Welcome to Issue 82 of the Statistics Teacher Network. There are three articles that touch on several valuable ideas you can implement in your schools and classrooms. The four associate editors and I have worked with the authors to provide you this timely and useful information. Here’s a brief synopsis of each article:

- You may recall in the last issue Rebecca Nichols’ description of the ASA’s U.S. Census at School program. In this issue, the article titled “Kicking and Screaming about Statistics: How soccer data can potentially alleviate statistical anxiety” by Juana Sanchez, Dianne Cook, Johnny Masegela, and Marela Kay Minosa describes the STATS4Soccer project that introduces a new dimension to teaching mathematics and statistics by applying the “learning-through-play” teaching concept. The program is similar to Census at School, but adds the dimension of data collection through physical engagement in a sport.

- In the article, “Harnessing the potential of outside resources in a first or second year statistics course,” two high-school students—Serena Blacklow and Cam O’Reilly—share their experiences with an independent study course and offer suggestions on what teachers can do to engage their students.

- The third article, by Alisa Izumi of Western Governors University discusses her experiences using concept-mapping to assess student understanding of introductory statistics concepts.

I think you will find all three articles worth the read! Please continue to contact me with your ideas for improving STN, suggestions for articles, new teaching techniques, and upcoming events relevant to our cause. Please email me at rpierce@bsu.edu

Best Regards,
Editor, Rebecca Pierce, Ball State University

Associate Editors
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Harnessing the Potential of Outside Resources in a First or Second Year Statistics Course

By Serena Blacklow and Cam O’Reilly

After spending the past year developing our own Beyond AP Statistics Independent Study, we want to share with you the aspects of our course that were most educationally beneficial.

We are high-school seniors who, after having completed AP Statistics, embarked on a journey beyond what our school—Buckingham Browne and Nichols—offers. Along the way, we learned not only about new statistical concepts such as ANOVA and logistic regression, but also about making the most of our opportunities to reach outside the classroom in our studies. Our supervisor told us the first day to approach this course like a graduate-level course. We found it most educationally enriching to expose ourselves to the professional world of statistics, whether through attending talks, meeting with experts or using professional-level software. We believe that these experiences would greatly enhance any statistics course.

I. Attending statistical talks

Attending a professional statistics presentation for an evening was one of our first opportunities to learn about statistics from a source other than our teacher or textbook. At the dinner preceding the American Statistical Association (ASA)-sponsored lecture, we sat down and chatted with a number of experts in various subfields. Before the presentation began, we already were immersed in learning as the statisticians around us eagerly explained how they applied statistics in their work. Many encouraged us to consider a career in statistics, citing personal employment history as examples of the varied job opportunities for a statistician in today’s world as data become more readily available. Though our teachers can give us the same encouragement, hearing it from a professional transforms the hypothetical into the achievable.

One software expert stressed the importance of reproducibility in computational processes—finding averages, fitting models, calculating standard deviations, and more. Although we frequently discuss reproducibility in the context of data collection, we never had considered thinking of it in the context of computer algorithms. Our discussion revealed a subfield of statistics that we had never previously considered, and the expert’s passion for this topic made a strong impression on us.

After the dinner, Nicholas Horton, a Smith College statistics professor, presented his recent study, “Adjusting for Nonresponse in the Occupational Employment Statistics Survey.” These data, income in different occupations, were interesting in part because income is a topic that we had seen in our daily newspaper. This discussion added a crucial element to our knowledge about statistics at a higher level since the strategies used to handle non-response are left unexplored in our textbook. Although one year of AP Statistics did not allow us to understand all of the presentation, we still had sufficient background to understand several of his approaches to adjusting for non-response. In general, being among a community of statisticians made us appreciate the problems these professionals face and their approaches to solving these problems.

II. Meeting with a statistical expert

After the conference, we decided to contact one of the statisticians we met there and he agreed to meet with us as a mentor. During our meetings, we discussed topics that we had researched extensively but still could not understand. We each gave short presentations on the topics, such as Prediction Intervals and Variance Inflation Factor, welcoming him to critique us. During our presentations, he asked us questions about our conclusions and offered his own input and opinions. As someone who uses the most complex statistical methods on a daily basis, he could introduce us to methods of analyzing data that were not explained in our textbook or by our teacher. His academic and professional experiences led him to prefer some methods over others, such as using a normal probability plot rather than a normal quantile plot to evaluate conditions for multiple regression. By sharing opinions, he gave us a multifaceted view of a topic that we would otherwise never have considered to be open to discussion.
One of the many advantages to meeting with a professional statistician is that he or she can present examples from his or her work. Our mentor provided us several of his papers to illustrate concrete examples of various concepts we discussed in our meetings. For example, after talking about experimental design, we read a paper from one of the studies he conducted for the American Medical Association on Risk Adjustment of Hospital Mortality.

This paper connected sensitivity and specificity—ideas we had touched on in our AP Stats class—to a professional study and introduced us to the c statistic and ROC curves, which we had not previously encountered. Since we could talk with one of the authors, we were now able to explore the study much further than the abbreviated description accompanying a dataset in a textbook. We suggest if you do get a chance to meet with a statistical expert, you ask him or her for their articles or papers to share with your students. These readings will serve as a catalyst for discussion, and students will have a much easier time finding answers to their questions about the study if the article’s writer of the article to meet with them.

While some points made in the articles were difficult to grasp, looking beyond interpretations given to us in the textbook can open up new ideas and challenge us. Through this process, we have learned that reading articles as supplements to what we are learning from a textbook is an eye-opening way to gather information and ideas.

Recently, our mentor introduced us to Median Polish. This method is similar to ANOVA, which compares group means and variances to test if one mean is significantly different from the rest. While ANOVA uses means and square deviations, Median Polish uses medians and absolute deviations. Our mentor taught us how to create a Median Polish model on a set of data by hand and showed through examples that the Median Polish model is advantageous because it is resistant to outliers. After comparing the residual plots from the two tests on the same data, we realized just how different these tests were.

Our meetings have left us thinking more about fitting models to data and being cautious when creating these models. Analyzing data is not a mechanical process—it takes planning and thought about what the data means before entering it in the software and receiving an “answer.”

III. Using the statistical software JMP

Another centerpiece to our statistics course was using the SAS software package JMP (John’s Mac Project). In AP Statistics we used the TI-Nspire calculator to create histograms and calculate p-values. JMP empowered us to explore the example and practice problems in our textbook as well as work on our independent projects. We liked using JMP better than the calculator because it had organized outputs and allowed us to compare graphs opened in multiple windows. Although a number of aspects of JMP are basic and intuitive, we encountered several issues along the way. The easiest solution to our troubles with JMP was to ask questions through the SAS online forum and thus hear the ways other statisticians solved similar problems. The JMP community was supportive and therefore answers were readily available.

One issue with many high-school courses is that students are discouraged from asking questions online for fear of copying someone else’s thinking. Rather than waste hours poring over user manuals or clicking random buttons, we were able to ask experts and implement their advice quickly.

Overall, using the JMP software has been another step in our statistics learning and using the JMP forum has made us more comfortable with searching for answers online. Solely being exposed to statistical software has opened our minds to the type of analysis that real statisticians conduct, regardless of its particular drawbacks.

Conclusion

How can you get your students excited about statistics and incorporate the professional world into your course? For us, attending statistical conferences, meeting with an expert, and using statistical software presented us with a variety of perspectives through which we can approach statistics and, to a larger extent, enhance our learning. We hope that you, too, will look at local statistics lectures, contact professors at nearby universities, or explore software packages to boost your students’ learning. It is important that subject matter is approached through not just one source. These multiple sources have provided us a balanced understanding of new statistical concepts and a glimpse into the ways of professional statisticians.

About the authors:

Serena Blacklow

I took AP Statistics during my junior year at Buckingham Browne & Nichols (BB&N), a prep school in Cambridge, Massachusetts. I already have found statistics useful in my science classes at the high-school level, so although I plan to pursue biology or engineering in college, I may minor in statistics. Outside of school, I often read scientific papers that mention statistical terms and software outputs that I can now recognize. I am also an avid rower and sports fan. As a result, I studied which variables correlated with two different responses: the number of points the New England Patriots football team scored per game in 2012 and whether it won or lost. In our course, we have not only explored new statistical concepts but also have approached learning in a new light.

Cam O’Reilly

The first experience I had with statistics was taking AP Stats in my sophomore year at BB&N. I was excited by the applicability of statistics to so many fields and enjoyed reading news articles with a statistical eye. For my senior year, I chose to pursue statistics for a second year as an independent study in collaboration with Serena. Under the supervision of Dr. Coons, we explored every avenue of learning we could find. One of my favorite topics was Multifactor ANOVA, which I applied to a set of times from the BB&N cross-country team. As a cross-country runner, I found it very intriguing to compare the effects of different courses to the individual effects of each runner on their time. Looking forward, I plan to pursue statistics at Dartmouth College. Although I have not yet decided on a college major or career, these experiences have shown me just how versatile statistical knowledge can be!
Kicking And Screaming About Statistics: How Soccer Data Can Potentially Alleviate Statistical Anxiety

Juana Sanchez, Dianne Cook, Johnny Masegela, Marela Kay Minosa

Schools and teachers face the challenge of including more statistics in the K-12 curriculum (Cohen, 2012). However, the Common Core State Standards (CCSS) include less statistics at the elementary school level than is recommended by the Guidelines for Assessment and Instruction in Statistics Education (GAISE) report and National Council of Teachers of Mathematics (NCTM) standards. The pedagogy proposed by CCSS also is very different from Levels 1 and 2 of GAISE and the more general guidelines of NCTM (Bargagliotti and Webb, 2011). GAISE and many school-curriculum reforms around the world aim to enhance the process of statistical inquiry across the curriculum from a very early age and to make young students statistically literate. In this paper we present the experience of South Africa in providing educators with alternative and creative means of teaching and facilitating statistics from a very early age via an integration of classroom activities and physical education.

In 2006, the Department of Education (DoE) of South Africa introduced Learning Outcome 4 (LO4), which introduces statistics at school level, particularly data handling and probability. The introduction of LO4 was cumbersome as mathematics teachers were not sufficiently trained in statistics. Statistics South Africa (StatsSA) in collaboration with DoE introduced a variety of projects that are focused on promoting statistical literacy among students and teachers.

Via the Census at School program (Census@School, 2001) StatsSA knew that about 70% of students liked soccer at grade 3 level and 71% of students liked mathematics in the same grade. As the students progressed through school, their interest in mathematics dropped drastically, mainly due to the apparent difficulties the students face when it comes to learning the subject. To address this issue, StatsSA came up with a way to make mathematics and statistics innovative and non-threatening to students.

Inspired by South Africa’s role as the host country for both the 2010 FIFA World Cup and the International Statistical Institute’s (ISI) biennial meeting in 2009, StatsSA recruited the assistance of Johnny Masegela, otherwise known as “Black Sunday” during his prime as the Orlando Pirates’ striker. Together, they came up with a way to marry the love for soccer and mathematics throughout a student’s school life. Hence, STATS4Soccer was born as the extracurricular activity for the Capacity Building Program of StatsSA, ISIhako 6. The goal of STATS4Soccer is to help students see “statistics from a different view other than just mathematics and to see how it can be applied to everyday events and activities.”

Some of the expected learning outcomes of STATS4Soccer are:

- If students practice mathematics and statistics outside the classroom, in a real environment, it is easier to comprehend mathematics and statistics in the classroom.
- If students realize that mathematics and statistics can indeed be fun and in fact is not difficult and scary, there will be more students enrolling for statistics courses at tertiary level, therefore producing more statisticians.
- Provide educators alternative and innovative means of integrating topics covered in Mathematics and Life Orientation.
- Provide soccer coaches information that will help them in training sessions.

The STATS4Soccer project introduces a new dimension in teaching mathematics and statistics. It applies the “learning-through-play” teaching concept in which mathematics and statistics are taken out of the classroom and applied on the soccer pitch. It consolidates learning that has taken place in the classroom using geometric shapes. It is oriented to LO4 for grades 3-12. It also addresses LO3, which focuses on Space, Shape and Measurement. The statistical topics covered during the exercises are: (a) data collection; (b) data capturing; (c) data analysis; (d) data representation; (e) statistical computation and descriptive statistics. It involves two requirements: personal investigation, and complex and meaningful context, not unlike the constructivist approach to learning (Libman, 2010). STATS4Soccer complements the MATHS4Stats program (Maths4stats, 2013).

The example set by StatsSA is relevant for educators not only because of the new CCSS but also because Science, Technology, Engineering and Mathematics (STEM) education outside of formal school environments via community/youth after-school programming is getting increasing attention in the United States (Bell et al., 2009) Albert (2003), Stephenson et al (2009), Lock (1998) and Lock (2006, 2010) recommend using sports data, and Neumann et al (2010), Lee (2013), Davies et al (2012), SportsAtSchool (2013), and Franklin et al (2007) advocate that data be collected by students as a motivator to engage them in statistics. In contrast with those authors’ proposals, in STATS4Soccer, the students actively are—in the literal sense of sports—partaking in data collection. So we asked ourselves: Could an innovative program like this help motivate school students in the United States to learn about statistics? Will working with real data like that collected by the STATS4Soccer activities create an
interest for both male and female students to become statistically literate? Can it be primarily about statistics, not only soccer?

In the rest of this paper we describe STATS4Soccer activities and exploratory analyses done in previous learning events that are appropriate for elementary-school students and beyond. Following that, we describe what we learned about the incidence of soccer interest in the United States and the potential geography within which using these activities might have most success. Finally, we present suggestions for additional classroom exercises with technology that are appropriate for middle- and high-school students.

WHAT IS STATS4Soccer?
The activities of STATS4Soccer have been captured in a YouTube video (STATS4Soccer, 2013). Before the start of the program, teachers involved are briefed about the layout and how it works. The Mathematics teacher and the Sports teacher will assist in manning the stations on a soccer field.

The field layout consists of seven different skills testing stations. The stations are designed with seven different geometrical shapes in mind, namely: trapezoid, octagon, equilateral triangle, pentagon, hexagon, isosceles triangle, and right angled triangle.

Each circuit has a purpose. For example, the isosceles sketch helps students gain foot speed agility. The different exercises test a player’s coordination, control, conditioning, dribbling and speed. “This is important in the development of soccer skills,” says Jonny Masegela. While a student runs the circuit with the ball, another student times the speed at which the player completes the activity and another enters the data in a spreadsheet. Table 1 describes the objective and distance covered by the seven circuits.

The facilitators lay out the different shapes with color ribbon attached to the ground and allocate the soccer training tools (speed hurdles, agility cones, agility poles, conditioning ladder) before the students assemble at the field. There are 49 students involved. This number consist of the following students.

- **Players**: 14 non-soccer players and 14 soccer players.

  There must be four players for each station at the beginning of the exercise. Seven players, one from each station, will perform the exercises simultaneously moving to the next station every time they complete the exercise. They will also be numbered—1 to 28—from the first to the last player and will follow that sequence through all the stations.

- **Timekeepers, data capturers and observers**: seven groups.

  Each of the seven stations is allocated three students charged with monitoring the time spent by each player to complete the exercise for that particular station (the timekeeper) and record the time of each player as recorded by the timekeeper (the data collectors).

### Table 1. Summary of circuits.

<table>
<thead>
<tr>
<th>Activity</th>
<th>meters</th>
<th>Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pentagon</td>
<td>141</td>
<td>Ball control and leg conditioning</td>
</tr>
<tr>
<td>Hexagon</td>
<td>130.8</td>
<td>Ball control</td>
</tr>
<tr>
<td>Trapezoid</td>
<td>188</td>
<td>Running (with ball) and dribbling</td>
</tr>
<tr>
<td>Octagon</td>
<td>96</td>
<td>Dribbling</td>
</tr>
<tr>
<td>Equilateral Triangle</td>
<td>92</td>
<td>Coordination and footwork</td>
</tr>
<tr>
<td>Isosceles Triangle</td>
<td>74</td>
<td>Agility and foot speed</td>
</tr>
<tr>
<td>Right Triangle</td>
<td>108</td>
<td>Body coordination</td>
</tr>
</tbody>
</table>

Figure 1. Two examples of circuits used in STATS4Soccer

Pentagon. Ball control skill and conditioning

Octagon. Tests dribbling skills.
The data is compiled in a combined Excel spreadsheet and students analyze it in the classroom.

For example, consider student number 2, male, age 14, who starts at the pentagon station. This student runs inside the perimeter of the pentagon circuit while kicking the ball, following the path indicated by the black dotted lines in Figure 1 (top panel). When the student arrives at each of the speed hurdles (in blue), he must leave the ball aside and jump the hurdles, as shown in Figure 1, upper panel. When finished, the data collector of that station enters student 2’s time, his grade, gender, age, and the number of mistakes made. Student 2 then moves on to the octagon station and student 3 starts the pentagon station. At the octagon station, student 2 must run along the octagon perimeter while kicking the ball. At alternate sides of the octagon, he must zigzag the ball through agility cones, as indicated in Figure 1 (lower panel). When done, the time taken and number of mistakes is written down by the data collector, student 2 moves to the next circuit and student 3 starts the octagon station. Figure 2 (right) shows a student capturing data from the octagon activity on the octagon data-entry sheet in the field (upper panel) and students entering the data into Excel and analyzing it (lower panel).

Although each activity’s data is handwritten in a spreadsheet specific to that activity, as indicated in Figure 2’s left panel, the final Excel spreadsheet combines data for all activities and students and may look like the one shown in Figure 3 (below), which contains data collected in 2009 during the ISIBALO/International Statistical Literacy Project (ISLP) International Statistical Literacy Competition.
At the 2009 ISI biennial meeting in South Africa, students from all over the world participated in the first ISLP competition (ISIBALO/ISLP, 2009) and a week of statistics activities tailored to students. The students also experienced STATS4Soccer (ISIBANE, 2009) and produce the data presented in Figure 3 for the statisticians and the players. After the field activities, educators from the United States and South Africa entered the data into Excel and used the data shown above in a classroom equipped with a computer for each student to expose the students to modern statistical graphing techniques and new statistical methods.

The data set in Figure 3 presents a variation of the usual scheme of players and number of participants described above, showing exceptions to the allocation of participants are possible and may be dictated by the circumstances. The reader interested in obtaining the data set may contact the main author.

The data set shown above consists of continuous variables (age, grade, time for each activity [in seconds] and discrete variables [ISI09, what role students played, the gender and grade]). With this data we are interested in the total time taken by the participants (speed) for each activity.

Before students were exposed to modern ggplot graphics and Excel, there was a heated classroom discussion about what was interesting to examine and graphs of the data were drawn on a paper pad attached to an easel, as indicated in Figure 4 (above). That preliminary analysis helped trigger statistical discussions and make the transition to the computer graphs and summary statistics smoother for most students.

At ISI 2009, Dianne Cook discussed with the students several potential research questions that could be answered with the data collected for soccer players and statisticians, such as:

- Who is the fastest?
- Who is the most consistent?
- Which activities took longer?
- Were some activities easy for everyone or were there any that had a lot of variation from athlete to athlete?
- Are older kids faster?
- Does grade make a difference?
- How do girls compare with boys?

The graph in Figure 4, left panel, showing average speed by player (the sum of all times taken in all activities divided by 7), was used to answer the first question without using the computers. This brought the discussion as to what was more appropriate to use: the mean speed of a student or the median speed? It turned out that the ranking of the player by speed varied considerably depending on whether the mean or the median was used. Students learned which player had the highest mean speed, but they realized that this student was not the most consistent (in other words, the student did not have the lowest standard deviation). In fact, a different student had a smaller standard deviation of speed than the fastest student. The graph on the right panel of Figure 4 helped illustrate the relation between age and speed, with different colors for the dots representing players and statisticians.
With graphs like those shown in Figure 5 (above) and corresponding summary statistics in Table 2, Dianne Cook showed students how to compare in a snapshot the speeds in all activities of two groups of students. The graphs showed that the students who were soccer players were more consistent (had less standard deviation, narrower color band) and faster (less mean speed overall, dots more to the left of the graph) than the students who were participating in the statistical literacy competition. The graphs and summary statistics also show that the octagon is the quickest and most consistently completed activity in the soccer players group, while the equilateral had those characteristics in the statisticians’ group.

In addition to the previous analyses, Dianne Cook showed students how they could look at the relation between age and speed and how that relation changed across several well-defined subgroups. This analysis entails looking at two factors: group (with two levels, statisticians or soccer players) and gender (also with two levels, male or female). Figure 6 (below) illustrates how she used ggplot to do these analyses and the conclusions extracted from the data.

The lines shown in Figure 6 are the simple regression lines. The dependent variable—speed—is computed by averaging the time taken by a student in all activities. The independent variable is age. For example, the regression line for the male soccer players group is:

$$\text{Expected Speed of Male Players} = 35.735 - 0.0338 \times \text{age}.$$ 

Thus, a soccer player one year older is expected to have 0.0338 seconds disadvantage in speed. Not only is this practically insignificant, but it also is statistically insignificant (p-value = 0.866). But we knew this because in Figure 6 the regression line has slope practically 0 and the majority of the observations are outside the 95% confidence band for the predicted values of speed at each age level. The R-squared, which tells us what percentage of the variability in speed is explained by age, is 0.0017, suggesting that age has no saying in the speed of the soccer player group.

On the other hand, Figure 6 indicates that there is a negative relation between age and speed of statisticians. In the plot for this

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**Table 2. Summary statistics corresponding to Figure 6: average time ± standard deviation by circuit and role.**

<table>
<thead>
<tr>
<th>Circuit</th>
<th>Soccer players (seconds)</th>
<th>Statisticians (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Octagon</td>
<td>23.36 ± 1.96</td>
<td>31.39 ± 6.35</td>
</tr>
<tr>
<td>Equilateral Triangle</td>
<td>24.54 ± 2.03</td>
<td>24.38 ± 2.98</td>
</tr>
<tr>
<td>Hexagon</td>
<td>30.58 ± 2.79</td>
<td>41.32 ± 8.33</td>
</tr>
<tr>
<td>Right Triangle</td>
<td>31.40 ± 1.86</td>
<td>34.03 ± 4.27</td>
</tr>
<tr>
<td>Isosceles Triangle</td>
<td>45.16 ± 4.26</td>
<td>41.74 ± 13.38</td>
</tr>
<tr>
<td>Pentagon</td>
<td>46.04 ± 3.12</td>
<td>52.58 ± 9.50</td>
</tr>
<tr>
<td>Trapezoid</td>
<td>47.60 ± 4.11</td>
<td>64.05 ± 16.85</td>
</tr>
</tbody>
</table>

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**Figure 5. Comparing average time ± standard deviation of two groups of students using ggplot (time in seconds).**

**Soccer Players**
Very consistent times among the athletes.

**Statisticians**
A lot more variation in times, and slower on average. Interestingly, the equilateral triangle was done almost as well as the soccer players.

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**Figure 6. Relation between age and speed for subgroups of students.**

**Soccer Players**
No association between performance and age. Not enough data to say anything about age.

**Statisticians**
A big difference between girls and boys. The boys show an improvement in performance with age. The girls show a decline in performance with age. But the relationships are fairly weak and with these few data, we might not be very confident about these relationships.
group, more statisticians fall into the 95% prediction confidence bands than do soccer players, leading us to believe that we should see a significant relation between age and speed in this group.

The regression line has slope -1.2213, suggesting that getting one year older makes the statistician 1.22 seconds faster on average. That is not only practically significant, but also statistically significant (p-value = 0.0271). R-squared is 37.11%, telling us that age plays a bigger role among the group that is not trained to play soccer than among soccer players, albeit age is not the only determinant of their speed:

Expected Speed of Male Statisticians = 56.78 – 1.2213 age.

When looking at the female statisticians’ regression line, we find that females are expected to become 3.68 seconds slower when they become one year older, and significantly so (p-value = 0.02).

Expected Speed of Female Statisticians = –7.58 + 3.68 age.

The SOCCER4Stats activities were good motivators for students to understand the difference between groups.

After a tutorial by Dianne Cook, Naomi Robbins showed students how to use their computers and Excel to do their own analyses of the data. Figure 7 (above) illustrates one of the analyses done by the students. Facilitators and teachers walked around the room discussing with students how to improve their analyses. South African students then presented the final analysis in front of hundreds of delegates attending ISI 2009.

The STATS4Soccer activities not only engage students in data collection on their own performance, the activities also challenge them to think about potential variables that might help explain soccer outcomes that will in turn help soccer scientists answer important research questions. Thus, the data helps students be involved in scientific enquiries conveying to them the notion that statistics is done to answer questions across the curriculum.

**WHY SOCCER?**

Statistics of soccer are not as well known as baseball or other sports data. Kaplan (2013) explains how people in soccer have historically paid little attention to statistics because the sport does
not lend itself to statistical scrutiny. It is difficult to quantify a player’s performance, as it involves the interaction with the passes and shots of all the other players on his/her team. Blalik (2008) claims that the very nature of soccer seems to defy statistical analysis and soccer scientists have not yet identified new statistics that correlate best with winning matches. “Part of the problem is that hundreds of events—and 20 players running about 10 kilometers each—lead to just a handful of goals, if any, in each match. In many cases, it’s hard to ascribe credit or blame for those goals to individual actions.” Thus, soccer presents itself as a scientific question that students need to solve not only with statistics but with knowledge they have from other areas of the curriculum, and their own experience in the field.

Not all students in our schools like soccer, thus what could be the incidence of implementing STATS4Soccer in schools in the United States? How widespread is the interest in soccer in the United States? According to the U.S. Census Bureau, in 2009 approximately 13.5 million people in the United States participated in soccer, with 8 million considered youth (ages 7 to 17). In addition to soccer being a sport played among the youth, it has something most major sports in the United States lack—a relatively high number of female participants (U.S. Census Bureau, 2012 a,b,c). Moreover, as conveyed in the map of U.S. High School Soccer Participation in Figure 8 (right), soccer participation at the high-school level is spread throughout the United States (NFSHSA, 2012). The data behind this map is obtained with a voluntary survey, thus the figures (participation in soccer activities per thousand participants in sports by state) must be interpreted with caution.

Now, why might participation in soccer—along with other sports—be largely attributed to youth (Handley et al, 1994)? In order to combat obesity in today’s youth, First Lady Michelle Obama created the program “Let’s Move” to help the youth of today follow a “path to a healthy future” (Let’s Move). Similar to this program, various youth sport organizations focused on promoting a healthier and more active lifestyle for today’s youth have been created.

The data indicates that interest in soccer is widespread in youth and growing. Thus, if not of interest to all students, learning by the use of soccer activities and data collected from students with STATS4Soccer can potentially create an interest in statistical language for girls and boys alike. It also could potentially bring the extracurricular activities of students and the school’s coach or activity advisor (for example, the soccer coach) into the classroom, and create interest in other parts of the curriculum.

**POTENTIAL ADDITIONAL ACTIVITIES FOR THE CLASSROOM**

Because the data collected in South Africa through the STATS4Soccer activities is observational, and was not planned to be collected in an experimental setting, the analysis done does not allow extracting causal conclusions. However, the data can be used to conduct different analyses. There is no room in one article to summarize all possible

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*Figure 8. U.S. high school soccer participation per thousand participants in sport by state.*
analyses, but to illustrate a few in Figure 9 (above), we indicate three other possible analyses also conducted at ISI 2009.

As indicated in Figure 9 (left panel), students could look at the distribution of speed by activity, having a histogram of the times for each activity to see if the times are consistent across players for that activity, whether there are some outlying values, and to compare the time across activities. Students also could look at the relation between speed and age by activity (middle panel). Jitter in the horizontal axis means that a small distance between overlapping dots was imposed to be able to see that there was more than one observation in that coordinate. Students also showed interest in the individual behavior across all activities (right panel). Thus, a plot of the time in all activities for each student can show similarities among students.

Other potential analyses that could be done with the STATS4Soccer activities may be dictated by the topic being discussed in the classroom and the interests of the students. For example, the regression analysis could be expanded to include more variables. Similarly, additional variables could be collected, and an experiment could be planned, such as allocating students randomly to do the activities with a prize at the end for the fastest, and other students being allocated to the activities but without a prize.

Teachers also could compare the test scores at the end of the quarter of those students who took part in the STATS4Soccer program, which obviously should be volunteer, and those who did not, to assess the incidence of this activity in student learning outcomes.

CONCLUSIONS

The main point we have tried to make is that all the areas covered by the Common Core standards can be studied with the SOCCER4Stat activities while keeping up with the GAISE and NCTM standards and the ideal of enhancing the process of statistical inquiry across the curriculum from a very early age and to make young students statistically literate. With STATS4Soccer, students see why it is necessary to collect the data and why what they learn in school helps them understand why the data analysis gives the results obtained.

STATS4Soccer, by bringing STEM education outside the classroom, not only enhances statistical skills of the students but it also benefits them through engaging with peers their own age. This enables them to learn while having fun and learning about each other.

REFERENCES


Lee, C. STATACT. http://stat.cmich.edu/statact/


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Using Concept-Mapping in Introductory Statistics
By Alisa Izumi, EdD, Western Governors University

Tell me, I forget.
Show me, I remember.
Involve me, I understand.

Chinese proverb

This proverb illustrates the importance of involving learners in learning activities, generating connections between what they already know and what they are being asked to learn, and constructing meaning from their experiences. As students become active participants in the knowledge-construction process, the focus of learning shifts from covering the curriculum to working with ideas (Scardamalia, 2002). Moreover, using manipulative tools “to think with” facilitates working with ideas and learning from that process.

For example, technology tools provide “the means through which individuals engage and manipulate both resources and their own ideas” (Hannafin, Land, & Oliver, 1999, p. 128). Some technology tools can extend memory and make thinking visible. Good examples include brainstorming and concept-mapping software such as Inspiration®. Other programs or simple paper-and-pencil models help to represent knowledge and facilitate communication. These tools enable learners to experiment with modeling complex ideas.

How do we model complex ideas?
Understanding a new concept involves a process of integrating new information with current models of the world. The learner creates a new and different world through cognitive construction—the act of creating and refining usable models. With this perspective on teaching and learning, the instructor plays a key role by providing experiences and helping students build complex mental models compatible with textbook explanations and their applications to daily life. These activities may take the form of a dialogue or conversation through which the instructor and the students suggest, construct and exchange ideas to produce mental models in basic statistics. Unfortunately, many students have difficulty building and communicating mental models of basic statistics. It may be that some of the difficulty in developing mental models is due to the inability of students to integrate text information with everyday usage. While statisticians understand complex statistical models, the knowledge of beginning students is often rote and not readily transferable.

One method for attaching meaning to statistics terms is through a concept map—a graphic representation of the knowledge of students. Having students create concept maps can provide insights into how students organize and represent knowledge. This can be a useful strategy for assessing both the knowledge possessed by students entering a program or course and their developing knowledge of course material. Concept maps include concepts, usually enclosed in circles or boxes, and relationships between concepts, indicated by a connecting line. Words on the line are linking words and specify the relationship between concepts. A science example can be seen at http://www.cmu.edu/teaching/resources/Teaching/CourseDesign/Assessment-Grading/OngoingAssessments/conceptmap.pdf

To produce a map, teachers ask students to create visual displays using Inspiration 9® (available at http://www.inspiration.com/), which is a computerized tool that inspires students to develop and organize their ideas. Students easily create and update concept maps to help focus their ideas. Visual-learning techniques, such as concept maps, help students process and retain new information by making connections between new and existing knowledge.

Through the process of creating and sharing concept maps, students provide very rich descriptions of current understanding. As a result, teachers can better engage students in conversation and progress their learning through a co-construction process. In this process, after students draw their initial concept maps, the instructor provides the students guidance crucial to the building of improved models. The guidance takes the form of a dialogue in which the instructor probes the ideas of students on an individual basis. Then, instead of presenting the students with the statistical model, as likely in traditional teaching, the instructor confronts the students with successive counterarguments and constraints that stimulate the students to review and modify their evolving ideas.

Are student relational concept maps produced on a computer a viable method to engage students and if so, what patterns might we see?

With concept-mapping programs like Inspiration 9®, teachers can gather student data on statistical understanding. When computer programs are not readily available, other options include sticky “Post-it” notes arranged on a poster board or a table top. However, the advantage of a software program is the ease of saving and filing student concept maps for later viewing and integrative discussions. At each class meeting, I ask students to reflect and share their understanding of statistical tasks from producing a student survey, then collecting and analyzing the data. We base class discussions on what students need in order to complete these tasks. When students finished tasks, each student
pair discusses and draws revised concept maps. The resulting concept maps produced at the end of the program evolved from some simple conceptions to more interlinked and complex ones.

Overview of the Study
As a SummerMath instructor at Mount Holyoke College, I evaluated student gains in order to direct ongoing instruction in central measures—a fundamental element of statistical reasoning. I was interested in how students developed models and how they employed these models to explain data generated from student surveys on teenage behaviors.

Participants
The two-week workshop class, consisting of a diverse mix of young women ages 14 to 17, met daily for 1.5 hours to provide a forum for discussing data collection, central measures and variability. The class of 14 female students had mixed ethnicities, grade levels, and abilities. The class size self-divided into higher and lower level students. Both groups contained 50% black-white racial mixtures. While there was some informality, the class content and general outline was directed by the instructor. Students were encouraged to talk, discuss and argue in both their small group discussions and in the full class interaction. Students were verbally encouraged to come with their prior conceptions, pieces of knowledge, beliefs and misconceptions.

During the workshop, students began and ended each meeting with conceptual maps linking their self-generated statistical topics. They investigated and discussed common themes of data collection, organization, presentation and analysis by creating and collecting student surveys. Using this hands-on approach, many of the real-life problems of randomness, sample selection and survey bias appeared. We discussed these issues along with how measurement choices may highlight certain opinions. We also discussed causal relationships through strong correlation, plausibility, replication, direct variation and computerized cross-tab analyses.

Data Collection
In a Mount Holyoke College computer laboratory, using an Inspiration 6® file, students provided a spatial display of their knowledge structures including the concepts, connections among concepts and the relationships underlying the connections. Each student pair moved the concepts in the boxes around the screen to depict how they were associated. The pair then connected the concepts by lines to show the linkages the student perceived. Finally, they entered words on the lines to describe the nature of the linkages or relationships.

Figures 1 through 4 (shown below), illustrate the change in concept maps from beginning to end of the workshop, one from a “low-ability” student and another from a “high-ability” student.

---

**Figure 1**

Low Ability (pre)

- majority
- survey
- percentage
- organizing
- data
- numbers
- name or category
Figure 2
Low Ability (post)

majority ⇒ survey ⇒ percentage ⇒ name or category

opinion race age ⇒ data

organizing ⇒ numbers ⇒ mean median mode

results

Figure 3
High ability (pre)

collection

numbers ⇒ records

opinions ⇒ percents

facts

differences

pie charts

ideas

race

PPL

data
Through the process of producing these maps, students develop self-regulatory processes that improve cognitive skills. These cognitive skills were manifested by the number of concept linkages, by the attention to details, and by sensitivity to informational feedback with the teacher. Table 1 summarizes the important features of the student concept maps from the preceding figures.

These examples are typical of students from the low-ability group. Pre-workshop concept maps were linear and showed limited knowledge by the few concepts represented. The post-workshop concept maps for these students illustrated more concepts arranged with a beginning hierarchical understanding. For students in the high-ability group, pre-workshop concept maps illustrated more concepts with correct terminology and spaced appropriately in the regions. However, usually the concepts were not linked. By post-workshop, these maps contained more concepts and an increased understanding of advance terms, like inference, and distinguishing more advanced statistical topics. For these students, the arrangements show a more advanced hierarchical understanding of the ideas.
As illustrated by these examples, having students create and revise concept maps may have a positive influence on their understanding of basic statistical ideas. That is—per the Chinese proverb cited earlier—by involving students in the creation of a concept maps a deeper and fuller understanding of the ideas occurs.

REFERENCES


ADDITIONAL RESOURCES


**ANNOUNCEMENTS**

**Give It Your Best Shot!**

*Introducing the Statistics2013 Photo Contest*

Get your camera and your photographic eye ready for the International Year of Statistics (Statistics2013) photo contest! If you are selected a winner, you will receive a cash prize for your best photograph.

This competition—called the Statistics2013 Photo Contest—is for secondary-school students (grades/years 7-12) around the world. The deadline for entries is December 1. Winners will be announced on or about December 16. Click here for contest criteria.

Contest organizers are seeking high-quality photos that most effectively illustrate the impact of statistics. Your submitted image must be an original photo taken by you and tell a story about how statistics advances the wellbeing of people in your country or our global society.

The Statistics2013 Photo Contest is organized by continents (except Antarctica). So, be sure to submit your contest entry in the appropriate continent.

In addition to a photo, you must submit a title for your photo as well as a maximum 50-word descriptive of you photo. These descriptive items, along with your photo, will be used by the international panel of judges to select the top three winning entries as well as up to three honorable mention winners on each continent.

Speaking of winners, the judges will select a top overall photo from among all the photos submitted for the Statistics2013 Photo Contest. This best photo will be awarded a prize of $800.

Additionally, the top three place-winners on each continent will be awarded cash prizes (U.S. dollars) as follows:

<table>
<thead>
<tr>
<th>Place</th>
<th>Prize</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>$350</td>
</tr>
<tr>
<td>Second</td>
<td>$200</td>
</tr>
<tr>
<td>Third</td>
<td>$150</td>
</tr>
</tbody>
</table>

To enter your photo, go to the Statistics2013 website and click on the Statistics2013 Photo Contest logo. Then complete the webform and attach your photo. Remember to enter a title for and a 50-word descriptive of your photo before submitting the webform. For more information, email Jeff Myers at Jeffrey@amstat.org.

**STEW in Search of New Editor and Additional Associate Editors**

The American Statistical Association (ASA)/National Council of Teachers of Mathematics (NCTM) Joint Committee on Curriculum in Probability and Statistics (K-12) is seeking an editor and additional associate editors for SStatistics Education Web (STEW). STEW (www.amstat.org/education/STEW) is an online resource for peer-reviewed lesson plans for K–12 teachers. Send nominations-including the nominee’s name, email address, and a brief description of his or her qualifications-to rebecca@amstat.org. The search committee will contact nominees to see if they are interested in applying. The deadline for nominations is April 1. Interested nominees must send their applications by May 1. Both nominations and applications should be sent to rebecca@amstat.org.

**World Stats Quiz**

(Flyer on page 20)

The Royal Statistical Society Centre for Statistical Education and Plymouth University have launched Stats2013AtSchool - a World Statistics Quiz for learners across the world. They are seeking international champions to promote this fun activity to teachers and school-aged learners. They also hope to build an international community of contributors who will help develop Stats2013AtSchool by contributing additional questions and resources. If you are interested in assisting with either activity, please register on Stats2013AtSchool or contact admin@CensusAtSchool.org.uk.

**Competition Winners for Best STatistics Education Web (STEW)/Census at School Lesson**

The ASA is pleased to announce the winners of the competition for the best new STEW lessons (www.amstat.org/education/stew) using U.S. Census at School data (www.amstat.org/censusatschool). The competition is sponsored by the ASA/NCTM Joint Committee on Curriculum in Statistics and Probability. We congratulate the following winners for their efforts to enhance statistics education in the schools:

**Grand Prize**

*Did I Trap the Median?*

Sarah Parks, Mathew Steinwachs and Rafael Diaz
California State University Sacramento, University of California Davis, CSU Sacramento

**Silver Medal Prize**

*Text Messaging is Time Consuming! What Gives?*

Jeanie Gibson, Mary McNelis and Anna Bargagliotti (for Project-SET)
Hutchison School, St. Agnes Academy and Loyola Marymount University

**Bronze Medal Prize**

*Types of Average and Sampling: “Household Words” to Dwell On*

Lawrence Mark Lesser
The University of Texas at El Paso

The second- and third-place winning lessons and other free K-12 statistics lesson plans are accessible at the STEW website (www.amstat.org/education/stew) and the resources page of the U.S. Census at School website (http://www.amstat.org/censusatschool/resources.cfm). The first-place lesson will be posted soon.
2013 International Year of Statistics and Free
International Statistics Education Resources

As part of the International Year of Statistics (Statistics2013), teachers everywhere can access in a wealth of statistics instruction tools and resources from around the world at www.statistics2013.org.

Also, you are invited to sign up your school to participate in Statistics2013. Hundreds of schools around the country and the world already are participating. Participation does not create an obligation for your school. To sign up, visit http://www.statistics2013.org/iyos/join.cfm.

Free Statistics Education Webinars

The ASA offers free webinars on K–12 statistics education topics at www.amstat.org/education/webinars.

This series was developed as part of the follow-up activities for the Meeting Within a Meeting Statistics Workshop. The Consortium for the Advancement of Undergraduate Statistics Education also offers free webinars on undergraduate statistics education topics at www.causeweb.org.

ASA 2013 Poster and Project Winners Announced

The ASA is pleased to announce the winners of the 2013 poster and project competitions. The competitions offer opportunities for students to formulate questions and collect, analyze and draw conclusions from data. Winners were recognized with plaques, cash prizes, certificates and calculators, and their names were published in Amstat News. To view the winning posters and projects or for more information, visit www.amstat.org/education/posterprojects.

2014 Poster and Project Competitions

Introduce your K–12 students to statistics through the annual poster and project competitions directed by the ASA/NCTM Joint Committee on the Curriculum in Statistics and Probability. The competitions offer opportunities for students to formulate questions and collect, analyze and draw conclusions from data. Winners will be recognized with plaques, cash prizes, certificates and calculators, and their names will be published in Amstat News. Posters (grades K-12) are due every year on April 1. Projects (grades 7-12) are now due on June 1. For more information, visit www.amstat.org/education/posterprojects.

PROJECT-SET

Project-SET is an NSF-funded project to develop curricular materials that enhance the ability of high-school teachers to foster students’ statistical learning regarding sampling variability and regression. All materials are geared toward helping high-school teachers implement the Common Core State Standards for statistics and are closely aligned with the learning goals outlined in the Guidelines for Assessment and Instruction in Statistics Education: A Pre-K–12 Curriculum Framework (GAISE) report.

Provide Feedback for Updating Curriculum Guidelines for Undergraduate Statistics Programs

The ASA is updating the curriculum guidelines (www.amstat.org/education/curriculumguidelines.cfm) for undergraduate statistics programs.

The working group, which consists of representatives from academia, industry and government, welcomes your input online at http://www.amstat.org/education/curriculumguidelines.cfm.

Albert Einstein Distinguished Educator Fellowship Program now accepting applications for the 2014-15 fellowship year

Applications due December 4

The Albert Einstein Distinguished Educator Fellowship Program is accepting applications for the 2014-15 fellowship year. The Einstein Fellowship seeks experienced and distinguished K-12 educators in fields of science, technology, engineering and mathematics (STEM) to serve an 11-month fellowship appointment in a federal government agency or U.S. congressional office. Applications are due December 4 and must be submitted through the online application system.

Information about this fellowship program, including eligibility requirements, program benefits, application requirements and access to the online application system can be found at http://science.energy.gov/wdts/einstein/.

Seeking Feedback for Careers in Biopharmaceutical Statistics Website

The Biopharmaceutical Section of the ASA (http://www.biostatpharma.com/) is seeking feedback from teachers and students about a new website/video to introduce students to careers in biopharmaceutical statistics.

Industry and government members alike realize there are more opportunities for statisticians than there are qualified candidates. The biopharm section started working on its website several years ago with the goal to design a site that would interest students in studying statistics. The site is still a work in progress and we would very much appreciate comments from you about how we can improve it. We are interested in whether you think that the site would interest your students. What items are missing? Do you think the site would benefit guidance counselors?

Please send your comments to Anna Nevius at Anna.Nevius@fda.hhs.gov or 301-258-0565 (home), 240-994-1748 (cell) or 240-276-8170 (work).

240-276-8170 (work).

Please send your comments to Anna Nevius at Anna.Nevius@fda.hhs.gov or 301-258-0565 (home), 240-994-1748 (cell) or 240-276-8170 (work).
Celebrate the international year of statistics by getting your school to take part in this fun initiative – probably the first world statistics quiz ever!

This Plymouth University-led innovative project, in partnership with the Royal Statistical Society Centre for Statistical Education (RSSCSE), aims to improve statistical literacy around the globe.

TAKE PART:
- Use – your computer or hand-held device
- Visit – www.stats2013atschool.org.uk/quiz/
- Have a go – 12 randomly selected questions based around statistics
- Achieve – your own personalised certificate

Attain 100% and get your school and country entered onto the league table on the www.stats2013.org website

Open to all school-aged learners – everywhere.
Lesson Plans Available on Statistics Education Web for K–12 Teachers
Statistics Education Web (STEW) is an online resource for peer-reviewed lesson plans for K–12 teachers. The lesson plans identify both the statistical concepts being developed and the age range appropriate for their use. The website resource is organized around the four elements in the GAISE framework: formulate a statistical question, design and implement a plan to collect data, analyze the data by measures and graphs, and interpret the data in the context of the original question. Teachers can navigate the site by grade level and statistical topic.

Lesson Plans Wanted for Statistics Education Web
The editor of STEW, Mary Richardson of Grand Valley State University, is accepting submissions of lesson plans for an online bank of peer-reviewed lesson plans for K–12 teachers of mathematics and science. Lesson plans will showcase the use of statistical methods and ideas in science and mathematics based on the framework and levels in the Guidelines for Assessment and Instruction in Statistics Education (GAISE). Consider submitting several of your favorite lesson plans according to the STEW template to steweditor@amstat.org.

For more information, visit http://www.amstat.org/education/stew.
Exploring Geometric Probabilities with Buffon’s Coin Problem
Written by:
Kady Schneiter
Utah State University
Kady.schneiter@usu.edu

Overview of Lesson
Investigate Buffon’s coin problem using physical or virtual manipulatives (or both) to make connections between geometry and probability, to identify empirical and theoretical probabilities and to discuss the relationship between them.

GAISE Components
This investigation follows the four components of statistical problem solving put forth in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report*. The four components are: formulate a question, design and implement a plan to collect data, analyze the data by measures and graphs, and interpret the results in the context of the original question. This is a GAISE Level B activity.

Common Core State Standards for Mathematical Practice
2. Reason abstractly and quantitatively.
5. Use appropriate tools strategically.

Common Core State Standards Grade Level Content (Grade 7)
7. SP. 1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

NCTM Principles and Standards for School Mathematics
Data Analysis and Probability Standards for Grades 6-8
Develop and evaluate inferences and predictions that are based on data:
• use conjectures to formulate new questions and plan new studies to answer them.
Understand and apply basic concepts of probability:
• use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations.

Prerequisites
Students will know the area formulas for squares and rectangles.
Students will have a basic understanding of probability.
Students will be familiar with the ideas of proportion and simulation.
Students will understand the terms “event,” “sample space,” and “estimate.”
Learning Targets
Students will define ‘geometric probability’, ‘empirical probability’, ‘theoretical probability’. Students will use simulation to identify an empirical solution to Buffon’s coin problem. Students will use area formulas to identify a theoretical solution to Buffon’s coin problem. Students will observe that as the number of trials increases, the empirical probability tends to approach the theoretical probability.

Time Required
Approximately 50 minutes.

Materials Required
• Coins and square grid (the length of the diameter of the coin should be less than the length of a square on the grid – possibilities are plastic lids on floor tiles, or pennies on graph paper). A blank grid for pennies is provided on page 9.
• Applet at www.math.usu.edu/~schneit/CTIS/BC.
• Pencil and paper for record keeping and note taking.
• Calculator.

Instructional Lesson Plan

This lesson plan involves two parts: an empirical investigation and a theoretical one. Each of these investigations follows the four GAISE components: (1) Formulate a Question, (2) Collect Data, (3) Analyze the Data, (4) Interpret the Data – answer the question.

Preparation: Divide students into small groups (2-4). Prepare a ‘coin’ and a grid of square tiles for each group (the length of the diameter of the coin should be less than the length of a side of a tile in the grid) but do not yet distribute these. Assign one member of each group to be the recorder.

Background: Georges-Louis Leclerc, Comte de Buffon (1707 – 1788) was a French mathematician and naturalist. His ‘coin problem’ is an early exercise in geometric probability, a field in which probabilities are concerned with proportions of areas (lengths or volumes) of geometric objects under specified conditions.

Examples of questions that deal with geometric probabilities are:
• What is the probability of hitting the bull’s eye when a dart is thrown randomly at a target?
• What is the probability that a six-color spinner lands on red?

Geometric probabilities can be estimated using empirical (experimental) methods or identified exactly (theoretical probability) using analytical methods.

Definition: An empirical probability is the proportion of times an event of interest occurs in a set number of repetitions of an experiment.
Throw 50 darts at the target.

5 darts hit the bull’s eye.

The empirical probability of hitting the bull’s eye is \( \frac{5}{50} = \frac{1}{10} \).

Spin the spinner 100 times.

Spinner lands on red 8 times.

Empirical probability = \( \frac{8}{100} = \frac{2}{25} \).

Definition: A \textit{theoretical probability} is the proportion of times an event of interest would be expected to occur in an infinite number of repetitions of an experiment. For a geometric probability, this is the ratio of the area of interest (e.g. bull’s eye) to the total area (e.g. target).

Area of target = \( \pi \times 6^2 \)

Area of bull’s eye = \( \pi \times 2^2 \)

Theoretical probability of hitting bull’s eye = \( \frac{\pi \times 2^2}{\pi \times 6^2} = \frac{1}{9} \)

Area of spinner = \( \pi \times 2^2 \)

Area of red section = \( \frac{1}{6} \pi \times 2^2 \)

Theoretical probability of landing on red = \( \frac{\frac{1}{6} \pi \times 2^2}{\pi \times 2^2} = \frac{1}{6} \)
Investigation (Empirical)

The GAISE Statistical Problem-Solving Procedure

I. Formulate Question(s)
Buffon’s coin problem: What is the probability that a coin, tossed randomly at a grid, will land entirely within a tile rather than across the tile boundaries? (Again, for the purposes of this activity, assume that the diameter of the coin is less than the length of a side of the tile.)

II. Design and Implement a Plan to Collect the Data
- Discuss as a class: how would we identify an empirical solution to Buffon’s coin problem? (This corresponds to prompt 1 on the task sheet.) (After students propose tossing coins at a grid, discuss details of the experiment: how many times will they throw the coin? How will the coin be tossed? Does skill matter? Will they count the times that the coin lands on a boundary or the times it lands entirely within a tile? Will each group do this the same way? What difference will it make if they do not? (For purposes of later discussion it will be helpful if everyone considers the event that the coin lands entirely within a tile.) Who will record the outcome of each toss? How will this count translate into an empirical probability?)

- Experiment: Instruct each group to conduct the experiment, as designed by the class. (Task sheet prompt 2.)

III. Analyze the Data
Instruct each group to use the data they gathered to compute an empirical probability of the event they considered.

IV. Interpret the Results
- Discuss as a class:
  o Summarize the probabilities on the board or other place where all students can see. Ask students what they observe about the empirical probabilities computed by the groups. (They are not all the same, most are similar, a few may differ by a lot, if the experiment were repeated different answers would be obtained).
  o Is it possible to get a more stable answer? (Yes, repeat the experiment more times, combine data from different groups).
  o Ask the students what they would expect to see if the coin could be tossed an infinite number of times. Why would they expect to see this? (Task sheet prompt 3.)

Investigation (Theoretical)

I. Formulate Question(s)
Recall Buffon’s coin problem: What is the probability that the coin, tossed randomly at a grid, will land entirely within a tile rather than across the tile boundaries?
II. Design and Implement a Plan to Collect the Data

- Discuss as a class: how would we identify a theoretical solution to Buffon’s coin problem? (Task sheet prompt 4.)
  (Just outline the process here: identify the shape of the region within the tile in which the coin must land to be entirely within the tile, look at the ratio of the area of that shape to the area of a tile. Students will work out the details with their groups in the next segment.)
- Explore: Again working in groups, ask students:
  - To formulate a conjecture about the relationship between theoretical and empirical probabilities. (Task sheet prompt 5.)
  - To identify the shape of the region within the tile in which the coin must land to be entirely within a tile. (This will be challenging for some students. The key is to consider where the center of the coin lands and how close the center can be to the edge of a tile while the coin is not on a boundary. The applet cited above can be used to direct students’ thinking. With the radio button ‘Show centers’ selected, for repeated coin tosses, the applet will mark where the center of the coin lands. The color of the mark differs depending on whether the coin crosses tile boundaries or not, thus the region of interest is clearly visible.)

III. Analyze the Data

Instruct each group to use their observations from the physical experiment, the applet, and the group discussion to compute a theoretical probability that a coin tossed randomly at a grid lands entirely within a single tile. (Task sheet prompt 6.)

IV. Interpret the Results

- Discuss as a class: Summarize students’ solutions on the board. Discuss observations about these solutions. Bring the class to a consensus about the solution. Observe that there is only one solution and it will not vary with further investigation.

Synthesis

- Discuss as a class: What seems to be the relationship between the empirical and the theoretical probabilities? (As the number of trials increases, the empirical probability tends to converge to the theoretical one). (Task sheet prompt 7.)

Assessment

Students will identify the theoretical solution to Buffon’s coin problem for a grid composed of non-square tiles. (The applet facilitates investigation with parallelograms and rectangles.)

Suppose you have a coin with diameter = 2 cm and a grid of rectangles with side lengths 4cm and 6cm as shown in the figure below.

- Describe how you would find the empirical probability that the coin would land completely within a tile when tossed at the grid. (Answer: toss the coin at the grid repeatedly, record the number of times the coin lands entirely within a tile, divide this count by the total number of tosses to obtain the probability).
- Find the theoretical probability that the coin would land completely within a tile when tossed at the grid. (Answer: 1/3).
• Describe the relationship between the theoretical and empirical probabilities. (Answer: As the number of times the experiment is repeated increases, the empirical probability will approach the theoretical probability).

Possible Extensions
Repeat the problem for grids with differently shaped tiles, for different sizes of tiles or coins, or ask students to suggest their own problems in geometric probability. The applet previously cited will allow students to experiment with differently shaped tiles, with tiles and coins of different problems, and with the chessboard problem described below.

The chessboard problem (Plus Magazine, 2004; Turner, 2006) is an interesting extension of Buffon’s coin problem in which the probability of interest is that of landing completely on a corner of a tile – this also can be investigated using a variety of grid shapes.

References


Exploring Geometric Probabilities with Buffon’s Coin Problem Activity Sheet

Definitions:
1. Geometric probability: a probability concerned with proportions of areas (lengths or volumes) of geometric objects under specified conditions.

2. Empirical probability: the proportion of times an event of interest occurs in a set number of repetitions of an experiment.

3. Theoretical probability: the proportion of times an event of interest would be expected to occur in an infinite number of repetitions of an experiment.

Investigation:
Consider the question: What is the probability that a coin, tossed randomly at a grid, will land entirely within a tile rather than across tile boundaries?

1. How would we identify an empirical probability that “a coin, thrown randomly at a grid, will land entirely within a tile rather than across tile boundaries?”

2. Work with your group to compute an empirical probability that the coin lands within a tile. Record your observations below:

3. What would you expect to see if the coin could be tossed an infinite number of times at the grid? Why would you expect to see this?

4. How would we identify a theoretical probability that “a coin, thrown randomly at a grid, will land entirely within a tile rather than across tile boundaries?” How is this question different from question 1?
5. Formulate a conjecture: what is the relationship between the empirical and theoretical probabilities?

6. Work with your group to compute the theoretical probability that the coin lands within a tile. Record your work below:

7. Compare the empirical and theoretical probabilities you found. How do your results relate to the conjecture you proposed?
Example Grid
OBJECTIVE
Write a STEW lesson plan that should:
- Demonstrate statistics concept(s) from the grades K-12 curriculum (K-5, 6-8, and 9-12 grade categories)
- Generate excitement about statistics
- Follow the STEW template, GAISE guidelines (www.amstat.org/education/gaise), and Common Core State Standards (www.corestandards.org)
- We especially encourage entries at the K-5 level

PRIZES
Three Prizes at $300 each

“Though we anticipate awarding a prize for the best lesson at each grade category, the ASA reserves the right to present fewer awards should entries not be of sufficient quality or to present awards to different grade levels.

ELIGIBILITY AND ENTRY DATES
ASA members, K–16 teachers, and students involved in education—undergraduate or graduate—are eligible to enter. Entries are due by March 1 and winners will be announced by May.

TO ENTER
Submit a Word version of the completed STEW lesson to steweditor@amstat.org
All entries will be considered for publication on the STEW website

JUDGING
Entries will be judged on the following:
- Completeness of STEW lesson plan (including compliance with the STEW format)
- Originality of the statistics lesson
- Incorporation of the GAISE guidelines (www.amstat.org/education/gaise), and Common Core State Standards (www.corestandards.org)
- Judging will be done by the STEW Editor and Associate Editors

FURTHER GUIDANCE
For a description of STEW, the STEW lesson plan template, and example STEW lessons, please visit www.amstat.org/education/stew. Additional questions can be directed to Mary Richardson, STEW editor, at steweditor@amstat.org or (616) 331-3364.
Complete a brief online survey (classroom census)

- 13 questions common to international students, plus additional U.S. questions
- 15–20-minute computer session

Analyze your class results

- Use teacher password to gain immediate access to class data.
- Formulate questions of interest that can be answered with Census at School data, collect/select appropriate data, analyze the data—including appropriate graphs and numerical summaries for the corresponding variables of interest—interpret the results, and make appropriate conclusions in context relating to the original questions.

Compare your class census with samples from the United States and other countries

- Download a random sample of Census at School data from United States students.
- Download a random sample of Census at School data from international students (Australia, Canada, New Zealand, South Africa, and the United Kingdom).

International lesson plans are available, along with instructional webinars and other free resources.

www.amstat.org/censusatschool

For more information about how you can get involved, email Rebecca Nichols at rebecca@amstat.org.
Bridging the Gap
Between Common Core State Standards and Teaching Statistics

Twenty data analysis and probability investigations for K–8 classrooms based on the four-step statistical process as defined by the Guidelines for Assessment and Instruction in Statistics Education (GAISE)

www.amstat.org/education/btg
Making Sense of Statistical Studies

Making Sense of Statistical Studies consists of student and teacher modules containing 15 hands-on investigations that provide students with valuable experience in designing and analyzing statistical studies. It is written for an upper middle-school or high-school audience having some background in exploratory data analysis and basic probability.

www.amstat.org/education/mssss