The
Statistics
Teacher
Network

www.bio.ri.ccf.org/docs/ASA/stn.html

Number 54 ASA/NCTM Joint Committee on the Curriculum in Statistics and Probability Spring 2000

Product Review

A Look at Statistics on One of Casio's Color Graphing Calculators

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The **Casio 9850GA PLUS** is an icon-driven, menu-driven color calculator, intended to be on a par with the TI-83. Three of these Casios can be purchased for about the price of two TI-83s. They have many of the same features as the TIs, and some different ones that are quite worthwhile.

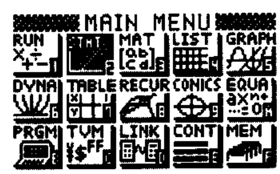
I found the manual to be adequately readable. As the instructions say, keeping three things in mind makes the calculator rather simple to operate:

1. pressing the **[MENU]** key brings up the main screen,
2. the functions **[F1]** through **[F6]** refer to the selections at the bottom of whichever screen is current, and
3. pressing **[EXIT]** backs procedures up one screen each time (but not to the main menu).

Also In This Issue...

As they say at Casio, "When in doubt, exit out!"

The **[MENU]** key displays icons for the fifteen basic topics, each icon having a symbol in the lower right corner telling which key to press next to initiate that particular topic. An alternative to locating the appropriate key is just to highlight the desired icon and press **[EXE]**.

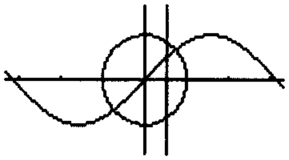


From the **[MENU]**, **STAT** is used for statistics and **MAT** for matrices. **LIST**, **GRAPH**, **LINK** and **TABLE** are just what they look to be. Other choices include **RECUR** (recursion), **PRGM** (program), **CONT** (contrast) and **MEM** (memory).

DYNA (dynamic graph) is an especially nice feature. It stores functions with variables for coefficients and draws multiple versions of the graph when you fix all the coefficients but one and give a range of values for that one. **EQUA** (equation) solves linear equations with up to six unknowns, as well as quadratics and cubics.

The Casio's display measures 127 by 63 pixels. There are three "built-in" windows for graphing. These are accessed, after selecting a mode, by choosing {V-Window}, that is, by pressing **[SHIFT]** **[F3]**. Happily, the user can save his own preferred window settings (maximum six). For early class work in graphing I personally like one with **Xmin** = -12.6 and **Xmax** = 12.6, which, when traced, yields -12.6, -12.4, -12.2, ... , 12.6 as the x-coordinates.

Not only are vertical lines (x = constant) easily graphed; they can be displayed on the same screen with functions and/or parametric equations!



When drawing overlapping areas, the Casio only shades in the intersection itself, rather using different types shading for each of the regions involved. Graphs can be drawn as blue, green or orange. The blue and green are almost indistinguishable, but the orange stands out.

Once in **GRAPH** mode, **[SHIFT] [F5] (G-Solv)** will display a function menu for finding roots, extrema, y-intercepts and points of intersection. One can also determine the y-coordinate for a given x, the x-coordinate for a given y, and the integral for a given range.

Zeros of a function may be found using the Solve function, **[F1]**. The syntax is: **Solve(f(x), n, a, b)**, where **n** is an initial estimated value, **a** is a lower limit, and **b** is an upper limit. If there is no solution between **a** and **b**, an error message appears at the bottom of the screen. In any event, you can edit by pressing the left arrow key, which returns you to the expression. **Solve(x^2-3, 1, 0, 3)** returns the larger solution of $y = x^2 - 3$.

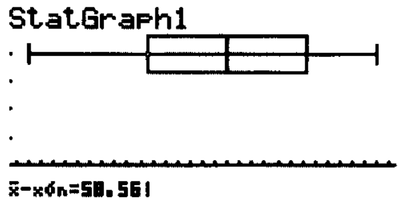
To access *maximum/minimum value* calculations, it's necessary to press **[F6]** and then **[F1]** for minima or **[F2]** for maxima. The syntax for minima on the interval $a \le x \le b$, with a precision $n = 1$ to 9, is **FMin(f(x),a,b,n)**. To determine the minimum value for $y = 4x^2 + x - 6$ on the interval $-1 \le x \le 3$ with a precision of $n = 6$ enter **Fmin ($4x^2$+x-6,-1,3,6)**.

Using the statistics menu of the Casio enables one to input statistical data into lists, find regression equations, determine the mean, maximum and other statistical values and perform various statistical tests on single-variable and paired-variable data. In addition, confidence intervals can be determined and distributions of data produced.

STAT features are accessed by selecting the **STAT** icon on the menu screen and then pressing **[EXE]**, or simply by pressing **[2]**. Either brings up the statistical data lists. Type in each number and press **[EXE]**; this automatically moves the cursor down one place in the list. The cursor keys move the cursor anywhere in the table to make corrections or to move to a different column.

Graphing Single-Variable Data

There are six graphs available for single-variable data: a histogram, two different box and whiskers graphs, a normal distribution curve, a line graph and a normal probability plot.



The **Med-box graph** encloses all the data in an area from the 25th percentile to the 75th percentile in a box, with a vertical line drawn at the 50th percentile. Lines (whiskers) extend from either end of the box to the minimum and maximum of the data. Data falling outside the box can be plotted if desired. The **Mean-box graph** shows the distribution around the mean, with a vertical line drawn at the point where the mean is located, and the box drawn so that it extends on each side of the mean by the standard deviation. The whiskers again extend to the extrema of the data.

(GRPH), or **[F1]**, is the first choice listed under the data table. After choosing this, press **[F6] (SET)**. Use the cursor keys to move through the categories, and press the appropriate function keys to choose one of the three graphing locations (**[GPH1]**, **[GPH2]**, **[GPH3]**) to use, which list is to be used, frequency, color of graph, and the like. **[EXIT]** out. Select the graphing location and the graph will be generated. (Before displaying a histogram or a broken line, the Casio allows the user to set the starting value and the pitch; however, just pressing **[F6] (DRAW)** accepts the calculator's choices.)

For any of the graphs, pressing **[F1] (1VAR)** will bring up all the statistical data: mean, sum of data, sum of squares, population and sample standard deviation, extrema, mode, and data mean ± population standard deviation. These values can be obtained directly by displaying the table of data and pressing **[F2] (CALC)** and then **[F1] (1VAR)**.

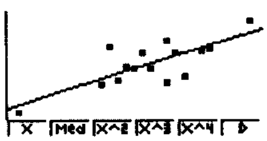
Graphing Paired-Variable Statistical Data

A scatter plot is very easily generated: after entering the data, pressing **[F6] (SEL)** and deciding which location to use, select **Graph Type**; sort through the other choices by pressing **[F6] (>)** three times; then press **[F1] (Scat)**. You can then specify which lists to use, the type of mark to make and the like. **[EXIT]** out. Press the function key for the location chosen, and the plot will be displayed.

Below the scatter plot are listed ten regression functions. (Only five at a time are shown; press **[F6] (>)** to display more.) Included are linear regression, the median-median fit, quadratic/cubic/quartic regressions, logarithmic, exponential and power regressions, sine and logistic regressions.

Pressing the function key for your choice displays the coefficients of the expression generated, along with r and r^2. You can then press **[F5] (COPY)** to store the expression as **Y1**, or press **[F6] (DRAW)** to graph immediately. Under the **[F6] (SET)** menu, a list can be specified for storage of the residuals.

Tucked in amongst the ten regression functions is **[2VAR]**. Pressing this displays the statistical data for each list: the mean, the sum, the sum of squares, population and standard deviations, and the extreme values. Also included is $\sum xy$, the sum of the cross products from the xList and yList data.



Performing Statistical Calculations

First, of course, input the data. Press **[F6] (SET)** to specify the number of variables and list(s) to be used. Again, use the cursor keys to move up and down the screen and the function keys to change selections on each line. **[EXIT]** out.

Press **[F1] (1VAR)** for data on the first list; **[F2] (2VAR)** presents data on both lists, one after the other. Use the cursor keys to scroll through the material.

Pressing **[F3] (REG)** from either the table screen or the data screen brings up the ten choices of regression formulas listed above. Again, press **[F6] (>)** for the last five choices (or to return to the first five).

Estimated values for x- and y- parameters, $\hat{x}$ and $\hat{y}$, can be calculated for linear, power, logarithmic and exponential regression graphs: after inputting the data and drawing the graph with **STAT Mode**, use **RUN Mode.** This necessitates returning to the Main Menu screen: press the gray **[MENU]** key, then **[1]**. In **RUN Mode**, to estimate $\hat{y}$ for a given x, say 40, press **40, [OPTN], [F5] (STAT), [F2]** ($\hat{y}$) **[EXE]**. $\hat{x}$ for a given y is found similarly.

RUN Mode is used to perform probability distribution calculations for single-variable statistics. Again, press **[OPTN]**, then **[F6] (>) [F3] (PROB) [F6] (>)** to display a function menu containing **P(**, **Q(**, **R(**, and **t(** . The latter can only be obtained immediately after performing single-variable statistical calculations. The four quantities are calculated using the following:

$$P(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{\frac{-s^2}{2}} ds$$

$$Q(t) = \frac{1}{\sqrt{2\pi}} \int_{0}^{t} e^{\frac{-s^2}{2}} ds$$

$$R(t) = \frac{1}{\sqrt{2\pi}} \int_{t}^{+\infty} e^{\frac{-s^2}{2}} ds$$

$$t(x) = \frac{x - \bar{x}}{\sigma_n}$$

Tests

Several tests are available: the **1-Sample** and **2-Sample Z Tests** work with the population means when the standard deviations are known. **1-Prop and 2-Prop Z Tests** checks the proportions of data from samples that satisfy certain criteria.

The **1-Sample** and **2-Sample t Tests** are used to see that samples are taken from the population. The **LinearReg t Test** calculates the strength of the linear association of paired data. Also available are the χ^2 **Test,** concerning the proportion of samples included in each of a number of independent groups; the **2-Sample *F* Test**, checking for changes in results if one of multiple factors is removed; and **ANOVA**, which tests that population means of multiple samples are equal. All of these can be accessed while the statistical data list is on the display by pressing **[F3] (TEST)**.

Confidence Interval

Pressing **[F4] (INTR)** while the data list is displayed produces two choices, **Z** and ***t***. Selecting either one yields choices involving the number of samples, or the proportions of the samples.

Distribution

[F5] (DIST) presents the user with seven choices. Under **[F1] (NORM)** are listed **normal probability density**, **normal distribution probability** and **inverse cumulative normal distribution**. **[F2] (*t*)** deals with **student-*t* probability density** and **student-*t* distribution probability**. Distribution probability can also be calculated for **chi-square, *F*, binomial, Poisson** and **geometric distributions**. The following distribution graph was generated via the **normal probability density** menu.

Closing Comment

I wrote this brief article to give the readership knowledge that a lesser expensive yet comparable calculator to the TI-83 exists, namely the CASIO 9850GA PLUS. I have enjoyed using it; they seem to have thought of everything!

Keep Us Informed...

The Statistics Teacher Network is a newsletter published three times a year by the American Statistical Association—National Council of Teachers of Mathematics Joint Committee on Curriculum in Statistics and Probability Grades K–12.

We need your letters, announcements, articles, and information about what is happening in statistics education! Please send hard copy, and, if possible, a disk written in standard ASCII text or a Word document to:

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Teaching Probability to Young Children

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Introduction

In teaching the topic of probability to young children, we adults often make broad assumptions that kids understand what "fair" is. Kids' ideas of what comprises "fairness" usually center around the *behavior* of the players. Young children judge a game "fair" if the other players follow the rules and wait for their turn.

So the notion of "fairness" must be examined and taught, before we can approach having kids examine broader ideas about probability. By the late third grade, early fourth grade level, our goal is to help children develop a method for determining the fairness of games. To do this, students need experience with examining many games.

We need to teach students to do the following:

1) Review the rules of the game carefully, paying attention to their meaning.
2) Examine the sample space.
3) State the chance of winning (and the chance of losing).
4) Determine if it is a fair game.

To effectively analyze a game without a teacher's prompts and scaffolding, students need to internalize the four steps stated above. This four-step procedure could be shortened to a simple rule to remember:

When facing unpredictable events: **review the rules, picture the possibilities, regard the risks, and be able to state your chance of success.**

The Lesson

This lesson is designed to be an "Interactive-Narrative-Adventure" lesson. The teacher should prepare sentence strips for **all bold statements** *and post them as the story is read. Tree diagrams and sample spaces can be prepared ahead of time or can be developed as part of the role-play. You may choose other enhancements (an overhead projector with pictures of a carnival, a magician, etc. and transparent number tiles to "tell" the story.) The story teller can pause at any [bracketed text] and elicit student responses, to increase the interactive role-play nature of the story. The narrative is told in two episodes. Part two will be in the next edition of "The STN."*

PART I

Outsmarting the Magician at the Carnival Games

Here is a story of kids at the carnival. Let's see how they "Fair" (so to speak).

That pesky Magician is back! He comes to the fair each year. He has card tricks and dice games and he enjoys surprising people so that they are in awe of his powers. He seems only slightly sinister . . . almost likeable . . . and it is unnerving that he uses dice and cards and things you thought you understood, but he always seems to win! You want to know more. You think there must be some trick to what he is doing, because in all your years, you've never seen someone repeatedly win at a game, unless there was a trick or it was "rigged." You watch him carefully . . . you're planning to challenge him soon.

You ask yourself:

- **How do I know if the game is fair or unfair?**
- **What am I up against and what can I do about it?**
- **How can I be sure the game is random so I know I have a fair chance for a win?**

How do I know if the game is fair or unfair? I'll start by reviewing the rules of the game and I'll observe the actions of the players.

The Magician puts out a challenge for a game. He calls out, "Step right up! Who will play the game?"

(The prizes are little beanbag characters—and you have an obsessive desire to add to your collection!)

The Magician continues, "Who will challenge me in a game? It's simple. There are cards numbered 1 to 6. I choose 1 and 2. You get 3-4-5-6. After I shuffle, you turn over 2 cards. If a 1 or a 2 shows, I win a point. If the two cards you turn over are yours, you win a point. When you get 3 out of 5 points, you win a beanbag character. Who will play?"

Wow! That sounds easy! And the prizes are terrific! You want to play, but you think you'll watch first. There are lots of people clamoring to play, and it's a little crowded, but you nudge in to observe. The cards are shuffled and fanned out face down.

[Option: read the narrative or role play the game with several students.]

The kid who is playing turns over a 6 (yes!) and a 2 (uh-oh) , Whose point is that? [1 point for the magician.] (Reshuffle and fan out.)

He turns over a 3 (yes) and a 5 (yes), Whose point is that? [1 point for the kid!] (Reshuffle and fan out.)

He turns over a 5 (yes) and a 1 (too bad), Now whose point? [1 point for the magician.] (Reshuffle and fan out.)

Then a 6 (yes) and a 4 (yes), and . . . ? [It's a tie game!] (Reshuffle and fan out.)

Last hand: Ack! The first card over is a 1—and the magician wins 3 out of 5. [Can you see why only one card had to be turned over to see whose point it was?]

He says: "Aw! Too bad, sonny. Want to try again?" But the kid shakes his head, and the next player steps up.

You watch carefully, but the first three rounds go right to the magician. Whew! What bad luck!

The next player tries. He plays 4 rounds and it's tied, 2 points to 2. The last round determines the winner. The crowd is really focused. He turns over a 3 (yes) and then a 5 (YES!!) and beats the Magician with 3 out of 5 rounds. Lucky kid! He gets that cute little purple character you had your eye on.

[If you role-played the above, resume the narrative here.]

You're ready to play now, but you can't get much farther up in line. Your friends join you, and you watch together. The next four people in line lose to the magician. Huh? Seems odd. You watch. Your turn is getting closer, but your friends decide that this isn't a fair game. Yes it is, you argue. You've watched it, and it's always the player who is selecting which cards to turn over, so it's not rigged in the shuffling. You saw a kid win. But you start wondering . . .

What am I up against and what can I do about it?

I'll make a picture of all the possibilities.

Your friends are feeling suspicious that this is an unfair game. Together you take action. Someone produces a notepad and begins sketching out what you have observed. Let's see . . .

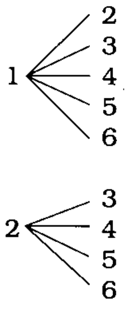
- The Magician has chosen: 1 & 2
- Your choices are: 3 - 4 - 5 - 6

It sounds like you have the advantage. Let's see what it looks like.

You must turn over two of your cards to win. Combinations are tricky. Let's diagram all the possibilities:

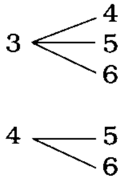
[The teacher can develop these tree diagrams with the class or have prepared diagrams to refer to in the story.]

Here are the winning matches for the Magician:



That makes 9 combinations for him to win.

Here are the winning combinations for the player:



That's only 6 combinations for the player to win! This means that "the cards are in his favor!" (Say! Maybe that's where that old expression came from!)

[If you elicited class participation for the above, resume the narrative here.]

Your friends were right! You are glad that you took their advice and made a diagram of the possibilities. Now your suspicions are confirmed! It is an unfair game! So . . . what should you do about it?

You decide to wait your turn—and you are next. When you are up, you confront the Magician with the situation (in front of the crowd, of course.) You point to the diagram *[point to the one you have posted]* you have made and show him that YOU have 6 out of 15 combinations to win, while HE has 9 out of 15 combinations to win. You tell him that you don't mind losing in a FAIR game, after all, that's what luck is all about. You pay your money and you take your chances. BUT, in the overall scheme of things, you expect that whoever plays should have a fair chance at the prizes. The game should be RANDOM. That means that when you look at things ahead of time, there's no way to tell who is going to win.

The Magician seems surprised that his game was not fair, and seems impressed with your mathematical ability to count the combinations and to explain things so well!

Now that his advantage has been exposed, the Magician becomes very accommodating to the crowd, and says: "I deeply apologize. Of course I want you to have a fair chance to win these prizes!"

So you offer new rules for playing his game. You tell him:

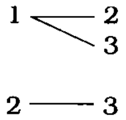
Keeping a game random gives all the players a fair chance for a win.

"Now," you propose, "Let's play it this way: You have number 1-2-3. I have numbers 4-5-6. If any pair of your numbers shows, you have 1 point. If you turn over anything else besides a pair of YOUR numbers, you score a zero. When it is my turn, I must turn over a pair of my numbers to get a point. If I do not turn over two of my own numbers, I cannot get a point. We get three turns each to make a round. Whoever is ahead at the end of the round, wins."

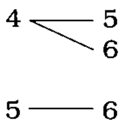
"Well, that's alright by me," he replies. "But can you show me that it is fair?"

"Sure," you say, as you sketch the possibilities: *[The teacher can develop these tree diagrams with the class or have prepared diagrams to refer to in the story.]*

Matches for you:



Matches for me:



[If you elicited class participation for the above, resume the narrative here.]

We each have 3 chances to make a point. Every other combination is worth zero.

He agrees and you play.

[Option: Role play this scene or read the narrative and simulate the cards. Elicit bracketed responses from the students.]

- The first turn for him shows a 1 and a 5. [0 points].
  (He reshuffles and fans out) . . . and you turn over a 3 and a 4. [0 pts. Tie!]
  (He reshuffles and fans out.)
- On his second turn, he flips over a 6 and a 2. [0 pts.]
  (He reshuffles and fans out)...and you turn a 3 & 1. [0 pts. Tie again!]

The tension builds and the crowd cheers you on as he reshuffles and fans out the cards.

- On his third turn, he selects a 1 and a 2! Ack! [He's now ahead with 1 point. You have to tie him, or you risk losing!]
  (Now there's the reshuffle and fan out) . . . and you turn a 4 (yes!) and a 1 (oh no!). [You have 0 points. You've lost!]

[If you role-played the above, resume the narrative here.]

The crowd says: "Aw!"

The Magician says: "Sorry, sonny. Do you want to play again?"

You say, "No, thanks! I think I'll wait for my luck to change. But you won, fair and square."

Your friends console you as you walk away, but you say, "Hey, that was fun. I'm going to come back later. You know, I still have a good chance of winning."

The Magician agrees, and we look forward to playing each other in lots of new games, because we each know that we have met our match. And we both also know:

In Games of Chance you take chances to win, but you also have a chance to lose. If you are able to state your chance of winning this will guide you in your decision to play the game.

When you state your chance of winning, you can also figure out your chances of losing. This can help you to decide what you are willing to risk when playing a game.

To play a game of chance, you have to cope with randomness. You never know what is going to happen in the next move or the next game. You can only predict what will happen in the general picture over time.

You can "predict" a general trend, but it's not the same as foretelling the future. All you have to go on is your best advice. If you're lucky, maybe the game will "play out" to be to your advantage.

To make an informed decision to play: "review the rules", "picture the possibilities", "regard the risks" and "state your chance of success."

END OF PART 1

Look for Part 2 in the next edition of "The STN."

Editor's Note: *There is a new NSF funded project being developed for a fourth graders target audience. It is a cartoon series and Web site called "Cyberchase" and will be broadcast in the fall of 2001. This is a math adventure series in which a team of young characters travels in cyberspace, solves mathematical dilemmas and compliments each other in their learning styles and problem-solving techniques. One of the episodes is devoted to exploring the fairness of games.*

Cyrilla Bolster and her husband, Carey Bolster, are two of the mathematics consultants for this project. Look for series previews in your area.

Statistics in the Classroom...

Introducing Descriptive Statistics and Graphical Summaries

Brad Warner,
United States Air Force Academy

"The average statistician is married to 1.75 wives who try their level best to drag him out of the house 2.25 nights a week with only 50 percent success. He has a sloping forehead with a 2% grade, 5/8 of a bank account, and 3.06 children who drive him 1/2 crazy; 1.65 of the children are male." — W. F. Miksch

Among the first topics in a basic statistics course are descriptive statistics and graphical summaries. This article presents a simple activity to introduce these topics.

As a teacher, it is often tempting to jump into explaining histograms, measures of center and spread, and other methods of summarizing data. Unfortunately, this approach can cause the students to miss the main concept and focus on the formulas for calculating statistics and the mechanics of producing graphs. They do not understand the purpose and reason behind descriptive statistics and graphical summaries. The activity discussed helps cement the main ideas of these topics and should be performed as an introduction.

To start the activity, select one student to be seated at the front of the classroom facing the class. You stand behind the student. Explain to the class that you are going to show them an object. Then, one word at a time, members of the class will give the chosen student an adjective describing the object.

The objective is for the chosen student to accurately guess the object in the minimum amount of adjectives. In addition, certain "taboo" words can be written on the board out of sight of the chosen student. These are words that define the object rather than describe it.

I typically bring a tennis ball into class for my object. My taboo words are "tennis" and "ball." I increase the excitement by telling the class that the current school record for describing the object is two words. Due to the competitive nature of the students, they want to tie or beat the record. Some typical adjectives used by the students are fuzzy, round, and green. It usually will not take more than 3 or 4 adjectives for them to succeed.

After the commotion diminishes, I explain to the students that a tennis ball is a very complex object. It is a composition of many different materials. It is not exactly a sphere. It has two or three colors. Yet with only a few adjectives we were able to convey all this information to our chosen student. We summarized a complex object with a few adjectives. Likewise, the purpose of descriptive statistics is to summarize, sometimes very complex, data sets with a few mathematical adjectives.

I then explain to the students that instead of adjectives we could have also drawn a picture of the object. The picture is not the object but a graphical summary of it. The drawing can be very crude or very detailed but the objective is to summarize the information in the object. Graphical tools such as box plots and histograms are our mathematical drawings of the data. The students typically respond that pictures convey more information than words; the "a picture is worth a thousand words" idea. This is precisely why we advise them to plot their data before performing any analysis.

This simple 10-minute exercise has been extremely helpful to my classes in conveying the purpose of descriptive statistics and graphical summaries.

Let us know your Zip +4!

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