From *SIAM News*, Volume 40, Number 9, November 2007

Flagrant Gerrymandering: Help from the Isoperimetric Theorem?

By James Case

Inspired by the gerrymandering problem from this year's Mathematical Contest in Modeling, I resolved to explore an idea for eliminating the most flagrant abuses of the redistricting process. I had long believed the idea to be my own, but a number of others* have suggested imposing an upper bound on the "isoperimetric quotient" of a legislative district.

The isoperimetric theorem, which dates back to antiquity, asserts that if A and P represent the area and perimeter of any planar figure, the dimensionless quotient $IQ = P^2/4\pi A$ must equal or exceed unity, with equality only for circles.† Figures with fractal boundaries, such as the Koch snowflake, can have infinite IQs. The real question in the gerrymandering context—to which I return later—is by how much IQ should be allowed to exceed unity.

Gerrymandering is as old as representative democracy. The U.S. Constitution provided that each state should send to the House of Representatives a contingent roughly proportional to the size of its population, relative to that of the nation as a whole. The geographic details of the partition into congressional districts was left to the states. Abuses began almost immediately, and were not confined to Massachusetts, where the original "Gerry-Mander" was a district shaped like a dragon (or salamander, whence the name), depicted in a cartoon published in the *Boston Sentinel* on August 8, 1812 (Figure 1). Concocted by the Jeffersonian party, of which then Governor Elbridge Gerry was a member, the plan was something of an embarrassment to Gerry, who was also James Madison's running mate in that year's presidential election. (Though they won the election, Gerry died in office two years later.)

Figure 1. *The original Gerry-Mander.*

Current gerrymanders are even more flagrant than the original. The fourth congressional district in Illinois (Figure 2) is a case in point. Its 116-mile boundary encloses an area of less than 40 square miles, giving it an IQ in excess of 27. In it, two predominantly Hispanic neighborhoods are connected by a long slender corridor (containing more than a mile of Interstate 294, which is legally uninhabitable), in order to meet the requirement of contiguity. Other states, including Georgia, Florida, California, and North Carolina, have congressional districts with even larger IQs, some (but not all) of which possess natural boundaries. Among the latter is Florida's eighteenth district, which includes the Florida Keys.

Figure 2. *The area in green is the fourth congressional district of Illinois.*

Not until 1962 did the Supreme Court begin to restrict the nature of the boundaries the states can interpose between congressional districts. That year, in *Baker v. Carr*, the court ruled that district boundaries should in future respect the principle of "one man, one vote," thereby extending to intrastate jurisdictions the "equal representation" clause of the Constitution. In 1982, Congress amended the Voting Rights Act of 1965 to encourage the creation of legislative districts likely to elect African American or Hispanic legislators. Yet the Supreme Court has consistently declined to enunciate any clear-cut standard by which the acceptability of a partition could be judged. Instead, beginning in 1986, it has issued a series of rulings intended to clarify the meaning of the 1982 amendment.

The Court has confirmed on several occasions that redistricting designed to give one political party an electoral advantage is not necessarily unconstitutional, although it can be in certain instances. In the 1993 case of *Shaw v. Reno*, the Justices declared that race can be taken into account, but cannot become the predominant influence when district boundaries are drawn. Indeed, in the 2004 case of *Veith v. Jubelirer*, the Court upheld (by a 5 to 4 vote) a Pennsylvania districting plan expressly designed to increase the number of legislative districts likely to be won by Republicans. So long as district boundaries do not discriminate on the basis of race, or flagrantly violate the "one person, one vote" principle, the Court did not find the advantage conferred by one party's redistricting plans on its own candidates to be worthy of judicial intervention.

To learn the IQ values that might be expected for impartially constructed congressional districts, I calculated the IQs of the 50 states (and the District of Columbia), using data obtained from the U.S. Geolog-ical Survey. After all, the states are the districts represented by senators. The 51 IQ scores varied from a low of about 1.29 to a high of about 40.23, with a mean of 4.17, a median of 2.46, and a standard deviation of 6.57.

*A recent example being Hermann Kremer and Eric W. Weisstein, "Isoperimetric Quotient," on the Wolfram Web resource MathWorld.

† Different authors define the IQ in different ways, some omitting the factor 4π, some using $\sqrt{IQ}$ or $1/IQ$, etc.

The relative sizes of the median and mean suggest that the distribution is significantly skewed toward larger values. A histogram of the smallest 40 scores looks decidedly unimodal, though still skewed toward the larger values, suggesting a gamma distribution with parameters $\alpha = 2/5$ and $\beta = 1$, or perhaps a log-normal distribution.

The ten states with the highest IQs—Alaska, Maryland, Florida, Hawaii, Louis-iana, Michigan, North Carolina, Virginia, Massachusetts, and Texas—all have extensive natural boundaries, such as rivers and coastlines. It could be argued that the presence of natural boundaries distorts IQ scores, and that—in the interest of comparability—such boundaries should be approximated by shorter polygonal arcs. When this is done to the Potomac River and Atlantic Ocean portions of Maryland's boundary, for instance, using rectilinear edges between 2 and 20 miles long, the state's boundary is reduced from the 1317 miles reported by the USGS to about 776 miles. This causes Maryland's IQ to fall from 14.17 to something smaller than 5. It is probable that similar reductions can be achieved by similar means for other states with extensive natural boundaries. If a precise standard is needed, fractal geometry furnishes an obvious candidate: Perform the measurements with a "ruler" several miles in length.

As might be expected, the IQs of the nation's 437 congressional districts are substantially larger than those of the states, with a mean of 6.59, a median of 4.85, and a standard deviation of 5.46. The overall range is the same as for the states, because Wyoming (IQ = 1.29) and Alaska (IQ = 40.23) have only one representative apiece. Again, the distribution is skewed toward higher values, suggesting a gamma (this time with parameters $\alpha = 3/2$ and $\beta = 1/5$) or perhaps a log-normal distribution. And again, the need to correct for natural boundaries arises—something would have to be done about it before binding legislation (or possibly a constitutional amendment) could even be contemplated. But even if it led to no legislation at all, an expanded version of the present pilot study—exploring (among other things) various methods of correcting for the effects of natural boundaries—could improve matters.

Flagrantly gerrymandered legislative districts—state and local, as well as national—are regularly challenged in court. A thorough study of the proposed sort, if carefully carried out and extensively publicized, would encourage judges to interpret any IQ in excess of (say) 5 as de facto evidence of an attempt to steal one or more elections. Such decisions seem likely to be appealed all the way to the Supreme Court, where they could conceivably be upheld. Whereas gerrymandering would hardly cease in the wake of such a finding, its utility as a means of distorting electoral outcomes would be significantly reduced.

James Case writes from Baltimore, Maryland.