Greetings, teachers! The first article of this issue, “Using Peanut M&M’s in an Introductory Statistics Class to Illustrate Bionomial Probabilities,” discusses a fun activity involving M&M’s candies. The activity can be easily tailored to suit students in grades 9–12.

The second article, “Are Education Graduates Ready to Teach Stats?” examines an important question many of us have asked ourselves. Many times, teachers are asked—or take it upon themselves—to teach statistics with minimal or no formal training. Three college students weigh in on this topic.


I encourage and welcome any articles or ideas you have for publication consideration. Please email me directly at dwebb@bemidjistate.edu.

Best Regards,
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St. Louis University

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Using Peanut M&M’s in an Introductory Statistics Class to Illustrate Binomial Probabilities

James J. Diamond, Jefferson Medical College

As part of an introductory statistics course I teach each fall, I introduce students to the binomial distribution. Like many teachers, I search for ‘experiments’ that will bring forth statistics principles without resorting to derivations or calculus.

This M&M’s experiment came from my speculation about the likelihood, or distribution, of various colors within a package of peanut M&M’s. Having assumed the various colors were distributed randomly, I sent an email to Mars, Inc. to see if this was true. To my surprise, it turns out that in the ‘population’ of peanut M&M’s, 20% of the candies are brown, yellow, red, or blue. Ten percent are green or orange. (At the time, the color distribution was as Mars, Inc. reported. The color distribution has since changed.) This distribution is shown in Table 1.

According to the representative I spoke with, “While we mix the colors as thoroughly as possible, the above ratios may vary somewhat, especially in the smaller bags.” This last comment really generated the concept that if I could have the class sample from the population of M&M’s, we could generate our own sampling distribution and compare it to the appropriate binomial distribution. Furthermore, with bags of different numbers of M&M’s, I could illustrate standard error and goodness of fit.

I started collecting M&M’s results in 2002 and continued until 2007. The students in my classes take graduate courses in public health and all have undergraduate degrees, but they have not taken any statistics courses beyond what they might have taken as undergraduates. My course has no prerequisites and does not include any calculus or derivations. Class size varies between 25 and 30 students per term.

Methods

After discussing the binomial and illustrating its various principles (e.g., mean, variance, and normal approximation), I asked the 2002 class to vote on its favorite color of peanut M&M’s from the list of colors already noted. Green won. I then distributed to each student one fun size package of peanut M&M’s. These packages were considered independent random samples of size 8 from the population. M&M’s are packed by weight, and the fun size is 21 grams.

This introduced a bit of noise, but I determined the modal count of candies for a fun size package was eight. If any student did not get exactly eight peanut M&M’s, he or she received another package, if there were fewer than eight. Over the years, only one student received a package with fewer than eight peanut M&M’s twice. This student then received another package that contained exactly eight candies, kept the original packages, and was asked to isolate them from the study.

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If any student received a fun size package with more than eight peanut candies, he or she was asked to open a small slit in the package and squeeze out one or more—without looking inside—until there were exactly eight left. This was considered a random process, and it was possible because students could feel the candies through the package and count the number inside.

When each student had a fun size package with eight peanut M&M’s, I distributed a small sheet of paper and asked them to open their packages and record the number of green candies in their ‘sample.’

Because the class voted on green as its favorite color, the statistical distribution we used to model this situation was a binomial distribution with parameter values of \( n=8 \) and \( p=0.10 \). The following questions were then posed:

How many green candies might one expect in any given package?

What was the expected number of green candies?

What was the modal number of green candies one might expect when examining lots of packages?

How likely were \( x \) number of green candies, where \( x \) varied between 0 and 8?

The students used the binomial table in their textbook to determine the likelihood of exactly 0, 1, 2, ..., 8 green candies. This experiment was repeated by the other classes, with green as the illustrative color.

### Results

Having collected data from several classes, I had results from 166 independent fun size samples, where the measured variable was the number of green candies out of the eight in each package. Shown in Table 2 is the sample space for the count of green candies, the probability of getting from zero to eight green candies, the expected number of samples having each value in the sample space, and the obtained number of green candies.

The probability of getting each count can be found in binomial distribution tables online or in the back of most introductory textbooks. The expected number of green candies out of 166 can be computed by multiplying the probability of getting a particular count by 166. For example, the expected number of samples having a count of two green candies is \( 0.1488 \times 166 = 25 \).

The M&M’s illustration was done early in the term. While we were discussing probability, however, students saw these concepts and applications in greater detail when we discussed inferential techniques later in the course. At that time, we discussed whether the obtained and expected frequencies were

### Table 1—Color Distribution of M&M's as Reported by Mars, Inc.

<table>
<thead>
<tr>
<th>Color</th>
<th>Brown</th>
<th>Yellow</th>
<th>Red</th>
<th>Blue</th>
<th>Green</th>
<th>Orange</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>20%</td>
<td>20%</td>
<td>20%</td>
<td>20%</td>
<td>10%</td>
<td>10%</td>
</tr>
</tbody>
</table>

### Table 2—Results of Binomial Replications Using Peanut M&M’s

<table>
<thead>
<tr>
<th>Count of Green Candies</th>
<th>Probability of Getting This Number of Candies</th>
<th>Expected Number of Green Candies (Out of 166)</th>
<th>Obtained Number of Green Candies (Out of 166)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.4305</td>
<td>71</td>
<td>52</td>
</tr>
<tr>
<td>1</td>
<td>0.3826</td>
<td>64</td>
<td>67</td>
</tr>
<tr>
<td>2</td>
<td>0.1488</td>
<td>25</td>
<td>41</td>
</tr>
<tr>
<td>3</td>
<td>0.0331</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>0.0046</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0.0004</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0.0000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0.0000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>0.0000</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
close and what it meant to be close; why the obtained and expected numbers were not exactly the same; and other principles that led to sampling distributions, decision rules, statistical hypothesis testing, and error rates. Students saw these data again when goodness of fit was discussed. Hence, the M&M’s technique was applied again later in the course.

**Discussion**

When I first thought about the peanut M&M’s, I was presuming that the proportion of colors in the population was the same. If they were, the class could still have voted on the target color and we could have still used the binomial model. So the voting did not really matter. However, it gave the students the opportunity to be more involved. In fact, one student told me he would never think about M&M’s in the same way again. As the instructor, I found the process engaging, and I believe most of the students did, as well.

In a way, the M&M’s idea is something like Ronald Fisher’s tea-tasting lady, who alleged she could tell whether the milk had been added to the tea or vice versa. Fisher was a statistician from the United Kingdom—one of the giants in the field. Many of the techniques we use today were developed by him, including the analysis of variance (ANOVA) and Fisher’s Exact Test. About 60 years ago, he wrote about how one could test whether the tea-tasting lady’s ‘ability’ was genuine. He discussed how many cups of tea one would expect the lady to correctly identify if she really did have the ability to discern the order in which the contents of the cup were added. It seems to me that the M&M’s example is in that same spirit.

**Note:** In 2001, David Salsburg, a professor at the University of New Haven, wrote *The Lady Tasting Tea: How Statistics Revolutionized Science in the Twentieth Century*, in which he discusses the contributions of applied statisticians such as Fisher. He goes into a detailed discussion of the tea-tasting lady. For more information about the book, visit [www.maa.org/reviews/ladytea.html](http://www.maa.org/reviews/ladytea.html).

**Further Reading**


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**U.S. Census at School**

The ASA recently launched the U.S. version of Census at School, an international project that engages students in learning about statistical inquiry and census taking. Volunteers are needed to expand this pilot program. For more information, contact ASA Director of Education Martha Aliaga at martha@amstat.org. [www.censusatschool.com](http://www.censusatschool.com)
Making Sense of Statistical Studies consists of student and teacher modules containing 15 hands-on investigations that provide students with valuable experience in designing and analyzing statistical studies. It is written for an upper middle-school or high-school audience having some background in exploratory data analysis and basic probability.

www.amstat.org/education/mssss
Are Education Graduates Ready to Teach Stats?

Nicole Keenan, Chris Moore, and Lisa Dobson, Ball State University

Introduction to statistics during high school was minimal. We opted for math courses other than AP Statistics, leaving us with only an Algebra II statistical background when pursuing a degree in secondary math education. As some may know, the typical Algebra II class covers statistics in about two weeks, touching on such topics as permutations, combinations, and card hands—more probability than statistics. This was enough to get us through the first year and a half of our degree program, but certainly not enough to prepare us for teaching statistics.

We weren’t prepared for a university-level statistics course, either. However, we were not required to do computations by hand and were given a classroom laptop to aid us. In the 16-week course, we used Beth Chance and Allan Rossman’s *Investigating Statistical Concepts, Applications, and Methods* and its supplementary CD. The class was taught with a more exploratory approach. For instance, instead of being told what the related terms meant, we learned how they were used and then assigned our own definition for each concept.

Throughout the course, we also carefully analyzed the correct way to conduct studies and experiments. With a fine-tooth comb, we learned how to use proper wording of results and conclusions while writing reports for two projects, four labs, and biweekly homework assignments. This learning process ensured we portrayed accurate information from our investigations. We also learned about normal curves and how to compute a $p$-value, standard deviation, and $t$-score, but the course was not the only facet of college that taught us how to better learn and teach statistics.

Tutoring other students—whether in high-school or college—provides an excellent way to gain experience. In the broad spectrum of math courses, there are students who need help with statistics. So, once again, we are exposed to confidence intervals, basic probability, standard deviation, and various graphs and charts. Helping others with their learning gives us the opportunity to teach some of the topics we learned in our course, which helps solidify the concepts.

In class, the primary goal was to arrive at the correct answer by typing in the right numbers in the appropriate places. When tutoring—and sometimes teaching—we have to answer conceptual questions and are therefore required to be able to express a deeper understanding of ideas and vocabulary.

Are We Ready?

Technically, the degree we will earn qualifies us to teach all secondary math courses. However, does obtaining the diploma mean we will be ready to take on all math topics a school board may ask us to teach?

On one hand, we are not prepared to teach an advanced high-school statistics class (i.e., AP Statistics) to the standard we have learned to expect from ourselves. We understand the procedures of statistics and probability, but not the meaning of the answers we calculate. If we don’t fully understand the concepts, how can we expect this from our students? Sure, we can teach a straight procedural course and get along just fine, but we don’t think a mathematics course should be taught procedurally.

On the other hand, we may be prepared and ambitious enough to take on an introductory statistics course and are definitely ready to teach an Algebra II class. For an introductory course, we are confident we can teach ourselves the concepts that will aid us in becoming successful mathematics teachers.

What Would Make Us Feel More Prepared?

To boost the confidence of pre-service teachers, college courses should emphasize conceptual understanding and focus less on procedural, numerical answers. It may be beneficial to switch to computer programs such as Fathom to make such a shift. In our experience, this program enables the user to actually conduct studies and simulations. By manipulating one number or a small portion of a study, the user suddenly sees that $p$-values, standard deviations, means, and all other computations can change drastically. Observing the results of different manipulations encourages more discovery-based learning.
The editor of STatistics Education Web, an online bank of peer-reviewed lesson plans for K–12 teachers of mathematics and science, is accepting applications/nominations for associate editors and reviewers. Those chosen will review lesson plans that showcase the use of statistical methods and ideas in science and mathematics based on the framework and levels of the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A Pre-K–12 Curriculum Framework. Associate editors and reviewers will be selected for each of the three GAISE levels. Interested persons should electronically submit a letter of interest and a current CV or résumé to the STEW editor, Scott Street, at STEW.Editor@gmail.com and copy Rebecca Nichols at rebecca@amstat.org.
In 2007, Utah revised its core curriculum for school mathematics. Among the changes was an increase in statistical content across grade levels, making the curriculum more consistent with recommendations from the National Council of Teachers of Mathematics and the GAISE report. While this is an important step in helping students become statistically literate, it was one that was seen with dread by many of the teachers affected by the new standards.

In my experience as a mathematics/statistics educator, I have found that many pre-service and in-service math teachers feel under-prepared to teach statistics. As stated in the GAISE report, “Statistics … is a relatively new subject for many teachers, who have not had an opportunity to develop sound knowledge of the principles and concepts underlying the practices of data analysis that they now are called upon to teach.”

In an effort to help teachers make the transition to the new high-statistics content core, the Utah Office of Education organized a committee to create a summer workshop for teachers. The committee consisted of me, an assistant professor in statistics education at Utah State University, and three high-school math teachers—all of whom had many years’ experience teaching AP Statistics. The expectation was that the workshop would help participants better understand statistics while providing them with resources and skills for teaching it.

One of the preliminary acts of the committee was to outline a common philosophy for teaching statistics and discuss how this would influence the course design. Our philosophy included the following points:

1. **Statistics is inherently relevant.** We are all confronted by statistical information on a regular basis. Therefore, in a course on statistics, we should emphasize relevance and capitalize on it to make the material meaningful for students. One of the best ways to do this is to use real data and, even better, student-generated data. Relevance was a guiding principle behind all we did in preparation for the course. We selected readings about the use of statistics in the media and planned activities in which we would look at statistical displays in newspapers, collect data from the group, and engage in simple experiments.

2. **Everyone has something to contribute.** The workshop was offered to math teachers at middle and high schools throughout the state. Participants ranged from 7th-grade teachers to those who had taught AP Statistics, and the statistics backgrounds of attendees were similarly disparate.
We expected participants would make important contributions through both their uncertainties and expertise. Participants who express doubts and confusion tend to benefit the group discussion, as tricky points are clarified and facilitators are enabled to tailor the material to the needs of the group. As participants share their expertise, they enrich the group experience by providing alternative explanations, help to less-informed colleagues, and lessons or ideas for lessons.

We scheduled time for participants to share their ideas and receive feedback. We found this added significantly to the learning and sense of community. To encourage contributions from the participants, we contacted them prior to the workshop to ask what they hoped to get out of the course and to request that they bring their own activities and lesson ideas to share with their colleagues.

3. Learning about statistics is fun. Hopefully, most statistics teachers are aware of this already, but many of the math teacher participants were not. The workshop title—Statistics Is FUNdamental—stressed the idea that statistics is valuable and stimulating. We planned activities that required lots of interaction: investigations, problemsolving, applications, and games through which participants were actively engaged in learning concepts of probability and statistics.

4. Participants should acquire resources and understanding. The workshop lasted five full days and covered the statistics content in the Utah core curriculum. We gave more or less attention to specific topics as the needs of the participants dictated, but we did try to cover everything, including data collection, data summary and display, distributions, probability, bivariate data, and appropriate pedagogy. Because of the breadth of the material covered, not all parts were directly applicable to what all teachers address in their courses. We emphasized the placement of each topic in the core curriculum, letting teachers know where they would see the concepts, but we also stressed the connections between topics, which helped participants see the big picture. For each activity developed for the workshop, we created an activity guide for the participants and gathered materials such as dice and attribute blocks to enable teachers to implement the lessons with their own students.

To illustrate how we put these principles into practice, I’ll describe some of the activities from the course. In one of the first activities, participants formulated questions about each other; the answers represented different types of data. We called this the ‘getting to know you’ data. Questions covered how many minutes it took to drive to the workshop and whether teachers of secondary students had attended. We used calculators to create graphical displays of data and searched newspapers to find and analyze examples of graphical displays. Participants also were given time to share activities they had used with their classes to teach data summary. This was a stimulating activity, as participants shared appropriate. Finally, we summed up the information through an instructor-led discussion of graphical data summaries in which we formalized definitions and procedures, made explicit appropriate uses of various methods, and discussed advantages and disadvantages of different plots for the same type of data (e.g., circle graphs versus bar charts for categorical data).

In follow-up activities, we used calculators to create graphical summaries of data and searched newspapers to find and analyze examples of graphical displays. Participants also were given time to share activities they had used with their classes to teach data summary. This was a stimulating activity, as participants shared appropriate.

2010 Poster and Project Competitions Need Entries

Introduce K–12 students to the world of statistics through the 2010 poster and project competitions, directed by the ASA/NCTM Joint Committee on Curriculum in Statistics and Probability. The competitions, now in their 21st and 24th years, respectively, offer opportunities for students to formulate questions, gather and display data, and draw conclusions from data.

Winners are recognized with plaques, cash prizes, certificates, and calculators (donated by Texas Instruments) and their names are published in Amstat News.

Posters judged in four grade-level categories (K–3, 4–6, 7–9, and 10–12) are due every year on April 1. Projects are due on April 1 for grades 4–6 and 7–9 and on May 30 for grades 10–12. More information about the poster and project competitions, including entry forms and two instructional webinars, is available at www.amstat.org/education/posterprojects/index.cfm.

Participate in National Lab Day

President Obama recently announced the establishment of National Lab Day, a day to bring together science, technology, engineering, and math (STEM) professionals and teachers who provide high-quality, hands-on, discovery-based lab experiences for students.

To learn how you can become involved, visit www.amstat.org/education/asa-nldresourcepage.cfm.
Recorded webinars (web-based seminars) on K–12 statistics education topics are free to view at [www.amstat.org/education/k12webinars](http://www.amstat.org/education/k12webinars).

This webinar series was developed as part of the follow-up activities for the ASA Meeting Within a Meeting (MWM) Workshop for K–12 Mathematics and Science Teachers ([www.amstat.org/education/mwm](http://www.amstat.org/education/mwm)), held in conjunction with the Joint Statistical Meetings. MWM and the webinars are part of the ASA’s outreach activities to enhance K–12 statistics education. The Consortium for the Advancement of Undergraduate Statistics Education (CAUSE) offers free webinars on undergraduate statistics education topics at [www.causeweb.org/webinar](http://www.causeweb.org/webinar).

As each participant received copies of the plans for the various activities and tools for implementing them (such as a simulator for the calculators), all left with resources to improve their statistics teaching. I implemented the graphics activity in one of my introductory courses, asking students to devise ways to represent class data. Because the students are encouraged to invent new methods and use familiar ones, students with a broad range of background knowledge could actively participate in the activity and contribute to subsequent discussion. The activity motivated students to think about the various ways of displaying data we use and why we use them.

Response to the workshop was positive: Participants were able to discuss, investigate, and understand the relevance of statistics in their lives; the learning experience for all was enriched as participants asked questions, shared their expertise, and discussed ideas for teaching; everyone had fun; and teachers at all levels said they understood statistics better as a result of the course and felt the resources they received made them better prepared to teach it.

Further Reading


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- Access to teaching resources, including webinars, the Statistics Teacher Network newsletter, and GAISE: A Pre-K–12 Curriculum Framework

- Information about upcoming events and products for K–12 teachers, including statistics education workshops, webinars, student competitions, publications, and online peer-reviewed lesson plans

Visit www.amstat.org/education for details.

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The American Statistical Association will participate in the USA Science and Engineering Festival, October 23–24, 2010, on the National Mall in Washington, DC.

We want your help to design hands-on activities to attract students to the ASA booth and stimulate an interest in statistics.

Competition rules and guidance at www.amstat.org/outreach/hands-on.cfm

**Grand Prize:**
$750 and a trip for two to Washington, DC

**Two Gold Medal Prizes:**
$500

**First Submissions Due April 15**