The Statistics Teacher Network

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Experimental Design in the Classroom

Linking Mathematics and Science Curricula with Statistical Design of Experiments

The world is multivariate and interactive. Chemical reactions depend on kinds and amounts of reactants and contaminants, temperature, mixing procedures, pressure, and catalyst availability, to name just a few factors. Plant growth depends on genetic factors, soil type, fertilization, pH, amount and type of light, amount and frequency of watering, temperature, and so forth. The industrial processes that give us technological wonders from genetically engineered drugs to computer chips are devilishly complex. Drug production can involve dozens of carefully controlled procedures; chip manufacturing often involves hundreds of steps, each requiring precise control of many factors. Environmental problems are affected by myriad factors interacting in complex ways, some of which are under human control and some of which are not.

Also In this Issue...

Experimentation is a principal tool through which scientists and engineers investigate the natural world. Science teaching emphasizes the experimental method from the earliest elementary school science activities, and formal laboratories are an essential component of all high school science courses. Because science labs involve measurement and the collection of data to study real phenomena, they provide a natural framework in which to apply statistical/data analytical ideas. Unfortunately, often variability is ignored entirely (dealt with as mistakes), or classical statistical methods are applied in a mechanical and stereotypical fashion (example: fitting a regression line to a bivariate data scatter without looking at a plot to see if a line is appropriate). This frequently leaves both instructor and students confused about such mysterious procedures. Far too often, students resort to "data fudging" to make results agree with the "right" answer. A greater corruption of the goals of science and math education can hardly be imagined.

To deal with the many factors that can simultaneously influence phenomena under study, science students are taught that they must hold all but one factor constant and vary just one at a time to determine its effect. This strategy, sometimes referred to as "OFAT"—for **One Factor at A Time**—is standard practice from beginning science labs to Nobel prize research. Surprisingly, we have known since 1920 that it is a poor strategy: a much better way to experiment is to **simultaneously** vary many factors in carefully structured patterns. This strategy depends fundamentally on statistical ideas and is known as Statistical Design of Experiments—DOE, for short.

DOE not only allows proper study of many factors and their possible interactions, but it also does so in a much more **efficient** way than OFAT. This means that it explicitly considers and deals with experimental variability both in the design and analysis stage of experimentation. This gives both a more accurate representation of phenomena (because it allows for multiple factor interactions, while one-at-a-time strategies cannot), and it also produces more information at less cost. These issues are decisive in industrial work, and that is why DOE is being more and more widely used in industry.

Traditionally, DOE has been considered an advanced topic, usually reserved for upper level undergraduate or graduate college courses and often requiring strong statistical prerequisites. However, as with many statistical topics, this need not be the case. There is a core of simple, fundamental, and very useful ideas that can—indeed, we believe, **should**—be taught to science students from the time they begin formal laboratories in high school. For approximately the past three years, we have been working at the Macomb Math Science Technology Center (MMSTC) to do this by teaching both teachers and students basic DOE and to help them integrate DOE into their curriculum.

We believe that our work to date demonstrates that DOE **can** be successfully taught at the high school level. Students have shown that they can master and use the ideas by developing and carrying out their own independent experimental investigations. Teachers involved agree that DOE allows more rapid and effective treatment of "standard" topics and encourages both broader and deeper creative exploration that was not previously possible. Perhaps most important, DOE builds bridges between science and math/statistics, displaying the relevance of statistical thinking to practical scientific problems. It has allowed science teachers new flexibility to go beyond rote lab procedures that emphasize "right" answers to permit students to design their own studies, do their own data analyses, and develop their own scientific "models."

From the math/statistics perspective, DOE allows students to learn and integrate math and data analytic activities into the science curricula. Basic algebra, graphical analysis, dealing with variability (not on an abstract but on a real, tangible level), and developing and interpreting mathematical models are all an essential part of the DOE activities.

Although all this can be done without a computer—even non-programmable calculators are quite sufficient—it is a natural tool to use in this work. Appropriate software (simple spreadsheets or basic statistical packages will do—one could even use BASIC as part of a programming project), can be used for both design and analysis. It is also possible to develop computer simulations using real physical models that allow one to do experimentation on the computer. This helps to show how issues of measurement, equipment, and experimental variability can affect experimental outcomes and how DOE and statistical methods combat this variability. This emphasizes the natural and productive role of statistical thinking and mathematical modeling in scientific work.

Because DOE has not heretofore been taught at this level, we have had to develop our own approaches and materials from scratch. Needless to say, we have made many mistakes, have learned a great deal from them, and know that as we gain more experience we shall find many better ways to do things. Nevertheless, we have developed a clear idea of what can be done and experience about how to do it. We are now trying to put it together, organize it, and bring it to a publishable form for others to use. We are presently seeking grants to help us do this.

We are anxious to share our ideas. We expect to have a short (10-15 mins.) videotape describing the project and showing some of the student activities. Contact Bob Peterson if you are interested in obtaining a copy. If you are interested in learning more specifics or have thoughts or experiences to share, please contact any of us at:

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Statistics at All Grade Levels

Using Birthday Data to Integrate Statistics into the K-12 Mathematics Curriculum

There have been more recommendations for changes in the teaching of mathematics in the past few years than at any time since the "new math" era of the 1960s. One of those recommendations is that teachers incorporate more statistics into their classes. We suggest that one way to fit in new topics is to integrate them with old topics, so that a single lesson may serve a double purpose. We will illustrate this with examples. Since one of the strongest recommendations is to have students work with real data, we start by discussing one data set that your students can gather and use.

Young children are usually very interested in birthdays—especially their own! Even college students take pride in being able to report the exact time of their birth—and a surprising number are able to do so. The idea of having the students generate a data set that concerns themselves encourages participation and interest. Thus, you might collect birthday data from your class. The birth year will not prove interesting, as the range will be small for most K-12 classes. A class data set that includes month of birth, exact time of birth, day of the month, and cumulative day of the year will be more interesting. The cumulative day of the year may be obtained by counting, by using a calendar that includes that information, or by entering the dates into a spreadsheet. (Entering a constant year value for each birthday into a spreadsheet, you can use the Julian Date function to obtain the numerical day of year values.) If 2/29 is one of the birthdays, you could decide to designate the cumulative day of the year for this entry as zero (0), so that the number representing March first would be the same whether or not you are looking at a leap year. The remaining birthdays could be represented using values of one through three-hundred sixty-five.

Even in the first grade, textbooks use a number line to illustrate the addition, subtraction, or comparison of two numbers. The textbooks we consulted did not have the students doing much of this themselves, but we think it is important for children to *do* as well as see. If we have them put more than two numbers on a single number line, we are on our way to a dotplot. Having students make this display gives them practice in using a number line, something we wanted them to know about anyway, before we started trying to teach them statistics. For the birthday data, we could make dotplots of the day of the month.

Once we have the numbers on a line, we have also sorted the numbers. Dealing with the relative sizes of numbers helps students to grasp the meanings of numbers in a way that computation does not. There is a whole branch of statistics called order statistics that is based on ordering or sorting numbers more than on calculating with them. For example, the maximum and minimum values of a set of numbers would be examples of order statistics. So would the median (middle number in the sorted data), and other things such as quartiles, percentiles, and deciles.

By the end of first grade, children are doing single-digit subtractions. The difference between the maximum and minimum values in the data is called the *range*. It gives us an idea of how spread out the numbers are. Our suggestion is to combine current lessons on plotting numbers on a number line, comparing sizes of numbers, and simple subtraction, into a lesson where those same skills are practiced in the course of describing some data. For the birthday data, students should be able to estimate the range of most of the variables. If the months are treated as integers between 1 and 12, the range will usually be 11. For the day of the year, we can estimate the range, but not as accurately.

The above exercise can be repeated at higher grade levels as other kinds of numbers are introduced. Even some college students think that $0.2 < 0.05$ or $-2 < -5$, and are quite unsure as to how 4/7 compares to 11/19. They may also have difficulty finding 3.426 on a number line, so the extra practice may be worthwhile. The time of birth is the one variable in the birthday data that is not an integer. If times are quoted in hours and minutes, we can consider them as fractions with a denominator of 60. We can change these to decimal fractions of an hour. We can also look at this as a units change. In every case, we get rational numbers.

In addition to using dotplots, we found that stem-and-leaf plots usually prove quite interesting, not only for studying the pictorial characteristics of the data, but also the concepts of place value and digit truncation. For a stem-and-leaf of the day of the month, we pick stem units of 0, 1, 2 or 3 (representing the tens place) and leaf units of 0 through 9 (representing the units digits). The plot, if done by hand, will have the units values listed to the right of the appropriate tens value. Stem-and-leaf plots can be generated by many of the common statistical software packages as well.

When it comes to statistics that you calculate, the mean is probably the best known. Virtually all the college students we see can calculate a mean, so we figure it must be being taught, and taught well. Children can begin doing this as soon as they have the component skills, which are addition of a column of figures and division. We would like to see the mean not only calculated, but placed on a number line with the data. This helps to give insight into what a mean is, and also provides an error check. For the birthday data, we should also be able to estimate what the mean will be, since the variables all have an approximately uniform distribution. This provides practice in the art of estimating, but students should also see some examples where there is an element of surprise. There will be a bit of surprise in the birthday data for many students; unless you have a very large class, you are likely to observe significant departures from expectation in statistics like the mean or even the range. These can lead to a discussion of the idea of sampling error.

Once the mean is placed on the dotplot, it becomes possible to ask questions like "How many points below average was the 3?" or "Which observation was farthest from the mean?" This reinforces subtraction ideas and leads to the concept of a *residual*. The difference between an actually observed data value and a summary of the data (such as the mean) is called a residual. This can be used as an introduction to negative numbers, since no prior knowledge of these is required to put a "-" in front of the residuals for points that are below average. If the students have prior exposure to operations on negative numbers, they can add up the residuals and find that they sum to zero. Encourage them to check this out for other data sets and other summaries to see if it always happens. One way to generate negative numbers from the birthday data is to have the class (or each student) pick a "hero" or famous figure whose birthday they would like to find out. (We use "hero" here to refer to an admired person of either gender.) Let the students research the birthday, and then define for each child a variable that is the day of the year they were born minus the day of the year their hero was born. Can you estimate what the mean of this variable might be? Another calculated variable of interest might be the number of days until ones birthday. This could be a non-negative variable, or we could define it as number of days to your **nearest** birthday, with negative numbers indicating observations where the last birthday is closer to today's date than the next is.

If your students know how to multiply negative numbers, then they can square the residuals and add up the results. Dividing the sum by one less than the number of observations gives the variance. The square root of this is the standard deviation. The interpretation is that the standard deviation is *a typical value for the residuals*. Thus the standard deviation measures how variable or "spread out" the data are. We have some reservations about teaching young children about standard deviations because the computation is lengthy and not very intuitive. However, we expect that the topic is likely to be taught simply because lots of teachers already know about standard deviations. We think that some of the more visual topics such as dotplots are probably a better choice for children. If standard deviations **are** taught, we strongly prefer that the children use the method shown above that includes the residuals. They should learn to interpret the residuals in terms of a dotplot, and learn to interpret the standard deviation in terms of the residuals. In particular, we advocate abstinence from the various "computational" formulae that are floating about, as these tend to hide the residuals as well as the meaning of the process.

There are some more intuitive measures of variability based on order statistics. We have already mentioned the maximum and minimum as order statistics, and their difference, the range, is another measure of variability. Another common order statistic is the median. This is based on sorting the data and then selecting the middle value. We can take this one step further and find the first and third quartiles, which are just the medians of the upper and lower halves of the data. The difference between the third and first quartiles is called the *interquartile range* (IQR). It is yet another measure of variability. You should be aware that there are many inconsistent definitions of quartiles in use. Pick one you like and stick to it.

It is important to note that order statistics typically involve little or no computation. They are based rather on sorting the data and counting to find mileposts along our trip through the (sorted) data. The mathematics topics that they reinforce are ideas of order and magnitude rather than computational skills. Lest they seem to be of interest only for the early elementary grades, consider making a dotplot and finding order statistics for the following sets of data: 3, $\sqrt{10}$, π, 2.718, 3.14141414..., 22/7, $10/\sqrt{10}$, $10/\pi$ or 0.005, 0.011, 0.01, 0.1, 1/20, 1/19, 1/9, 1/11.

So far we have considered data that are primarily numerical. Since month is a categorical variable, one could report the total number of observations and the percentage of the total represented by each category (month). A study of these percentages is usually interesting. A bar chart is the typical choice for displaying categorical data, but a dotplot differs only in artistic detail. Note that the distribution of the months may not be very uniform for small data sets. Further work for younger students might include exercises dealing with the calendar such as the order of the month names, or the length (in days) of particular months.

So far we have considered one variable at a time. We can also look at relations between variables by plotting one against the other. You might try day of month versus day of year, and day of month versus month, both of which generate interesting patterns that may appear to be identical or to be no pattern at all until you get an appropriate scale. If your class were to choose a single hero to compare their birthdays to, these data can be plotted against day of year.

In precalculus, we often compare the graph of f(x) with the graph of f(x+a), af(x), or f(ax). We can do the same with data. We can shift the day of year data by taking something other than 1 January as our starting point—say the first day of school or the first day of summer vacation. We can multiply I(y) or I(x) by a constant by changing units—we could use time of birth in hours or minutes or we could express day of year as decimal fractions of a 365-day year. At least one of the patterns should be found to be approximately periodic. It might make a good example for a trigonometry class, especially as it looks quite different from any trig function.

For all the things we have suggested, the apparent pattern may change as we increase the number of observations. You can increase your database by pooling birthdays from past classes or by pooling the data from several different current classes. You can also open it up to family members, Presidents of the United States, or all the teams in the NFL—whatever appeals to your class. To make a bridge to calculus and college, you can ask students to discuss how some of the graphs might look if you let the sample size increase without bound.

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Book Review

Exploring Measurements (Teacher Edition)

by Peter Barbella, James Kepner, and Richard L. Scheaffer
Palo Alto, California: Dale Seymour Publications, 1995.
$15.95, 37 + 120 pages, spiral bound paperback.

Exploring Measurements is the fifth book in the "Quantitative Literacy Series", published by Dale Seymour. The teacher edition contains introductory material, quizzes, answers to quizzes, and an annotated version of the student edition.

When I was asked to review the newest addition to the QL family my first thought was: Do students need to know the stuff in the first four books before *Exploring Measurements*? After reading the book, the answer to my question is "well, sort of".

As the authors state in their overview, *Exploring Measurements* makes use of graphical and numerical summaries of data, and relies heavily on the concept of simulation. Therefore students need to have some prior experience exploring and producing data. Two of the books in the original QL series, *Exploring Data* and *The Art and Techniques of Simulation* contain virtually all of the prerequisite information required to handle the new fifth book, but it is not required that students obtain their experience from the QL series. Any solid grounding in displays, averages, and some hands-on coin flipping or dice rolling activities would most likely be sufficient.

A third original member of the QL Series is *Exploring Surveys and Information from Samples*. Many of the concepts in *Exploring Surveys* are also contained in *Exploring Measurements*. The newer book emphasizes measured variables, for which center and spread make sense, as opposed to the counts and proportions found in *Exploring Surveys*.

The remaining original member of the QL Series is *Exploring Probability*. Probability certainly lies at the foundation of simulation and the notion of a sampling distribution, but the manipulative skills developed by *Exploring Probability* are not essential to *Exploring Measurements*.

Because of the prerequisite experience with elementary data analysis, and the strongly statistical nature of the material, *Exploring Measurements* is best suited as a text or supplement in a statistics course, probably at the tenth grade level or higher. If chosen carefully by the teacher, individual activities from the book may also be used to motivate or reinforce concepts in courses ranging from Algebra I to college level statistics.

Exploring Measurements is divided into three sections: "Summarizing a Data Set", "Patterns that Arise in Repeated Sampling", and "Estimating an Unknown Population Mean". A broad outline of the topics in the book includes: measuring center, measuring spread, computing means and standard deviations, random samples, distributions of means, and estimating means. I think the level of the book is appropriate for high school students, and I especially like the development of some statistical topics that have been traditionally hard to explain. For example, the material on "Estimating the Variation in a Population" is very clear, although the authors call it optional since it is not required to understand the remaining topics in the book. I also found the explanation of population parameters through the use of "ideal" samples fascinating. Throughout *Exploring Measurements*, the authors use clear graphical displays to get their points across and also provide easily understood explanations of quite difficult statistical concepts.

Like the earlier QL Series books, the material in *Exploring Measurements*, flows in and around Applications which involve active participation by the students. The realistic nature of the activities will keep the students interested and create opportunities for cross-disciplinary connections as well. Some of the applications in Section III: "Estimating an Unknown Population Mean" in particular, put students in position to compute useful estimates and make informed decisions based on what they learn from data.

One of my favorite examples is the page full of rectangles accompanying Application 17. Students take samples of the rectangles, initially using their own judgement and eventually choosing the rectangles randomly, with the goal of estimating the average of the areas of rectangles on the page. I have used this example in my own courses with great success. It provides an extremely clear illustration of sampling variability and bias.

There are some minor problems with the book. First, many of the questions are intentionally open ended. Although solutions are provided, many of them contain the words "answers may vary". While such exercises are certainly realistic, they can be hard for teachers to handle unless they really have the material at their fingertips. Some of the exercises contained in the discussion of means and medians are interesting because students are asked to decide which of the two measures of center is more appropriate. I would have liked the authors to first ask the students if measuring center makes sense at all for a particular dataset, and then decide which measure of center would be best. I was a bit confused by Applications 11 and 12, which involve comparing two columns of information after converting them to standardized scores. The data in these applications would be better analyzed using proportions or scatterplots, in my opinion. The shortcomings of the book are minor compared with the potential benefit of students learning the concepts (not just the formulas) of statistical analysis. I heartily recommend *Exploring Measurements*, to anyone who wants to teach students to summarize and simulate measurement data. The realistic examples and lucid explanations make *Exploring Measurements*, following in the footsteps of the original QL Series books, a great resource for high school students who want to learn statistics.

Reviewed by Tom Short
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Software Review

DataScope and Prob Sim

Although several software products are available to assist students in entering and analyzing data, few programs have been developed to help students develop a conceptual understanding of statistics. Cliff Konold at the University of Massachusetts, Amherst, received an NSF grant a few years ago to develop software and curriculum materials for secondary level students to use in learning basic concepts of probability and statistics. Assisted by Craig Miller, he produced two programs: DataScope and ProbSim, now available from Intellimation Library for the Macintosh.

DataScope is an easy to use package that offers a spreadsheet for entering data and tools for representing and analyzing data. By clicking at the top of a column of data, a variable may be selected for individual analysis or used as a grouping variable to analyze a different variable or variables. It is very simple for students to generate multiple histograms or boxplots to compare groups of data, to identify interesting points or outliers, or to analyze bivariate data using scatterplots and fitting regression lines. Although this program does not offer the sophisticated techniques found in Minitab, Data Desk or other programs used in college classes, its advantage is that it takes minutes to learn, is very simple to use, and encourages students to judge relationships by examining different visual representations of data. The curriculum materials provided with DataScope include five one-week lab units which help students formulate questions and explore four data sets that are included with the software.

Prob Sim was designed to help students develop an understanding of probability concepts by helping them construct models to represent different probability problems, select parameters, run a random generator to produce simulated data, and analyze the results.Four one-week lab units are offered with teaching materials that support students as they learn to build models to use in generating and analyzing data.

I have used both of these programs in my introductory college statistics classes for several years while the programs were being developed, and have found them to be an excellent way to get my students, who are often computer novices, quickly involved in entering and analyzing data. I have used many of the lab activities written by Konold to help students develop important concepts of descriptive statistics and probability.

At the beginning of my course, students usually design a survey which we then use to collect a large set of data from the class. These data are entered on DataScope and used by students as we learn about different topics in statistics. Using the class data, students use DataScope to explore different features of histograms and to determine the relative advantages of displaying data sets in histograms, tables, or boxplots. They use histograms and boxplots to compare groups of students based on different variables (e.g., gender, year in school, those who plan to go to graduate school, etc.).

Later in the course we use Prob Sim to model a variety of problems involving random generators such as coins, dice, and cards. I often have students first use these devices in class, such as tossing 5 coins 25 times and counting the times they get different numbers of heads. Then we move to the computer and students quickly appreciate the power of the computer in easily modeling and simulating data for thousands of trials of the same experiment.

In a recent article in Mathematics Teacher, "Teaching Probability through Modeling Real Problems" (April, 1994) Konold gives some examples of activities using ProbSim. He describes the "One Son" problem, based on the Chinese policy of limiting each family to one child. Students are asked to speculate on what should happen if each family was allowed to keep having children until they had one son. After making predictions about the average number of children in a family and the ratio of births of girls to boys, students set up a model and generate data to estimate the answers to these questions. The results are counterintuitive, and often lead students to try to better understand what is going on in this problem.

These two Macintosh programs may be purchased from Intellimation (phone: 800-346-8355). The price for "DataScope: A data analysis tool for testing statistical concepts and validations" is $85.00 and "Prob Sim: The probability teaching tool" is sold for $59.00. Together the two programs are being offered for $125.00 or $549.00 for a site series.

Reviewed by Joan B. Garfield
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From the Editor

Application forms and information for the sixth annual poster and ninth annual project competitions are available from Cathy Crocker, Director, ASA Center for Statistical Education, 1429 Duke Street, Alexandria, VA 22314-3402. The deadline for entries for both competitions is April 15. Encourage your students to enter, not only in hopes of winning a prize but as excellent hands-on statistics activities that bring additional life and meaning to classroom material.

The video tape that was mentioned in the first article on design of experiments of this issue is available by contacting Bob Peterson. I viewed it recently and although it does not go through the details of any particular experiment, it is very effective in describing why design of experiments is the orientation that should be followed in bringing statistics into the mathematics and science school curriculum. I found it to be very interesting.

Please don't hestitate to send articles that you would like to share on successful classroom statistics activities. Also, if you are interested in reviewing books or software for STN, let me know. Thanks.

Keep Us Informed...

The Statistics Teacher Network is a newsletter published three times a year by the American Statistical Association—National Council of Teachers of Mathematics Joint Committee on the Curriculum in Statistics and Probability.

We need your letters, announcements, articles, and information about what is happening in statistics education! Please send hard copy, and, if possible, a disk written in standard ASCII text to the editor:

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