MULTIVARIATE STRATIFICATION AND SAMPLE ALLOCATION SATISFYING MULTIPLE CONSTRAINTS FOR SURVEYS OF POST-OFFICE MAIL VOLUMES

1. BACKGROUND
The U.S. Postal Service (USPS) conducts two nationwide surveys to continuously determine mail volumes and other mail characteristics by classes and subclasses of mail. Both surveys select samples on a quarterly basis. One of these is the Revenue, Pieces, and Weights (RPW) survey, which produces national-level estimates for a large number of mail classes and subclasses. RPW has a two phase design. In the first phase, the Primary Sampling Units (PSUs) are individual post offices; and in the second phase, the secondary sampling units are physical locations where clusters of mail can be subsampled before exiting the USPS. The other survey is the Origin-Destination Information System (ODIS), which produces subnational estimates for a smaller number of mail classes and subclasses as well as the length of time from when a piece of mail is postmarked until it has been through the final processing before delivery. ODIS has a single stage design, with the PSUs being physical locations where clusters of mail can be subsampled before exiting the USPS.

For both surveys, the universe or population of interest is all mail being delivered by the USPS. In 1994, the USPS repartitioned the universe of mail into Mail Exit Points (MEPs). The MEP-based frame is shared by both RPW and ODIS. MEPs represent the secondary sampling unit in RPW and the PSU in ODIS. Information available for each MEP in the universe is the approximate expected daily volume by mail shape, the expected daily volume of Priority mail (a specific mail class), and special service mail—called accountable mail. Also available are the estimated data collection time and travel time for each MEP. The mail shape volumes are used to predict specific mail class volumes of interest. These predicted mail class volumes along with the data-collection-time estimates provide the opportunity to utilize multivariate stratification and allocation techniques to improve the efficiency of both RPW and ODIS.

This paper describes our work in stratifying the MEP sampling frame and determining the stratum sample sizes for RPW and ODIS. Our general approach consisted of the following steps:

Step 1. Predict average daily mail class volumes for each MEP.
Step 2. Create clusters of MEPs based on predicted mail class data and MEP-level cost data.
Step 3. Combine clusters to form strata.
Step 4. Predict stratum variances among sample units.
Step 5. Determine stratum sample sizes via multivariate optimal allocation.

Section 2 of this paper defines additional USPS terminology. Section 3 discusses stratification of the MEP sampling frame (steps 1 through 4), and Section 4 discusses sample allocation (step 5). Section 6 presents our conclusions.

2. TERMINOLOGY
USPS delivers mail between different geographical locations. Consequently, geography is an important element of USPS's management information systems. These systems produce reports at various levels of a hierarchical geographical partitioning of all of USPS's post offices and processing facilities. The lowest level of this hierarchy is an ODIS area, consisting of a small group (possibly one) of geographically contiguous 3-digit ZIP Code areas. There are 733 ODIS areas. Based on patterns of mail distribution, the ODIS areas make up 197 plant-areas, which USPS has organized into 87 Customer Service Districts (CSDs). In turn each CSD is assigned to one of 10 Customer Service Areas (CSAs).

USPS also categorizes Post Offices with respect to the amount of revenue they generate. This is done by means of a Cost Ascertainment Grouping (CAG) code, which groups post office revenues in descending alphabetical order from A (largest post offices) through J (smallest post offices). The RPW sampling procedure, in turn, groups CAGs into three super-CAGs: A-and-B, C-and-D, and E-through-J.

ODIS and RPW sample both locations within post offices and delivery days—that is, days that mail can be delivered. The delivery days for a four week period constitute an Accounting Period (AP), which in turn are combined into Postal Quarters (PQs). PQ1, PQ2, and PQ3 each consist of three APs, and PQ4 consists of four APs.

3. STRATIFICATION
This section describes the procedure we used to stratify the MEP sampling frame. Section 3.1 describes the stratification variables. Our evaluation of the described...
stratification procedure on test data sets produced candidate stratifications in which some strata contained a very small number of MEPs. This led us to developing an additional method of selecting sample units, which we describe in Section 3.2. The final step in the stratification procedure (called Step 4 in Section 1) is the prediction of stratum variances among sample units. Because Step 4 controls Step 3 (combining of MEP clusters, however, we discuss Step 4 in Section 3.3 and then discuss Steps 2 and 3 in Section 3.4.

### 3.1. Stratification variables

In developing the MEP sampling frame, USPS staff recorded the following information for each MEP:

- $\bar{X}_i^{(1)}$: estimated average daily volume of Priority mail in MEP i
- $\bar{X}_i^{(2)}$: estimated average daily volume of accountable mail in MEP i
- $\bar{X}_i^{(3)}$: estimated average daily volume of letters and cards in MEP i
- $\bar{X}_i^{(4)}$: estimated average daily volume of flat mail in MEP i
- $\bar{X}_i^{(5)}$: estimated average daily volume of parcels (that can be machine processed) in MEP i
- $U_i^{(1)}$: indicator (1=yes, 0=no) of whether MEP i is an originating RPW unit
- $U_i^{(2)}$: indicator (1=yes, 0=no) of whether MEP i is an APO/FPO (i.e., overseas-military mail) unit
- $U_i^{(3)}$: indicator (1=yes, 0=no) of whether MEP i is a special delivery unit
- $C_i^{(test)}$: estimated test time for MEP i (recorded to the nearest tenth of an hour), and
- $C_i^{(travel)}$: estimated travel time for MEP i (recorded to the nearest tenth of an hour).

The MEP information variables $\bar{X}_i^{(s)}$, $s=1,...,6$, break down the average daily mail volumes in a MEP by shape class, whereas the objective of ODIS and RPW is to estimate average daily mail volumes by mail class. Some mail classes are shape based, however, and for those that are not, ODIS records both shape and mail class of each sampled piece of mail. Consequently, we were able to predict for a MEP its average daily volumes by mail class as follows:

- $\tilde{Y}_i^{(1)}$: predicted average daily volume of mail in MEP i
- $\tilde{Y}_i^{(2)}$: predicted average daily volume of accountable mail in MEP i
- $\tilde{Y}_i^{(3)}$: $s=3,4,5$.

where

- $r = 3$ for First-Class mail,
- $r = 4$ for parcel post mail,
- $r = 5$ for other fourth class mail,
- $s = 3$ for letters and cards,
- $s = 4$ for flat mail,
- $s = 5$ for parcels that can be machine processed,
- $s = 6$ for irregular parcels, and
- $\pi_s$: he proportion of shape class s mail that is mailed at mail class r.

We estimated $\pi_s$ from historical ODIS data, using the combined estimate for a ratio (i.e., ratio of two totals, each based on simple expansion). For ODIS, we combined $C_i^{(test)}$ and $C_i^{(travel)}$ (both measured in hours) into a single measure of data collection cost (measured in days) as follows:

$$C_i^{(ODIS)} = \frac{\pi(C_i^{(test)}) - 0.5 \cdot FLOOR \left[ C_i^{(test)} / 3.5 \right]}{C_i^{(test)}}$$

where $\pi(C_i^{(test)})$ is 0 or 1 depending on whether $C_i^{(test)}$ is less than or greater than 2.5 hours, and FLOOR(x) is the smallest integer greater than or equal to x.

For ODIS, the MEP stratification variables were $\bar{X}_i^{(1)}, \bar{X}_i^{(2)}, \bar{X}_i^{(3)},$ and $C_i^{(ODIS)}$. For RPW, the MEP stratification variables for MEPs in first-phase-unit post offices were as follows:

- If any of the unit-type indicators--$U_i^{(1)}, U_i^{(2)}$, or $U_i^{(3)}$--was equal to 1, the MEP was assigned to an associated pre-specified stratum.
- Otherwise, the stratification variables were $\tilde{Y}_i^{(r)}$, $r=1,...,5$, and $C_i^{(test)}$.

### 3.2. P and Q sampling

Prior to the availability of the MEP sampling frame, the delivery-unit day was the sampling unit for ODIS and the second-phase sampling unit for RPW. We carried this method of sampling, which we call P sampling, over to the MEP sampling frame--replacing a delivery-unit day with a MEP day--with one exception. The exception is the method of sampling, called Q sampling, used in what we call the Q stratum.

To explain the difference between P and Q sampling, we need some notation. Let:

- $M_h$: the number MEPs in stratum h of the MEP sampling frame,
- $D$: the number of delivery days in the Accounting Period,
- $N_h$: the total number of MEP days associated with stratum h of the MEP sampling frame, and
- $n_h$: the number of MEP days sampled from stratum h of the MEP sampling frame.

For P sampling, $1 < n_h < N_h$ and the sample unit is a MEP day. Thus, the same MEP can be selected more than once for mail testing on different days. For Q sampling, on the other hand, $n_h = N_h$, the sample unit is a delivery day, and the MEPs serve as second-level strata. One delivery day is selected from each and every MEP in...
The advantage of Q sampling is that the sampling variability contains only variability among days and does not contain variability among MEPs. The disadvantage is that if $M_h$ for the Q stratum is large, then the Q-stratum sample size (i.e., $n_h = M_h$) may be larger than necessary. Thus, there exists $M_h^\ast$ such that if $M_h < M_h^\ast$ then Q sampling will be more efficient than P sampling.

### 3.3 Prediction of stratum variances among sample unit

After we stratified the MEP sampling frame (described in the next section), we predicted stratum variances among sample units for each mail class. Because the data used for stratification, however, were MEPs' predicted average daily mail volumes, using the stratification data to calculate stratum variances would not have accounted for all the variation among MEP-days. Based on experience, USPS's statistical staff expected that within a MEP the coefficient of variation of daily mail volumes by mail class would be approximately 0.7. Strand (1994) shows that this allows one to predict approximately the stratum variances among sample units. For each major estimation variable, we calculated for P-sampling strata:

$$\delta_{h}^2 \text{ - predicted variance among MEP days in stratum } h$$

$$= \frac{1}{M_h} \left( \sum_{k=1}^{M_h} (c_h-1)Y_{h, k}^2 - \frac{1}{M_h} \left( \sum_{k=1}^{M_h} Y_{h, k} \right)^2 \right)$$

and for Q-sampling strata:

$$\hat{\delta}_{h}^2 \text{ - predicted average variation among days within MEPs in stratum } h$$

$$= \frac{1}{M_h} \sum_{k} c_h Y_{h, k}^2$$

where $c_h = 0.7$ is the coefficient of variation, and the subscript $(h)$ has been added to the stratification data to denote assignment to stratum $h$.

### 3.4 Strata Construction

We used multivariate clustering methods to stratify the MEP sampling frame. Our approach was similar to that of Julien and Maranda (1990), who stratified the list frame for the Canadian National Farm Survey by doing the following:

- Normalize each stratification variable to a mean of zero and a variance of one.
- Apply k-means clustering to the multivariate normalized data to create a large number of clusters. In these clusters, sample units are similar to other sample units in the same cluster but dissimilar to sample units in different clusters.
- Use Ward's clustering algorithm to combine clusters into strata.

Ward's algorithm is a multi-step, agglomerative clustering procedure that starts with $n$ clusters and at each step selects two clusters to combine, so that after $n-1$ steps there is only one cluster. The two clusters that the algorithm selects at each step to combine are those that minimize the resulting within-cluster variability.

When we applied Ward's algorithm to clusters of MEPs, each step of the algorithm produced a candidate stratification. We then selected one of these stratifications on the basis of the estimation variances resulting from a fixed-cost optimal allocation to the associated strata. This is explained in more detail below. The reason we performed k-means clustering once and then collapsed to various candidate stratifications was that the k-means clustering is computationally intensive, whereas agglomerative clustering of the k-means cluster means is not.

**ODIS strata.** For ODIS, we separately stratified each of the ten CSAs (in PQ2/FY95 and PQ3/FY95) or each of the 197 plant-areas (in PQ4/FY95). In each such area, we applied k-means clustering to the four ODIS stratification variables to create 50 clusters. When clustering available test data sets (that were derived from the delivery-unit frame and not the MEP frame), we discovered that a small number of the resulting clusters contained a large number of sample units, and many clusters contained very few sample units. Moreover, the clusters containing very few sample units often did not combine with other clusters until the number of strata was quite small.

Consequently, we decided to assign to the Q stratum those clusters that contained a number of sample units less than a prescribed number, called the Q-stratum cut-off. Different values of the Q-stratum cut-off and different numbers of strata after combining clusters resulted in several candidate stratifications. We compared these stratifications in terms of the sampling variances for the area-level (CSA or plant-area, depending on area-level of clustering) estimated total mail volumes for the three major estimation variables when there was a fixed-cost optimal allocation of tests to the candidate strata.

We used the following formula to calculate the candidate sampling variances:

$$\hat{V}_r = \left( \sum_{h} \frac{N_r^2}{n_h} - N_r \right) \hat{\delta}_h^2$$

where

$$\hat{\delta}_h^2 \begin{cases} 
\delta_{h}^2 & \text{for mail class } r, \text{ if stratum } h \text{ is not the Q stratum ,} \\
\hat{\delta}_h^2 & \text{for mail class } r, \text{ if stratum } h \text{ is the Q stratum .}
\end{cases}$$

Because we were interested in making plant-area estimates, we then intersected the cluster-based strata
with plant areas to create, what we call, allocation strata.

**RPW Strata.** For RPW, the cluster-based strata resulted from clustering six stratification variables. The allocation strata were the cluster-based strata intersected with CSDs.

### 4. Allocation

**ODIS allocation.** For ODIS, the USPS statistical staff specified target precisions for First-Class, Priority, and parcel post for each plant-area. They also specified a maximum for total data collection costs for each plant-area. In the event that it was not possible to achieve the target precisions without exceeding the specified maximum total costs, USPS was willing to modify the specified precisions.

To determine the sample sizes for the ODIS allocation strata, we used a modified version of the SAS program CHROMY_GEN, described by Zayatz and Sigman (1995). The CHROMY_GEN program implements the algorithm described by Chromy (1987) for finding the minimum-cost stratified sample that satisfies multiple precision constraints. Chromy’s algorithm is iterative, and the CHROMY_GEN program stops iterating based on a stopping rule described by Causey (1983). Another feature of the CHROMY_GEN program is that it calculates for each precision constraint the 10% shadow prices (described in Bethel (1989)), which indicate the approximate decrease in survey cost resulting from a 10% increase in the constraint’s target coefficient of variation.

The modified version of CHROMY_GEN that we used for the USPS work allowed for the definition of cost-constraint domains. These were collections of strata associated with a specified maximum domain cost, $C^*$. The modified CHROMY_GEN program first performs the calculations in the unmodified program; next calculates the total cost, $C$, for each cost domain; and then reduces the domains’ sample sizes from $n_i$ to $(C^*/C)n_i$ if $(C^*/C) < 1$.

For PQ2/FY95, we ran the modified CHROMY_GEN program for ODIS with 591 precision constraints, 197 cost constraints, and 2,140 strata. This number of constraints and strata resulted from enforcing one cost constraint and three precision constraints (for first class, priority, and parcel post) in each of the 197 plant areas. The resulting sample size was 32,385 mail tests. All of the cost constraints were satisfied by either using the cost-domain feature of the modified program or by modifying some of the precision constraints based on the shadow prices. All the precision constraints were also satisfied (based on the predicted variances), though some had to be modified to satisfy cost constraints.

**RPW allocation.** The following were the differences between the allocation procedures for ODIS and RPW:

- For RPW, the USPS statistical staff specified target precisions and maximum costs at the CSD level, whereas for ODIS this was done at the plant-area level.
- The allocation strata for RPW were the cluster-based strata and the prespecified strata intersected with the CSDs. Unlike the procedure for ODIS, we predicted for RPW the stratum variances for each allocation stratum. In calculating these predicted variances, we weighted the data by the first-phase sampling weights. (See Section 5.3.)

For PQ2/FY95, we ran the modified CHROMY_GEN program for RPW with 435 precision constraints, 87 cost constraints, and 1,824 strata. The resulting sample size was 13,153 mail tests.

### 5. Survey-Design Operations for PQ2/FY95 Through PQ4/FY95

**5.1 Prediction of MEP Average Daily Volumes by Mail Class and Prediction of Strata Variances (Steps 1 and 4)**

**ODIS.** The ODIS data from AP4/PQ2/FY95 were analyzed to compare predicted stratum variances to the strata variances from the sample. This early analysis was conducted in preparation for the PQ3/FY95 sample selection process. The variances were fairly comparable, except for strata populated by only one frame unit at the plant-area level and the strata for military gateways, which are special plant-areas having different mail characteristics than other plant-areas. Generally, the sample data indicated that the strata CVs for First-Class mail were slightly lower than the assumed 0.7. Priority Mail CVs were close to 0.7, and parcel post CVs were slightly larger. Starting in PQ3 and continuing through PQ4, we assumed a CV of 0.6 for First-Class, 0.7 for Priority Mail, and 0.85 for parcel post mail. This had the effect of allocating more tests to MEPs with priority and parcel post mail. PQ2/FY95 data were analyzed to compare MEP-level mail volumes estimated from sample data to the predicted mail volumes used in stratification. Each of the predicted mail-shape and mail-class MEP volumes was plotted against the sample mail-shape and mail-class MEP volumes by strata and by plant-area, resulting in the expected linear relationship.

In general, the methodology described in section 3.1 and 3.3 to predict MEP volumes and strata variances seemed to work effectively.

**RPW.** The same modifications to the expected strata CVs were made in RPW beginning with the PQ3/FY95 sample selection process.

**5.2 Creating Clusters Using MEP Predicted Mail Class Volumes and Cost Data and Combing Clusters to Form Strata (Steps 2 and 3)**

Ongoing repartitioning of the universe of mail on the sampling frame made it desirable to re-stratify the population of MEPs each postal quarter. This helped ensure efficient stratification.

**ODIS.** In PQ2 and PQ3 we clustered at the CSA level and set the number of cluster-based strata for each
were developed at the CSD level without regard to the management problems. In PQ4, cluster-based strata could fluctuate, sometimes significantly, from one postal quarter to another, which created significant Workload imbalances between assigned workload and available data collection resources. Maximum cost constraints were decreased until the number of tests was practical for each plant-area.

Though the allocation results indicated decreases in sample sizes for a number of plant areas, we constrained the magnitude of decreases in sample size for reasons of staffing, providing the USPS with sufficient diagnostic information, and predicting strata variances. The sample selection procedure for PQ2/FY95 involved modifying the allocation software to allow a decrease in the sample size for a plant-area to 95% of the respective maximum cost constraint. After analyzing the sample data from AP4 of PQ2/FY95 with regards to strata variances, decreases in sample sizes were set to 85% of the respective plant-area's maximum cost constraint for PQ3 and PQ4 sample selections.

The minimum number of iterations required for the Chromy algorithm was 10. After 10, there were no significant changes at the strata, plant-area level to...
The allocation process distributed more tests away from panel offices in the major cities (generally CAG A and B offices) and into panel offices in more remote locations (generally CAGs C through J offices) in the CSD. This was opposite the effect obtained in the ODIS allocation. This resulted from using adjusting predicted mail-class volumes adjusted by their first phase inflation factors to calculate predicted strata variances resulting from the two-phase design.

The allocation procedure is a little more difficult in RPW in that there are two groups of MEPS processed, the special MEP types in prespecified strata and all other MEPS. The maximum cost constraints were reduced by the appropriate amount to process the non-special MEP type group. The results of the two allocation processes are merged back together and any final adjustments necessary are made before the selection of individual MEPS.

6. RESULTS AND CONCLUSIONS

For ODIS, plant-areas which have developed many MEPS around single mail shape processing streams have improved or maintained their level of precision with a reduction in sample size. Sample sizes have been decreased approximately eight percent nationally. However, the total number of mail pieces recorded from sampling in a postal quarter has increased without incurring added data collection cost. With the implementation of the construction of cluster-based strata at the plant-area level and as additional plant-areas develop more single mail-shape MEPS, indications are that sample sizes can be decreased 15 to 20 percent nationally over recent historical levels while maintaining current precision levels. Conversely, precision may be increased while holding data collection costs constants, which is a more likely scenario considering the large number of management functions dependent upon ODIS data.

For RPW, precisions of the national estimates for mail class and subclass estimates have been maintained with a reduction in the sample size of approximately three percent. Similar to ODIS, increasing the development of single mail-shape MEPS along with the construction of cluster-based strata at CSD level should improve precision with the current level of mail testing. Furthermore, we now have the flexibility of targeting mail tests to specific mail classes, such as Priority Mail, if different precision requirements are desired, while controlling sample size at the CSD level.

Though there is strong evidence that the benefits of the new stratification and allocation strategy has improved the mail class estimates, this has not been universally realized in all plant-areas and CSDs. There are two major reasons for this:

• First, the extent to which MEPS have been developed around single shape mail processing streams varies among plant-areas and CSDs. Mail shape (e.g., letter and parcel shape mail) is correlated to mail class. Where it is operationally feasible from the data collection perspective, MEPS are to be designed so that predominately one mail shape is being tested. This allows strata to be developed more efficiently around mail class volumes, since MEPS developed directly around specific mail class volumes is not often feasible.

• Second, for ODIS the CSA stratification is not efficient for all plant-areas within each CSA, and for RPW national stratification is not efficient for all CSDs due to geographic and hence mail processing differences.

The first issue is aggressively being worked on by the USPS through continued communication and education with plant-area personnel, resulting in frame updates. The second issue has been resolved by changing the strategy of stratification in PQ4/FY95 from the CSA to the plant-area level for ODIS and national by super CAG to CSD level for RPW.

We conclude that the development of a MEP-based frame for both ODIS and RPW is producing benefits from the data collection perspective and also in precision of the reported estimates. Key to realizing these benefits is the flexibility that the stratification and allocation software provide in adapting to different MEP compositions geographically throughout the USPS.

7. REFERENCES