43.01	+0.12
90	43.18	+0.03	43.06	+0.03	42.97	+0.08
120	43.17	+0.02	43.04	+0.01	42.92	+0.03
150	43.18	+0.03	43.03	0.00	42.88	-0.01
180	43.19	+0.04	43.02	-0.01	42.85	-0.04
210	43.18	+0.03	43.01	-0.02	42.82	-0.07
240	43.17	+0.02	43.02	-0.01	42.82	-0.07
270	43.17	+0.02	43.03	+0.00	42.86	-0.03
300	43.19	+0.04	43.07	+0.04	42.92	+0.03
330	43.21	+0.06	43.11	+0.08	42.99	+0.10

Table 4: Effect of a 2 m/s wind on lanes 1, 4, and 8, with time corrections from windless

According to these data, for a runner running in lane 4 the most favorable direction is θ = 210°, that is, a wind blowing toward the southwest, where there is a slight advantage over running in windless conditions. A wind angle range anywhere between 180° and 240° also provides a slight advantage. For lane 8, the advantage is obtained when the wind angle lies between 150° and 270° degrees, with an optimal reduction of 0.07 seconds in the 210°-240° range. Lane 1 realizes the least detrimental effect when the wind blows at an angle from either 120° or 240°-270°, although the overall time is slower for all wind angles. In windless conditions, the time differential between lane 1 and lane 8 is 0.26 seconds, with the advantage going to the outer lane. This is a consequence of the slower velocities due to the smaller bend radius for the inner lanes, as modeled by Greene. We consider the radius effect with the interaction in the table.

The same data is displayed in Figure 2, with smoothed curves joining the data points. It is clear that for each lane there are two significant directions, with roughly opposite compass points, where there is a clear advantage or disadvantage if the wind blows in one of the directions.

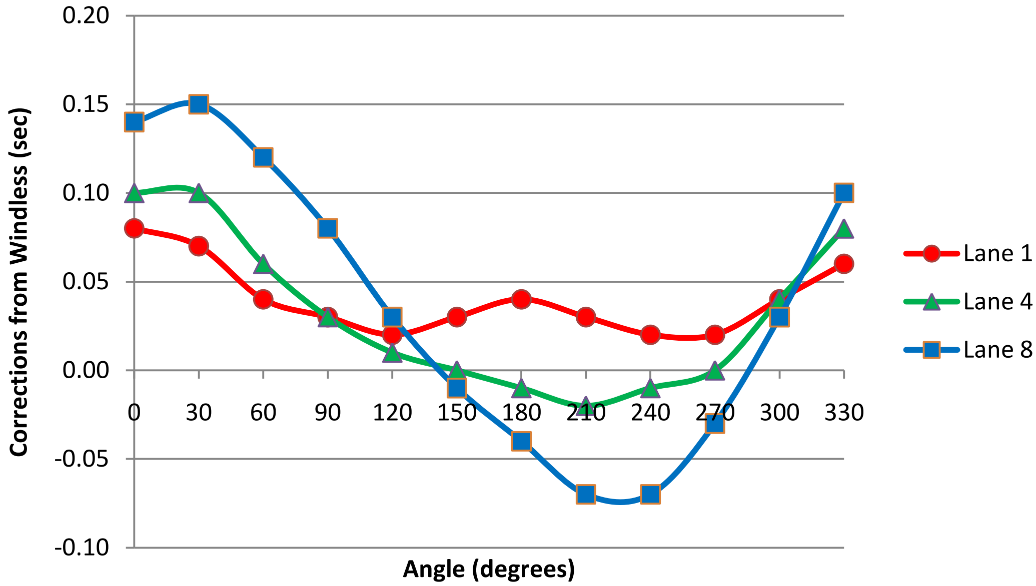

Figure 2: Effect of 2 m/s wind on lanes 1, 4, and 8

Although lane 8 seems to be the most favorable lane in windless conditions as well as in 2 m/s winds, most athletes prefer lanes 3, 4, 5, and 6. In championships these are allocated to the top-speed runners. The reasons why most athletes prefer these lanes are mostly strategic. Most runners prefer to see the progress of the other runners so they can pace their own effort. They are most visible from lanes 1 and 2, but the tighter bends incur a greater disadvantage than the strategic advantage. A runner's goal is to beat competitors rather than to achieve the shortest time, and that influences strategy (for example, recall Usain Bolt's 100m Olympic performance in 2008, when he slowed down significantly just before the finish line but still won the race).

8. Effects of Altitude and Air Density

Altitude has an effect on the performance of an athlete because of its effect on the aerobic energy system. There is a second more important factor, the air density. Races run at higher altitudes favor runners because air density is less at higher altitudes. We replace air density ρ in the expression for α, $\alpha = \dfrac{\rho C_d A}{2M}$ with the air density ρ_H, at an altitude of H meters above sea level, which is related to the air density at sea level ρ_O by $\rho_H = \rho_0 e^{\frac{gH}{R(T+273)}}$, where T is the air temperature in degrees Celsius, and the gas constant is $R = 287\, J \cdot kg^{-1} \cdot K^{-1}$ (in joules per kilogram per degree Kelvin). In our model, $\rho_O = 1.184\ kg/m^3$ and $T = 25°C$, and the altitude, H, changes according to the model. Table 5 contains the time corrections for lanes 1, 4, and 8 on the standard track at various altitudes and under windless conditions.

Altitude (m)	Lane 1 (43.15)		Lane 4 (43.03)		Lane 8 (42.89)	
	Time (s)	Correction	Time	Correction	Time	Correction
0	43.15	0	43.03	0	42.89	0
500	43.08	-0.07	42.96	-0.07	42.82	-0.07
1000	43.01	-0.14	42.88	-0.15	42.75	-0.14
1500	42.95	-0.20	42.83	-0.20	42.69	-0.20
2000	42.89	-0.26	42.77	-0.26	42.63	-0.26
2500	42.84	-0.31	42.71	-0.32	42.58	-0.31

Table 5: Time corrections for lanes 1, 4, and 8 for different altitudes in windless conditions

The time corrections are almost identical across the lanes at each altitude. This to be expected, as the only effect on race times due to lane geometry comes from the physical mechanics of negotiation of tighter bends, which should not be affected by altitude. The largest time correction was a significant 0.32 seconds, which can easily make a difference in whether a world record is broken or not, and is why it has been accepted that altitude-assisted performances should be noted for races at altitudes higher than 1000 meters, and, according to the International Amateur Athletic Federation, these performances are marked by including an ‘A’ notation.

9. The Equal Quadrant Track

Although the IAAF standard track is the norm for track dimensions, the IAAF accepts other tracks. One type of track is the equal quadrant track, which is a 400 meter track with 100 meter bends and 100 meters along each straightaway, measured along lane 1. The other type of track is the non-equal quadrant track, which is a 400 meter track, measured along lane 1, with two curved ends of equal radius and two straights equal in length but longer or shorter than the bends. Finally, there is a double-bend track, which is also a 400 meter track measured along lane 1, with two straights of equal length and two curves that are formed with different radii for each bend. Our model has only been applied to the IAAF standard track, but we will now examine how the wind affects a runner’s performance on an equal quadrant track, illustrated in Figure 3.

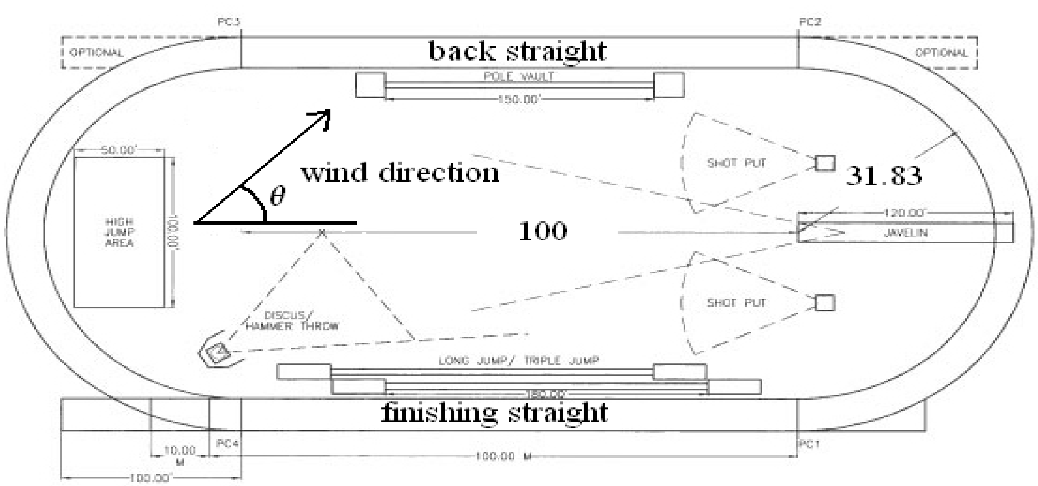

Figure 3: Equal quadrant track

We choose the equal quadrant track because before the IAAF standard track became so popular, the equal quadrant track was preferred. However, today the standard track is by far the most widely used design for a number of reasons, including a wider turning radius that favors runners and enhances performances, lessens injury, and allows greater flexibility in placing fields, especially soccer pitches, within the track oval.

Since the equal quadrant track differs from the IAAF standard track only in its dimensions, all the equations will remain the same, except for the wind velocity. Changes to the wind velocity functions can be easily determined, and the simulation run with virtually the same code. Using the standard track data from Ward-Smith and Radford (2002), but now for an equal quadrant track, we have the lane radii in Table 6.

Lane	**Radius (in meters)**
1	31.83
2	32.95
3	34.17
4	35.39
5	36.61
6	37.83
7	39.05
8	40.27

Table 6: Turning radii for the lanes on an equal quadrant track

Analyzing as before, we obtain in Table 7 new expressions on different portions of the track for the relative wind velocity v_ω.

Relative Wind Velocity v_ω	Region of Application
$v_\omega = u_\omega \cos\left(\theta + \frac{200 - s}{r_i}\right)$	First bend: $0 \le s \le 200 - \pi r_i$
$v_\omega = -u_\omega \cos\theta$	Back straight: $200 - \pi r_i \le s \le 300 - \pi r_i$
$v_\omega = u_\omega \cos\left(\theta + \frac{300 - s}{r_i}\right)$	Second bend: $300 - \pi r_i \le s \le 300$
$v_\omega = u_\omega \cos\theta$	Main straight: $300 \le s \le 400$

Table 7: Relative wind velocity around the IAAF equal quadrant track

10. Wind Effects on the Equal Quadrant Track

Using the same equations, except with a modified definition and domain for v_ω, we are able to produce model results for an equal quadrant track. Once again the finish time is increased by the 0.15 second reaction time. Table 8 shows the results of a 400m race in lane 4 for an equal quadrant track with no wind.

Distance (m)	Equal Quadrant Model Time (s)	IAAF Standard Model Time (s)
50	6.13	6.10
100	11.02	10.99
150	15.91	15.89
200	20.98	20.95
250	26.34	26.70
300	31.86	31.76
350	37.39	37.32
400	43.09	43.03
Finish Time	43.24	43.18

Table 8: Comparison of times for equal quadrant and standard tracks (windless)

The final time in the equal quadrant track is 0.06 seconds slower than the IAAF standard track simulation, due to the smaller radii on the bends. This causes the runners to spend more time on the ground than in the air, which increases their time. An equal quadrant track has longer straights than the IAAF standard track, which gives the runners on an equal quadrant track an advantage to be able to run faster on the straights. However, this does not outweigh the disadvantage that they have on the bends.

Now that the model has been modified for an equal quadrant track, the effects of the direction of a 2 m/s wind can be computed, and are displayed in Table 9 for lanes 1, 4, and 8, in terms of race times and corrections relative to the windless time of 43.24 seconds.

Wind Dir. (θ°)	Lane 1 (43.25)		Lane 4 (43.09)		Lane 8 (42.93)	
	Time (s)	Correction	Time	Correction	Time	Correction
0	43.33	+0.08	43.20	+0.11	43.07	+0.14
30	43.31	+0.06	43.19	+0.1	43.07	+0.14
60	43.29	+0.04	43.16	+0.07	43.05	+0.12
90	43.27	+0.02	43.12	+0.03	43.00	+0.07
120	43.27	+0.02	43.10	+0.01	42.96	+0.03
150	43.28	+0.03	43.09	0	42.93	0
180	43.28	+0.03	43.09	0	42.89	-0.04
210	43.28	+0.03	43.08	-0.01	42.87	-0.06
240	43.26	+0.01	43.08	-0.01	42.86	-0.07
270	43.27	+0.02	43.10	+0.01	42.90	-0.03
300	43.28	+0.03	43.14	+0.05	42.96	+0.03
330	43.31	+0.06	43.18	+0.09	43.03	+0.10

Table 9. Effect of a 2 m/s wind on lanes 1, 4, and 8, with time corrections from windless

The same data are displayed in Figure 4, smoothed.

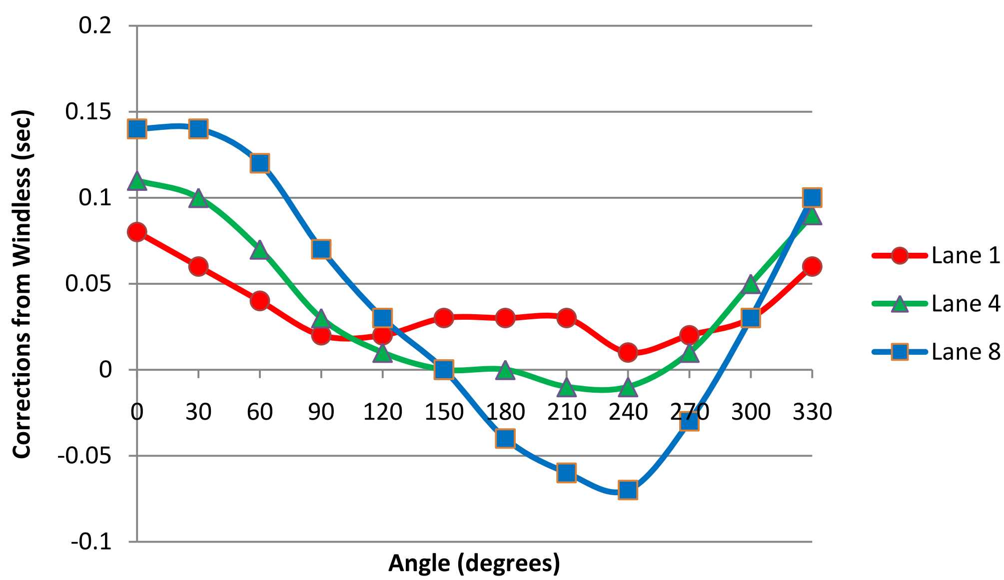

Figure 4: Time correction factors for a 2 m/s wind on an equal quadrant track

The results are similar to those for the standard track. The optimal wind direction for lane 4 is still in the 210°-240° range, for lane 8 at 240°, and lane 1 remains disadvantaged at all wind angles, although the best race times are achieved at angles of either around 120° or 240°.

11. The Ancient Greek Olympiad Track

One of the purposes of mathematical modeling is to predict results that are either too expensive, too dangerous, or simply impossible to achieve by testing. We decided to apply our working model to another track configuration, the track geometry used by the ancient Greeks in the early Olympic games, and revived in the restoration of the Panathenaic Stadium in Athens in 1895 for the modern Olympics (see Figure 5). Our model can answer questions about how much time differential between performances on an ancient track and a modern track can be attributed to the track geometry alone, disregarding the many other variables such as fitness, training, equipment, track surface, etc.

Figure 5: The Panathenaic Stadium in Athens

A track with this geometry has two straights of 180 meters each and two bends of radius 6.37 meters, each contributing 20 meters. All that is required is a change of the parameters governing the strength of the relative wind speed as a function of the distance around the track, analogous to the adjustments made in moving from the standard track to the equal quadrant track. Table 10 provides a comparison of the times in windless conditions for lane 4 in the Panathenian track with the IAAF standard track.

Distance in meters	Std. track time in sec	Pan. track time in sec	Std. time per 50 m	Pan. time per 50 m
50	6.10	6.15	6.10	6.15
100	10.99	10.91	4.89	4.76
150	15.89	15.79	4.90	4.88
200	20.95	21.06	5.06	5.27
250	26.70	26.69	5.75	5.63
300	31.76	32.04	5.06	5.35
350	37.32	37.57	5.56	5.53
400	43.03	43.28	5.71	5.71
Finish time (s)	43.18	43.43	43.18	43.43

Table 10: Comparison of lane 4 (windless) times; IAAF standard vs. Panathenian track

As might be expected, although the longer straights on the Panathenian track permit more of the race to be run at peak speed, the extremely sharp bends cause slowdowns that more than compensate, leading to an increase in race time of 0.25 seconds. Table 11 shows the effect of a 2 m/s wind on lanes 1, 4, and 8 on the Panathenian track.

Wind Dir. ($\theta°$)	Lane 1 (43.84)		Lane 4 (43.28)		Lane 8 (43.00)	
	Time (s)	Correction	Time	Correction	Time	Correction
0	43.93	0.09	43.38	0.10	43.08	0.08
30	43.91	0.07	43.38	0.10	43.09	0.09
60	43.87	0.03	43.35	0.07	43.08	0.08
90	43.85	0.01	43.32	0.04	43.06	0.06
120	43.86	0.02	43.31	0.03	43.07	0.07
150	43.88	0.04	43.32	0.04	43.08	0.08
180	43.90	0.06	43.31	0.03	43.06	0.06
210	43.80	0.04	43.28	0.00	43.02	0.02
240	43.86	0.02	43.25	-0.03	42.97	-0.03
270	43.85	0.01	43.25	-0.03	42.95	-0.05
300	43.87	0.03	43.29	0.01	42.97	-0.03
330	43.91	0.07	43.35	0.07	43.03	0.03

Table 11: Effect of a 2 m/s wind on lanes 1, 4, and 8, with time corrections from windless (Panathenian track)

We plot the correction factors induced by the wind, smoothed, in Figure 6.

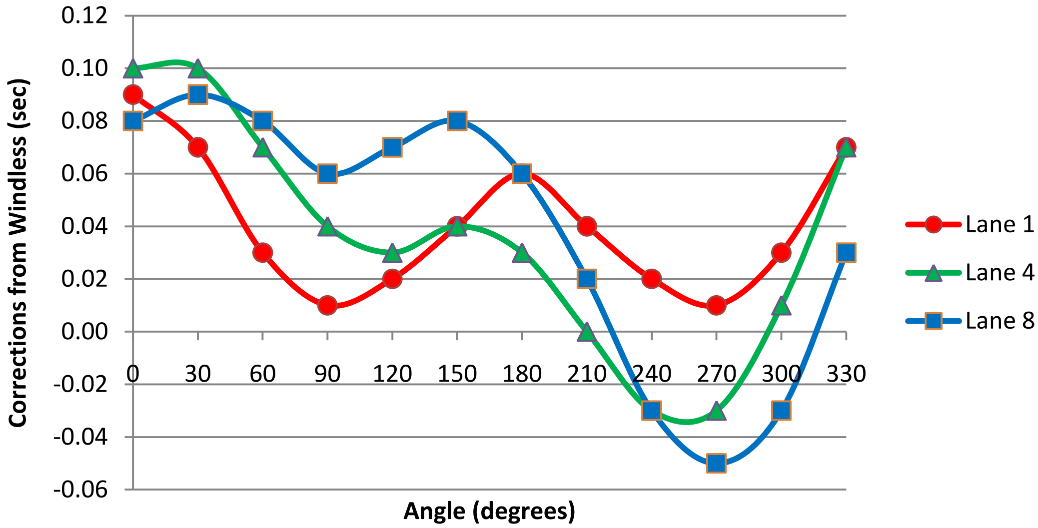

Figure 6: Time correction factors for a 2 m/s wind on the Panathenian track

12. Summary of Results

A comparison of results from the IAAF standard track and the equal quadrant track shows similarities as well as differences. The biggest disadvantages to the runner on both tracks in windless conditions are in lane 1, since the tighter bends force the runner to spend more time on the ground and less time in the air. The effect is greater for the equal quadrant track, since the radius of lane 1 is 31.83 m, instead of the 36.80 m radius of lane 1 for the IAAF standard track. Lane 8 appears to be the fastest lane on both tracks, having a shorter distance around the first bend than any other lane, and a wider radius around the second bend, permitting the runner to stay in the air longer than the other runners and thus producing a faster time than in any other lane. Although the radius of lane 8 on an equal quadrant track at 40.27 m is less than that of the 45.25 m radius of lane 8 on the standard track, which would seem to give an advantage to the standard track, the disadvantage of the standard track is that it has much shorter straights. On the standard track each straight is run for 84.4 meters, whereas on the equal quadrant track each straight is run for 100 meters. The more gradual (faster) bends of the standard track overcome the disadvantage of its shorter straights, providing a time that is 0.06 seconds faster in lane 4 under windless conditions. Although 0.06 seconds seems like a rather brief interval in many applications, it can feel like an eternity in a sprint race.

With a constant 2 m/s wind, the results indicate advantages and disadvantages. Lane 1, for example, offered no advantages from any wind direction, and a loss of 0.08 seconds on either track with a 0° headwind on the main straight. There is a slight advantage in lane 4 with a wind direction varying from 210° to 240°, providing a time correction of -0.02 and -0.01 seconds on the standard and equal quadrant tracks, respectively. Lesser advantages occur on both tracks with wind directions approximately between 180° and 250°. The largest wind advantage is found in lane 8 on each track, with a wind direction from 210° to 240° providing a time correction of -0.07 seconds. Lesser time advantages occur in lane 8 with a wind direction approximately between 150° and 280°.

Since the Panathenian track is no longer in competitive use, our interest in examining it was simply to observe what kind of advantages or disadvantages might be predictable from our model, and whether or not they have any relationship to what one might predict from the physics and geometry of the situation. The amount of time spent on the bends is minimal in comparison with the other two tracks, supporting an argument that reducing the headwind on the straights would serve the runner best. The model indicates an optimal time in all three lanes (1, 4, and 8) for a southerly wind of 270°, with another lesser local minimum time for a northerly, or 90° wind, both blowing perpendicular to the runner for most of the race. The corollary to this is that wind directions parallel to the straights, at about 0° or 180°, provide the worst conditions, as might be expected. The data illustrate a principle that holds for all three tracks, namely that the most benefit occurs for the runner facing a wind direction of 240°, while the worst outcome is associated with a wind direction of about 30°. One interpretation of this is that if a wind must be encountered, and particularly on a portion of the track which requires more energy (like the bends), it is advantageous to have the wind at the runner's back earlier in the race, and then to have the headwind later. If one considers two races that are run parallel to the wind, race A consisting of a mile with a tailwind, then a mile with a headwind, and race B with the headwind first and then the tailwind, runners in race A have an advantage over those in race B because the energy gain from a tailwind is known to be about half the size of the energy loss due to a headwind. This means that a headwind early in the race has a greater effect in decreasing a runner's energy for the remainder of the race, as opposed to the benefits reaped from an early tailwind.

As with the IAAF standard track, we tested the effects of altitude on the equal quadrant track and Panathenian track. The results were entirely consistent with the thesis that the higher the altitude, the faster the times. The average altitude time corrections were -0.09, -0.18, -0.27, -0.35, and -0.42 seconds for altitudes of 500 meters, 1000 meters, 1500 meters, 2000 meters, and 2500 meters.

We have shown that wind direction can significantly affect the performance of a runner in a 400m race. When sprinting, the disadvantage of a head wind is greater than the benefit of a tail wind of the same magnitude, so wind direction plays a role in the performance of a runner to the extent that a record may be set without wind assisted conditions on one day, only to be broken on the next day under a wind of the same velocity but from a different direction.

The effect of altitude on race times is also significant and predictable. Finally, altitude has a major effect on the overall time in both tracks, the biggest time corrections being for the equal quadrant track. Our model, inspired by Quinn, shows that other important factors are at stake, including lane selection, wind speed and direction, location and altitude, and even track geometry. Running in track and field events and winning is not as simple as being in shape and generating a strategy.

13. Directions for Possible Future Work

Quinn's 2004 model considered winds of constant speed and direction throughout the stadium. The architecture of the stadium also affects on how wind changes, depending on the placement of the bleachers, their height and width, and other factors that might produce erratic winds. Creating a model for these circumstances could result in more accurate outcomes, but there are many other variables not accounted for. In addition, improvements on the performance of a 400m run are not attributable solely to wind and altitude.

For example, lane 8 is the theoretical fastest lane, which indicates that it should be the favorite choice of top sprinters. Anyone who has watched a major track event knows that in fact the goal for most runners is to have a lane assignment in one of lanes 3 through 6. There are other strategic factors at work here, involving seeing more easily where the competition is, avoiding the curb adjacent to the inner track, being unable to see the competition at all, and being of a disposition which prefers leading to catching up, or vice versa.

Other factors exist that affect a runner's performance, including physical condition, nervous tension, health, clothing, personal issues, etc. Measuring and modeling them range from difficult to impossible. Another factor that has been overlooked, as Keller (1973) states, is that the goal of runners is to beat competitors rather than to achieve the shortest time, which influences their strategy.

Although many of these factors appear to be hopelessly non-quantifiable, it is the job of the applied mathematician to ferret out what the most significant factors are, and to discover a method to account for them in the model. Perhaps this work will inspire others to look for ways to improve it, or generate a completely new model, or turn mathematics to a description of other sports which have thus far been ignored, in the hope of obtaining a better understanding of all the factors that go into a world record performance.

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