Statistics—Making Sense of Data

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Statistics—Making Sense of Data offers Advanced Placement Statistics teachers a new alternative to the more traditional textbooks currently available. According to the course description the purpose of the Advanced Placement course in Statistics is to introduce students to the major concepts and tools for collecting, analyzing, and drawing conclusions from data. In order to accomplish this goal students should be exposed to the four following themes:

1. Exploring Data: Observing patterns and departures from patterns,
2. Planning a Study: Deciding what and how to measure,
3. Anticipating Patterns: Producing models using probability and simulation,
4. Statistical Inference: Confirming models.

Most of the traditional textbooks available accomplish this goal by a formula driven approach, which allows a student to conduct statistical tests, but without fully understanding the process of choosing the most appropriate statistical model for the data. With the capabilities of current graphing calculators, this approach encourages a student to blindly apply statistical tests without fully understanding what they are doing. Most textbooks currently available do not allow students to experience first hand the uncertainty and variability that arises when one attempts to measure something. This can only be accomplished by a hands-on approach, which allows students to produce their own data and observe first hand the variability that occurs. This is done in Statistics—Making Sense of Data by using simulation as an integral part of the modeling process throughout the textbook. Students are introduced to simulation early in the book and use it throughout to solve a variety of problems. Simulation becomes the glue that connects the four central themes in the Advanced Placement course description and allows students to produce and confirm their own statistical models. In most textbooks simulation is added as an afterthought or is not covered at all.

The textbook is divided into three parts: I—Describing Data (chapters 1-3); II—Statistical Modeling (chapters 4-10); and III—Estimation and Hypothesis Testing (chapters 11-15).

Chapters 1-3 introduce students to the graphical and numerical techniques needed to describe and summarize a variable’s distribution, starting with one variable data and then looking at the relationship between two variables ending with a look at linear regression. All the most current types of graphs are covered. The authors include the mean absolute deviation as another measure of spread in addition to variance and standard deviation. My students found that this helped them better understand what the variance measures. The approach used to introduce the line of best fit requires the student physically to go through a
three-step process. They first visually determine the line of best fit, then use the mean absolute deviation to find the least mean error regression line and finally develop the least-squares regression line. Although time consuming, this approach allows students to feel and see what the least-squares regression line measures. The idea of correlation is also developed differently than in most books. It is described in terms of covariance, which I feel allows students to better understand what the correlation coefficient is measuring.

It is in Part II—Statistical Modeling that this book changes directions compared to other textbooks. Chapter 4—"Modeling Expected Values" introduces the student to a five-step simulation model that will be used throughout the rest of the book. Both open-ended (eg. geometric distribution) and closed (eg. binomial distribution) simulations are done. Simulations are then used to introduce probability in Chapter 5 from an experimental approach first, followed by the more traditional theoretical probability formulas. At this time I would include sections 14.1 and 14.3 which cover some of the more advanced probability topics (eg. conditional probability). Instead of developing the different probability distributions found in most textbooks at this time, the authors jump right into an informal introduction to hypothesis testing in Chapter 6—"Making Decisions." Chapter 7 introduces the chi-square goodness of fit test by first using a simulation to determine if the data collected came from a fair die or not. This idea is then generalized to the standard question asked when using the chi-square test—how good does the observed distribution fit the expected distribution. Chapter 8 introduces some of the more common discrete and continuous probability distributions finishing with a detailed look at the normal distribution and some excellent applications of the normal distribution. Having been exposed to the chi-square test earlier, students are asked to determine if a set of data appears to fit a particular probability distribution by using the goodness of fit test. This is the type of thinking that is expected on the free response questions on the Advanced Placement Exam. 1 would include two more sections from Chapter 14, (sections 14.2 and 14.4) which deal with the mean and variance of the different probability distributions covered in Chapter 8. In the next edition I would like to see Chapter 14 broken up and put back in with the appropriate probability topic. Chapters 9 and 10 complete the section on Statistical Modeling by looking at how to produce or obtain data. Chapter 9—"Measurement" looks at the problems associated with obtaining an accurate, precise, and unbiased measurement. Chapter 10—"Sampling and Experimental Design" gives a detailed look at the different types of designs used to produce data.

Part III—Estimation and Hypothesis Testing covers the traditional confidence intervals and hypothesis tests suggested in the Advanced Placement course description. Chapter 11—"Estimation" formally introduces the central limit theorem and develops the general form of a confidence interval and then looks at the different types of intervals. Hypothesis tests are covered in Chapter 12 and include all the traditional significance tests as well as introducing a bootstrap hypothesis test (a common method practiced by statisticians). Having been exposed to informal hypothesis tests in Chapters 6 and 7, and familiar with setting up a five-step simulation model, you will not need to spend as much time on hypothesis testing as in the past. In Chapter 13—"Correlation and Regression," the authors return to look at the relationship between two variables. Hypothesis tests are developed for both the slope of the linear regression line and the correlation coefficient. Nonlinear relationships are looked at by transforming one of the variables and then looking at the linear regression line for the transformed variables. Multiple linear regression is also discussed and this exposure allows the student to realize how different variables affect each other. Chapter 14 covers a more formal approach to theoretical probability and the appropriate formulas needed to calculate theoretical probabilities for the different probability distributions presented. I feel that this material could be better presented if it were split up and inserted in the appropriate chapters earlier in the book, but it is not much trouble to insert it when you need the material. Chapter 15 closes the book with a treatment of analysis of variance and a more in depth look at multiple regression, both topics are not covered in the Advanced Placement course description but could be looked at after the Advanced Placement Exam has been taken.

One of the weaknesses of the text is the lack of computer printouts, although they are currently working on a computer supplement as well as a graphing calculator supplement for the text. Another weakness I see is the lack of questions at the end of each section, although
What's the same

1. Authoritative exposition—simple, direct, and full of insights, from someone who knows how statistics is practiced. Here's an example. In section 7.2 on Random Variables, YMM display a graph of sample means converging to a population mean, as sampling proceeds from $n = 1$ to $n = 10,000$. The $n$-axis is logarithmic, to sharpen the impact of the picture. In the sub-section[2] "Thinking about the law of large numbers," YMM talk about a "law of small numbers," which means that "we expect even short sequences of random events to show the kind of average behavior that in fact appears only in the long run." YMM conclude with this: "Much of the psychological allure of gambling is its unpredictability for the player. The business of gambling rests on the fact that the result is not unpredictable for the house."

2. Superb exercises, culled from real experience. Cobb (1987) reminded us to "Judge a statistics book by its exercises, and you cannot go far wrong." When I first used Moore & McCabe (1993), I enjoyed the narrative immensely, but it was the exercises that really got me. Every one, it seemed, conveyed something interesting or useful, and, as often as not, about contextual matters as well as about statistics. There we, my students and I, gained insights into the seasonal character of public demonstrations during the Vietnam war (IPS3, p.36/1.34); into death rates among women from lung cancer and breast cancer (IPS2, p.86/1.106—my students still are surprised to learn that more women die from the former than from the latter); into data summarized by one of my students as the "fat chickens don't cut it" data (IPS3, p.125/2.17); into the "spheres vs. vein" data set (IPS3, p.172/2.56) which suggests that some medical research seeks to replace effective invasive procedures with equivalent non-invasive procedures; into the rural/city pollution data (IPS3, p.176/2.64), which helps us see what kind of evidence might lead to a decision to shut down a pollution-monitoring site; and into the Archaeopteryx data set (IPS3, p.110/Example 2.4), which offers a first glimpse of the information available in fossils.

Moore's data sets are rich and can lead imaginative students in many directions. Most of them survive in YMM, where they have been augmented by others selected by Dan Yates. And to my surprise (because actually adding something to either IPS textbook is not easy), Yates has actually improved on the exercises...
What's missing/quibbles

1. Normal probability plots (NPPs) are introduced on pp. 94-96. But the answers at the back of the book and in the draft version of the Teacher Resource Binder rely on stemplots and back-to-back stemplots to assess normality. So what's the point of including NPPs?

2. Missing from chapter 5 is the matter of causation in the case of observational studies. This is so well done in Moore (SCC4, p. 328) and in the Decisions Through Data videotape (Unit 16) that it's a pity it didn't find its way into this book. AP students need to know that inferences from observational studies are fraught with difficulty, because this will help them assess the many instances in which inferences are improperly reported in the popular media.

3. Cumulative distribution functions are nearly invisible. Moore recently distributed a message to the apstat-l list asserting that "cumulative frequency distributions are ... only marginally useful and should be omitted." Though I do not doubt his judgment, it is a fact that "cumulative frequencies" are explicitly mentioned in the current Course Description's Outline of Topics. YMM users who want to include this topic in their course might examine Iman (1995), where they are systematically used.

4. I consider the work on linearizing transformations (Chapter 4) to be somewhat limited. After all, it involves only a single transformation, the logarithm. In my opinion, something could be gained by considering Tukey's "ladder of powers" for a variable v: v, v^2, v, v^5 = \sqrt{v}, \log(v), v(-.5) = 1/\sqrt{v}, v(-1) = 1/v, etc. Students would have more tools at their disposal, and would be able to use more flexible strategies in the model-building process. (See Tukey (1977) for a first encounter with the ladder.)

Quibbles aside, YMM is a fine blend of Moore-ishness, simulations, and activities. It is a particularly apt choice for new AP teachers, who may not be ready to wield the multiple resources used by some experienced teachers of the course.

References

Do you like pepperoni or sausage on your pizza? (And if you are vegetarian which would you choose for a friend?)

Do you like having your picture taken: yes or no?

Do you like to do homework: yes or no?

Most of the questions were chosen to split the class on issues that border on the ridiculous. This was intentional. I had asked the teachers for ideas for this questionnaire and I included some of their suggestions even though I thought that the outcome would probably split the class along racial or gender lines. Several did indeed do this, and while such questions did give a 50-50 split I did not use them since I wanted the students to focus on the mathematics and not on issues external to the class.

Six large graphs were posted on the walls. Each blank graph had a horizontal axis for test scores and a vertical axis for score count. The charts were labeled “Pepperoni”, “Sausage”, “Picture–Yes”, “Picture–No”, “Hwk–Yes”, “Hwk–No”.

Prior to the beginning of the course I had asked the Onaway teachers for test scores. I emphasized that I didn’t want names or the subject but that I did need scores from the same subject and on the same material. Two different teachers provided scores from their classes. I checked the data to make sure that they weren’t bimodal and that the combined scores were normal. I wrote the individual test scores on 48 separate slips of paper and put them in a bowl.

All of the data sets that I used for the class problems are based on real data. The data are suitably coded and/or scaled in order to protect proprietary information, however, the origin of the data was a real problem in which I was involved. I think it is very important to have data from real problems to work with and that it is also very important to take the time to explain to students what the data meant and why they were gathered.

As a student I would often wonder about the why of some of the problems I was asked to solve. In particular, I was always asking my teachers about the use of the mathematics that I was learning. I do recall that, while I enjoyed mathematics, I found it even more satisfying when I could connect my math to the world around me.
The students were told that the test scores were real and that we were going to treat the drawn scores as grades that they themselves had received on a number of tests. Each student drew four test scores and plotted the "grades" based on the results of their answers to the questionnaire. To minimize confusion I had the names of the six pepperoni lovers written above the pepperoni graph and similarly for the other categories.

After plotting, the class split into 6 groups of 2 students each. Each group computed the averages for all six histograms. Calculators were used by all of the students for this exercise. Their calculators did not have a summation button so they did the add and divide the old fashioned way. The calculator that I was using to check their numbers did have a summation button and I permitted some of the student groups to use mine just so they could use and see these functions on "a real statistical" pocket calculator. We had already computed averages in the earlier classes so this exercise merely helped to re-enforce earlier efforts.

With the calculations completed I asked for answers and summarized the findings on an overhead slide. The histograms with the largest averages were those for pepperoni, homework—no, and picture—yes. I mentioned that people who scored higher on a test were usually thought to be smarter than people with lesser grades. As a result, I announced that it was obvious from just looking at the averages that if a student ate pepperoni, didn’t do their homework, and liked to have their picture taken they were smarter than people who didn’t share these attributes. The uproar was all that I could have hoped for.

After everyone had calmed down I finished the session by emphasizing that "Extraordinary claims require extraordinary proof." I reminded the class that we had just made some astonishing claims and had not provided a single shred of proof. I explained that the proof or lack thereof would be the focus of the next three sessions.

Fourth Session—The Shape of Things—
The Elephant and the Blind Men

I began this session by telling the students that I was thinking about something and that I wanted them to guess what it was. I gave them the first hint—the object was gray. I asked them if this was enough information. They, of course replied that it wasn’t. When I asked why not they all pointed out that many things were gray and they proceeded to list a number of these items. I then gave a second hint—big—and we repeated the process. Finally after working our way through “gray,” “big,” “animal,” “lives in Africa,” and “has a snout” we had narrowed it down to a hippo or an elephant. When I added “the snout is flexible,” we reduced my item to an elephant.

I then told the story of the elephant and the blind men. The blind men were all describing the same object to one another but since each man had only one word or phrase to describe the elephant the end result was that no one understood what it was that they were touching. In order for the blind men to comprehend the elephant it was necessary for them to combine all of their word and phrases to form a description of the elephant.

I produced a picture of an elephant that I had drawn (Figure 1) and I asked them where the elephants “average” might be if we were to assume that by “average” I meant the point where I would have to lift if I was going to balance the elephant in my hand. I then produced a picture of the middle slice of the elephant and I announced that this what we had just described (Figure 2). I turned our discussion around and asked them if, by just looking at this slice, they could decide that they were looking at an elephant. The answer was, of course, that they couldn’t. I agreed and I pointed out that this slice could be anything—for example it could easily be the side panel on a truck (Figure 3).

I posted the total M&M histogram and the denim fabric histogram (see STN 49) and reminded the class that these very different looking distributions had the same average.
Consequently, histograms were like elephants—a single word or number does not provide an adequate description of either. Since this was the case, I asked the students what additional information they thought they would need in order to describe the two histograms to someone who had not seen either one. After some discussion about peak shapes we began to discuss the width of the distribution.

The graphs from last time were posted and I asked the students how they might get a measure of the width of the various distributions. I suggested that perhaps one way to do this would be to take the differences between the average and all of the individual data points and then add up these differences. To allow the students to do this in a timely fashion, I had made up a sheet for each distribution and I had already entered the numbers in the form of

\[ \text{individual data point - average = blank.} \]

The students had to fill in the blanks and add up the column of answers.

Since I ranked the data points from smallest to largest, the first answers were all negative numbers. I knew this would be a first and I had newspaper clippings of winter weather forecasts to help illustrate the fact that they already knew about negative numbers—now they just were going to have a chance to work with them. When they added up the column of differences they were a bit startled to discover that the answer was 0. Since the distributions obviously did not have zero width this was a puzzle. I told them that part of the problem was the negative numbers and that we would have to learn a new kind of addition which we would try next time.

During this discussion I made repeated references to the shape and size of the distributions and I talked about distributions with similar shapes and sizes. I took the graphs of two distributions that were drawn from the same lot of material and introduced the idea of comparing averages by looking at the overlap of their respective scatterplots. We treated the histograms as elephants and I moved these histograms back and forth on a common plot and asked the students to tell me when they thought one could say that a person was looking at two distinct elephants as opposed to an elephant and its shadow.

It was obvious that this last abstraction was taking some time to sink in but by repeatedly moving the distributions and talking about similar and different the class began to grasp the idea of similar or different distributions. While I could see that some of the students were beginning to understand the point I was making it was also obvious that I was going to have to revisit this issue in Lesson #5.

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**Faculty Development Workshop**

*From PRE-STAT*

*An NSF Faculty Enhancement Project*

**Where and When?**

Appalachian State University July 25–July 31, 1999, Boone, NC.

**Who is it For?**

College faculty who are involved with the preparation of middle or secondary level mathematics teachers and have an interest in improving the statistics education components of their curriculum. Previous participants have included faculty from mathematics, mathematics education, statistics and professional education.

**What is it About?**

The goal of the PRE-STAT project is to improve statistical education in the middle and secondary schools. The workshop focuses on statistical concepts, pedagogy and curriculum for these grade levels.

**What Does it Cost?**

Room and board, materials, and tuition are provided by the project. Participants must provide their own transportation.

**Where do I Get Additional Information?**

Mike Perry  
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Boone, NC 28608  
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E-mail: perryml@appstate.edu  
Web site: www.prestat.appstate.edu

Invitations may be offered as early as March 15, 1999, but applications will be accepted through June 15.
From the Editor...

Apologies! There was a typo in the Web site address I gave last month. It should be: www.bio.rtc.cf.edu/docs/ASA/stn.html. Note that the address is case sensitive. Check it out and give us some feedback on what you like and don't like about it. Past articles will continue to be added.

How terrific it is to see over 30 sessions devoted to statistics on the NCTM annual meeting program in San Francisco, April 22-24! Of particular note are three very exciting Conferences Within a Conference:

#53 AP Statistics for Beginners featuring Al Coons, Dan Yates, Jim Bohan, and Gloria Barrett.

#473 AP Statistics for Experienced Teachers with Al Rossman, Fred Djarg, Chris Olsen, and Paul Myers. Beth Chance will preside.

#935 Quantitative Literacy and Statistics Across the Grades K-12 and into AP Stats. Sue Kirby and Linda Young (elementary), Maria Mastromatteo and Jerry Moreno (middle school), Henry Kronendonk and Jeff Witmer (high school data driven math), Gretchen Davis and Roxy Peck (ap stats). Featured speakers are Dick Scheaffer and Jim Landwehr.

All are admission by ticket. Hope to see you at one of them!

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Keep Us Informed...

The Statistics Teacher Network is a newsletter published three times a year by the American Statistical Association—National Council of Teachers of Mathematics Joint Committee on Curriculum in Statistics and Probability Grades K–12.

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