From *SIAM News*, Volume 40, Number 2, March 2007

The Finite Element Method in EEG/MEG Source Analysis

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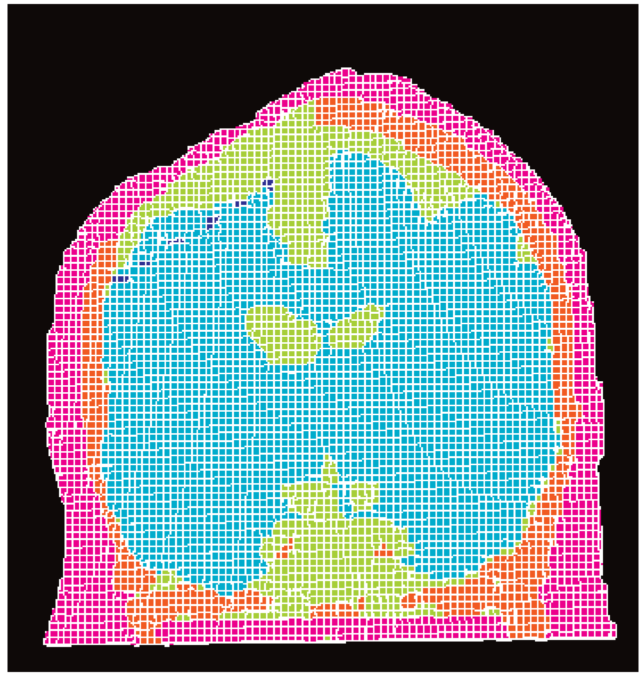

Figure 1. *2-mm geometry-adapted finite element hexahedral mesh of a segmented computer tomography and magnetic resonance data set for a patient with medically intractable epilepsy. The tissue layers—skin, skull (with the surgical opening visible), cerebrospinal fluid, and brain—and the depth electrodes are shown in different colors.*

Electro- and magnetoencephalography (EEG/MEG)-based source reconstruction of cerebral activity (the EEG/MEG inverse problem) is an important tool both in clinical practice and research and in cognitive neuroscience. Methods for solving the inverse problem are based on solutions to the corresponding forward problem, i.e., simulation of EEG/MEG fields for a given primary source in the brain with a *volume-conduction model* of the head. The associated differential equations for the forward problem are the quasi-static Maxwell equations. The primary sources are electrolytic currents within the dendrites of the large pyramidal cells of activated neurons in the human cortex, generally formulated as a mathematical *point current dipole*. Such focal brain activation can be observed in epilepsy (interictal spikes), or it can be induced by a stimulus in neurophysiological or neuropsychological experiments, e.g., somatosensory or auditory evoked fields.

Realistic volume-conductor modeling for accurate solution of the forward problem begins with segmentation of the tissues of the head; conductivity values are assigned in a second step. The tissues vary in conductivity and can also be inhomogeneous, e.g., the human skull, and anisotropic (with conductivity showing directional dependence), e.g., the skull and brain.

The finite element (FE) method is often used for the forward problem, because it allows realistic representation of the complicated head volume conductor. In the case of a point current dipole in the brain, the singularity of the potential at the source position can be treated with the "subtraction dipole model"; the model divides the total potential into the analytically known singularity potential and the singularity-free correction potential, which can then be approximated numerically with an FE approach [5]. For the correction potential, existence and uniqueness proofs of a weak solution in a zero-mean function space and statements about FE convergence properties have been given [5]. Beyond the subtraction dipole model are direct FE approaches to the total potential; these approaches are computationally less expensive, but also mathematically less sound if the point dipole is seen as the most realistic source model. They use either partial integration over the point source on the right-hand side of the weak formulation, approximating the source singularity by means of a projection in the function space of the FE trial-functions (partial integration dipole model; [4]), or approximation of the point dipole by an even smoother monopolar primary source distribution (St. Venant dipole model; [4]).

A prerequisite for FE modeling is the generation of a mesh that represents the geometric and electric properties of the volume conductor. An effective meshing strategy will achieve both acceptable forward problem accuracy and reasonable computation times and memory usage. Surface-based Delaunay tetrahedral tesselations are often used because of their ability to represent tissue boundaries in a smooth and regular way [3,5]. Hexahedral elements are also used; the hexahedra exploit the spatial discretization inherent in segmented medical tomographic data, and good performance has been achieved with them in recent accuracy studies [4,5]. A geometry-adapted node-shifting approach was developed to avoid the stair-like approximation of curved tissue boundaries that occurs with regular hexahedra; its use has led to significant reductions in field topographic and magnitude errors, despite the detrimental effects of deformed elements [5]. Adaptive methods preclude the use of lead field bases (see below) and lose efficiency in solving the inverse problem.

An important question is how to handle the computational complexity of FE modeling with regard to the inverse problem. The longtime state-of-the-art approach was to solve an FE equation system for each anatomically and physiologically meaningful dipolar source (each source results in one FE right-hand side vector). Iterative solvers were used, among them the successive over-relaxation or the preconditioned conjugate gradient (CG) method, with preconditioners like Jacobi (Jacobi–CG) or incomplete Cholesky. More recently, algebraic multigrid (AMG) solvers, used as a preconditioner for the CG method, have proved more efficient than solvers tried earlier. Specifically, large speedups have been achieved with a parallel AMG–CG method for an anisotropic FE head model as compared with a standard Jacobi–CG method on a single processor [1]. Still, repeated solution of FE equation systems with a constant geometry matrix for thousands of right-hand sides (the sources) was the most time-consuming part of the inverse localization process and limited the resolution of the models.

Another very efficient concept for reducing the computational complexity of the problem is reciprocity. The reciprocity theorem for the electric case states that the field of the lead vectors is the same as the current field produced by feeding a reciprocal current to the lead. This means that we can switch the role of the sensors and the dipole locations. Recently, for efficient computation of the FE-based EEG and MEG forward problem, an even easier principle, as-sociativity with respect to matrix multiplication, was applied [2]. Using this principle, which is closely related to reciprocity, EEG and MEG lead field bases can be defined, i.e., matrices with rows corresponding to sensors and columns to FE nodes. The exact definition of the lead field bases depends on the dipole model approach described above. For each model of the head, one has only to solve large sparse FE systems (one for each sensor) to find the lead field bases for both EEG and MEG; this can be performed efficiently with the parallel AMG–CG solver with block-right-hand sides. Each forward solution is then reduced to multiplication of the lead field basis by an FE right-hand side vector, with the latter dependent on the chosen dipole model as well. Exploiting the fact that the number of sensors (maximally about 400) is much smaller than the number of reasonable dipolar sources (tens of thousands), the lead field approach will be faster than the state-of-the-art forward approach by a large factor and can be applied to inverse reconstruction algorithms in both continuous and discrete source parameter space for EEG and MEG.

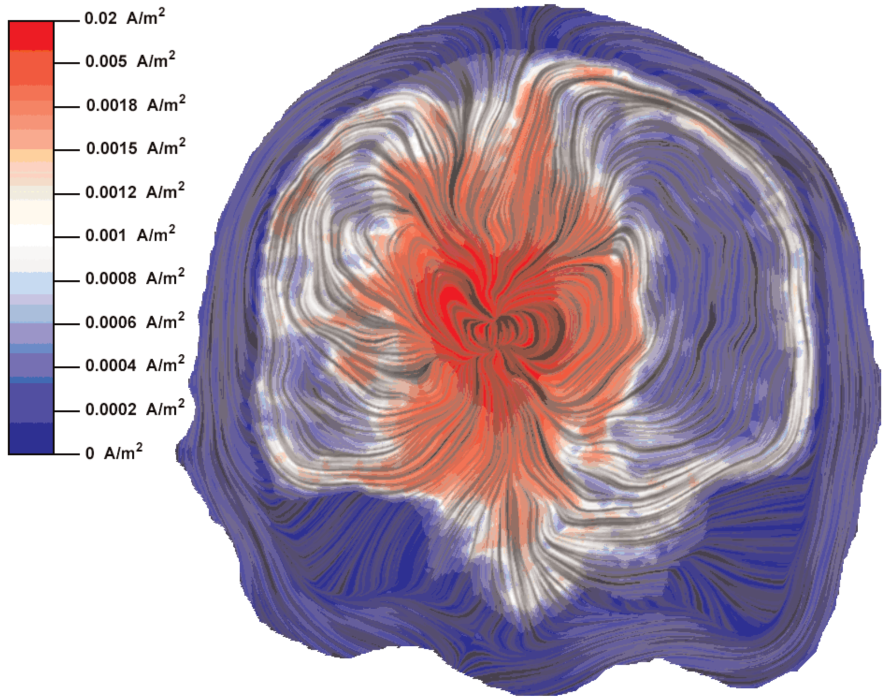

Figure 2. *Volume currents for a thalamic dipole source computed in a finite element volume-conductor model and visualized on a coronal cut through the model. Copyright 2007; reprinted from [3] with permission from Elsevier.*

FE dipole modeling approaches and mesh-generation techniques have been validated in spheroidal volume-conductor models for which analytical or quasi-analytical solutions exist. For the EEG, a quasi-analytical solution was presented for a point-dipole source in a volume-conductor model consisting of an arbitrary number of concentric/confocal anisotropic layers of different conductivities. For the MEG, it appears that the magnetic field outside the head is completely independent of the conductivity profile, provided that the conductor is spherically symmetric. An analytical formula has been derived for this model. Spheroidal models, in addition to their continued frequent use in source analysis routines, serve as validation tools for the FE approaches described here [4,5]. They can also be used to study the relative accuracy of different FE solvers [5].

FE analysis has revealed that skull anisotropy has a smearing effect on the forward EEG computations and no effect on the MEG, while brain anisotropy causes return currents to flow parallel to the fiber tracts [3]. The deeper a source lies and the more it is surrounded by anisotropic tissue, the greater is the influence of this anisotropy on the resulting EEG and MEG. Surgical opening of the skull, moreover, influences the forward problem for both EEG and MEG and needs to be modeled appropriately.

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