

A Classification of Fallacious Arguments and Interpretations*

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This attempt to cover mathematics, discussion, probability, etc., in one classification makes it seem likely that a natural linear ordering is impossible. Hence the cross-references.

In Part 2, I have made much use of the list of 'dishonest tricks' in Appendix I of Ref. 3. Dr. Robert H. Thouless and English Universities Press have most generously granted permission for this classification to be published. I am also indebted to the Admiralty for permission to publish.

- 1 Fallacies in pure mathematics. (See Refs. 1 and 2.) (Presumably all subclasses of 2 are potentially applicable here.)
 - 11 Misleading geometrical diagrams.
 - 12 Misunderstanding of a word. (Cf. 228 below.)
 - 13 Misunderstanding of a rule, e.g. by ignoring its conditions of validity. (Cf. 24 below.) (E.g. $0 = \log 1 = \log (-1)^2 = 2\log (-1) = 2\pi i$.)
 - 14 Misunderstanding of a notation. (E.g. (i) $c(a + b) = ca + cb$, therefore $f(a + b) = f(a) + f(b)$, (ii) What does $\sum_{i < j} a_{ij}$ mean?)
 - 15 Relying too much on *scientific* induction (in pure mathematics.)
- 2 Fallacies in discussion apart from probability. (See Refs. 3 to 12.)
 - 21 Psychological.
 - 211 Suggestion.
 - 211.1 By repetition.
 - 211.2 By implication of prestige or of relative prestige.
 - 211.21 By confident manner.
 - 211.22 By prestige.
 - 211.23 By pointing out that one's opponent is unqualified or bad. (Not relevant to his logic though it is to the reliability of his testimony. Likewise in 211.22. Headings 211.23 and 211.25 are arguments 'ad hominem'.)
 - 211.24 By false credentials.
 - 211.25 By falsely claiming that one's opponent is unqualified or bad.

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- 211.26 By 'blinding with science' (e.g. 'figures prove', photographs of bearded scientists with test tubes, slide rules and white jackets.)
- 211.27 By affectation of failure to understand.
- 211.3 By appeal to authority alone, unbacked by reason. (Francis Bacon's *Idola Theatri*.)
- 212 Emotional reasoning.
- 212.1 Angering an opponent to make him argue badly.
- 212.2 Overcoming resistance to a doubtful proposition by stating a few that are easily accepted emotionally.
- 212.3 Statement of doubtful proposition in such a way that it fits in with the thought-habits or prejudices of the hearer.
- 212.4 Commending or condemning a proposition because of its practical consequences to the hearer, when the public good is supposed to be under discussion.
- 212.5 Special pleading. (Use of an argument, otherwise unacceptable, that supports one's own case.)
- 212.6 Special counter-pleading. (Denial of an argument, otherwise acceptable, that supports one's opponent's case.)
- 212.7 Use of emotionally toned words. (I am firm, you are obstinate, he is pigmatic. 'Irregular conjugation'.)
- 22 Linguistic
- 221 212.7
- 222 Emphasis of wrong word in a sentence ('accent'.)
- 223 Failure to distinguish between verbal and factual questions.
- 224 Forgetting that each person talks a slightly different language.
- 224.1 Figure of speech interpreted literally.
- 225 'Taking-for-granted-ism', or the 'expositor's fallacy'. (The familiar assumption by an expositor that what is familiar to him is also familiar to his audience.)
- 226 Putting forward a tautology (such as that too much of a thing is bad) as if it were a factual argument.
- 227 Making a statement in which 'all' is implied, but 'some' is true. (E.g. 'red-headed people have bad tempers'. A quantitative statement is usually preferable.)
- 228 Equivocation = double talk = changing the meaning of a word during the course of an argument, without admitting it.
- 228.1 Confusion of whole and part.
- 229 Amphiboly = fallacy due to use of incorrect grammar.
- 229A Long-windedness and filibustering.
- 229A.1 Stating an argument in so much detail and rigour that it is almost impossible to understand.
- 23 Change of proposition.
- 231 Confusing the issue = diversion to another question, or to a side-

issue, by irrelevant objection, or by facetiousness. (Includes the whole of class 211.)

231.1 By attributing prejudices or motives to one's opponent. (Cf. 211.23 and 211.25.)

231.2 *Tu quoque*. ('You're another': irrelevant counter-attack.)

231.3 By *correctly* accusing one's opponent of committing an error when the point at issue is too small to matter. (A form of quibbling.) For an example, see Ref. 21.

232 Exaggerating the opponent's position, or provoking the opponent to exaggerate by contradiction or misrepresentation.

233 Lies (includes damned lies, but not statistics.)

24 Oversimplification.

241 *Post hoc, ergo propter hoc* (after this, therefore because of this).

242 Oversimplification by the omission of definitions (cf. 225.)

243 Oversimplification by definition, or demand for oversimplification by definition.

244 Cliché thinking (e.g. 'everything is relative', or 'men are descended from monkeys', as summaries of the theories of relativity and evolution.)

245 Everything is black or white ('verbal quantisation'). (Many arguments can be terminated with the remark 'It is all a question of degree'.)

246 Questioning a real distinction by reference to a continuous gradation.

247 *Plurimum interrogationum* = multiple question. (E.g. to ask an accused man, 'What time was it when you met this woman?' before it is agreed that he met her at all.)

25 Logic.

251 Evading a sound refutation by use of a sophistical formula (one of the most blatant being 'the exception proves the rule').

252 *Non sequitur* = 'it does not follow'. (Arguments more or less of the form: 'A, therefore B', when B does not follow from A. The argument must explicitly use a word like 'therefore' or 'since' I think. Most fallacies can be put in the *non sequitur* form.)

253 Faulty syllogism. (See Ref. 11.)

254 Argument in a circle, e.g. ' $B \rightarrow C \rightarrow A$, therefore A', where B is itself implicitly justified by assuming A.

255 Begging the question (e.g. ' $A \rightarrow B \rightarrow A$, therefore A'.)

256 Smuggling assumptions into a *logically* correct argument. ('Everybody knows', 'surely you agree that', 'since it is obvious that', 'since', when the assumptions are not obvious.)

257 Completely irrelevant analogy. ('Virtue grows when watered by War's red rain', which, as a form of argument, is equally valid or invalid if the word 'virtue' is replaced by 'vice'.) (Cf. heading 252.)

- 3 Probability, statistics, and the theory of rational behaviour. (Cf. Refs. 13 to 18.)
- 31 Design of experiment.
- 311 Sample biased.
- 311.1 Consciously (especially by form of wording of a questionnaire).
- 311.2 Carelessly. (This class constitutes a continuous grading between the previous and following classes.)
- 311.3 Unconsciously. (E.g. (i) in a Social Survey, the people who ask the questions may select people who look sympathetic; (ii) medical discoveries at the five percent significance level. How about all the non-significant and unpublished experiments to test the same thing? As one physician said 'Make haste to use the remedy before it is too late'.)
- 312 Sample too small. (E.g. (i) some, but not all, atrocity stories, (ii) seeing periodicities in too short a time series.)
- 313 Sample qualitatively incomplete.
- 313.1 Incomplete 2 by 2 contingency table.
- 313.2 Incomplete 2 by 2 by 2 contingency table. (E.g. The death rate in the American Navy during the Spanish-American War was less than that among civilians in New York City during the same period. It was argued that it was safer to join the Navy.)
- 313.3 Other examples.
- 32 Analysis of the evidence.
- 321 Theoretical blunders (subdivisions of 321 are very far from being exhaustive).
- 321.1 Adding up percentages of parts, for arriving at an overall percentage.
- 321.2 'Mean of square = square of mean'.
- 321.3 Overestimation of the strength of an argument.
- 321.31 *Reliance* on imperfect analogy: if $P(A/B \text{ and } C)$ is near 1, it does not follow that $P(A/B \text{ and } D)$ is near 1 (where $P(X/Y)$ means the probability of X given Y .)
- 321.32 The use of speculation as if it were decisive, without factual backing. (Assuming $P(H)$ is near 1 because H is simple. E.g. Archimedes's argument that the universe must be spherical because the sphere is the only perfect shape.)
- 321.33 Generalizing from too few special cases.
- 321.34 Fallacy of the consequent. (Drunkardry causes destitution, 'therefore' destitution proves drunkardry. It is only a 'factor in favour' in the terminology of Ref. 18.)

- 321.4 Blunders in decision theory.
 - 321.41 Accepting a position *only* because it is a mean between extremes, perhaps justifiable if there is no further information. (This often involves 321.3 and is then fallacious.)
 - 321.42 Allowing for utilities, but not for probabilities. E.g. the argument that we should not make efforts against X , which is evil, because there is a worse evil, Y , against which our efforts should be directed. (A little effort may remove X .)
 - 321.43 Allowing for probabilities but not for utilities. E.g. recommending that one should devote one's time to activity A rather than B , because A is more likely to be successful, and ignoring that B is much more important. (What really counts in 321.42 and 321.43 is 'expected utility'.)
 - 321.44 'There is much to be said on both sides, so no decision can be made'. (Not all large numbers are equal.)

- 322 Statistic inadequate. (Too much information lost in the 'reduction of the data'.)
 - 322.1 Average, without population standard deviation.
 - 322.2 Average, with population standard deviation but for very skew distribution. (E.g. average income as the measure of the standard of living in a country. The median would be better.)
 - 322.3 Point estimate without estimate of its standard error, or of significance level. (Useful for covering up small samples.) (E.g. the assumption that the winning team is the best team.)

- 323 Unjustifiable assumptions.
 - 323.1 Assumption that every distribution is normal. (It used to be said that the physicists thought the mathematicians had proved it, and *vice versa*.)
 - 323.2 Independence.
 - 323.21 Assuming it, incorrectly, i.e. that $P(A \text{ and } B) = P(A) \cdot P(B)$.
 - 323.22 Assuming that independence in pairs implies complete independence. (Not a common fallacy.)
 - 323.23 Assuming that a correlation of zero implies independence. (Correct when the joint distribution is normal.)
 - 323.3 Regression.
 - 323.31 Assuming that the regression curve of y on x is the same as that of x on y .

- 323.32 Assuming that regression is relevant when what should be estimated is the functional relationship between two variables.
- 323.4 Fallacy of typicalness (Ref. 18, p. 67). (E.g. if a die is known either to be unbiased or to give a 6 half the time, then, in ten throws, to get precisely five 6's does not give the maximum factor in favour of bias.)
- 323.5 Precision fallacy. (See Ref. 17.) The assumption that no statement is worth publishing unless it is fully objective (independent of personal judgment) and exact. There is also the opposite fallacy of assuming that anything in print must be worth saying, however vague.
- 323.6 Fallacy of accepting a hypothesis that is too far-fetched, after looking at the evidence. (There are no known clear-cut rules for deciding which hypotheses are too far-fetched. It is a question of judging utilities and initial probabilities.)
- 323.7 Fallacy of assuming that a statistical test and hypothesis (and perhaps all hypotheses) must be formulated before looking at the sample (or at the result of any scientific experiment.)
- 323.8 Uncritical use of Bayes's postulate. (Helped to get inverse probability into disrepute.)
- 323.9 Assumption that an autocovariance must decrease monotonically to zero.
- 323.9A Assumption that a source of noise is band-limited to too narrow a band, and hence misapplying the 'sampling theorem'.
- 33 Presentation of the results of the analysis.
- 331 Psychological.
- 331.1 Tone of voice or expression (cf. 212.7.) ('Half the so-called people in the world are below average intelligence'.)
- 331.2 Spurious accuracy for concealing the smallness of a sample. (Cf. 211.26.)
- 331.3 Mis-use of graphs.
- 331.31 Inadequately labelled. (Ref. 13, p. 50.)
- 331.32 Details magnified out of context. (Ref. 14, p. 85.)
- 331.4 Pictures.
- 331.41 'One-dimensional pictures'. (Depicting relative sizes, of say expenditures, by money-bags whose linear dimensions are proportional to the expenditures. See Ref. 13, pp. 66-73.)
- 331.42 Misleading perspective. (Ref. 14, p. 86.)
- 331.43 Misleading use of maps. (Ref. 13, p. 103.)
- 332 Terminology.
- 332.1 Confusion of kinds of probability. (See Ref. 16.)
- 332.2 'Expected' values (depends on the audience whether this expression is misunderstood.)

- 332.3 'Average' used loosely, possibly meaning median.
 - 332.4 Confusion of sample and population in notation. (Cf. 14.)
 - 332.5 Change of subject, e.g. meaning shifting with time. Includes case of changing methods of collecting data, or changing 'indices'.
 - 332.51 The assumption that of two things one must be better than the other. It may depend on the choice of index. (The simplest examples are obtained by taking all coefficients zero except one.) (Utilities are different for different people.)
 - 332.6 Confusion of an increase measured by a percentage with that measured by the percentage of a percentage. (E.g. 'the dividend has been increased by ten percent'.)
 - 332.7 'Period' of a time series, and 'cycle'. (What is the 'Trade cycle'?)
- 333 Presentation too incomplete.
- 333.1 Sources of information not specified.
 - 333.2 Sample size omitted.
 - 333.3 Comparison with what? (E.g. 'the pound is now worth only ten shillings'.)
- 34 Uncritical use of standard statistical techniques (includes misinterpretation of the presentation.)
- 341 Correlation does not imply causation, either way round. (Cf. 241.)
 - 342 Assumption that if two bivariate populations each have correlation coefficient 0, then a mixture of them must too. (The deduction is correct if the means are the same in the two populations.)
 - 343 Statistically significant but not important.
 - 344 Forgetting that a statistical statement is statistical.
 - 345 Assumption that a statistical average over a population of people (e.g. the number of suicides) must remain roughly constant irrespective of anything that can be done. (Possibly a case of being hypnotised by the phrase 'law of large numbers'.)
 - 346 Rejection of a hypothesis because of a tail-area probability close to 1. The initial probabilities of the alternative hypotheses that would explain the event are usually low, apart from the hypothesis of a mistake in arithmetic! (For this, and later sub-classes of 34, see Ref. 15.)
 - 347 Small tail-area probabilities (used uncritically.) (The technique might encourage the unwary to overlook rival hypotheses.)
 - 348 Maximum-likelihood estimation, ignoring initial probabilities and utilities.
 - 349 Large-sample theory, or asymptotic properties of statistics. (How large?)
 - 349A Likelihood-ratio method used for small samples. (See Ref. 19.)
 - 349B Unbiased estimates can take impossible values.
 - 349C Confidence intervals are supposed to protect the reputation of the

statistician in the long run, but can lead to such absurdities that there would not be a long run.

- 349D Fiducial distributions: need not be unique. (Ref. 20.)
- 349E Errors of the first and second kinds ignore questions of 'robustness'. i.e. insensitiveness to minor modifications in the model.
- 349F Significance tests: a large enough sample would lead to the rejection of almost any hypothesis.
- 349G The use of random sampling numbers. As soon as you notice anything peculiar about the random numbers that have been used in an experiment, the precision and objectivity of the interpretation are impaired.
- 349H Does decision theory cover ordinary inference? (In scientific work the utilities are often vaguer than in industrial applications.)

35 Error detection.

- 351 Have any errors been committed under any of the above headings (or others that the reader may have inserted)?
- 352 Are the conclusions surprising (to trained common sense)? If so they need more critical examination.

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