A Probabilistic Characterization of Shark Movement Using Location Tracking Data

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Data and motivation

- Our data come from a 2011 paper testing a VPS (acoustic telemetry) tracking transmitters implanted in 22 gray smooth-hound sharks in the tidal basin of a wetlands in Orange County, CA.
- Data are the 2-D coordinate locations of the sharks and the times they were recorded; the time gaps Δ_t between observations were unequal.
- From domain knowledge, biologists believe shark behavior largely consists of either **foraging** (slow, meandering movement while feeding) or **transiting** (faster movement between feeding areas).
- Since the animal's full trajectory—of which we have discrete realizations—depends on the movement, which depends on the behavior type, we can build a model to both **estimate the movement path** at times it is not observed, and **infer behavior type** from the movement.
- Inferring spatial distributions of behavior types is interesting as a way to identify ecologically-sensitive feeding areas.

Gray smooth-hound shark (*Mustelus Californicus*)

- Gray smooth-hound sharks are benthic (bottom-feeding) predators that typically grow to a length of about 4–5 feet.
- The Bolsa Chica wetlands have a southern outlet to the Pacific Ocean. Sharks occasionally leave the wetlands; their locations are then out of range of the receivers and thus unobserved.

State-space models (SSMs)

- **State-space models** (SSMs) are a general probabilistic method of using observations $\mathbf{y}_t$ to sequentially model time (t)-evolving unobserved variables ('states') $\mathbf{x}_t$.
- In robotics or tracking, $\mathbf{y}_t$ are often sensor or location measurements.
- Observations $\mathbf{y}_t$ are assumed to have some measurement error, and $\mathbf{x}_t$ are noise-free values. Here, $\mathbf{x}_t$ will include the animal's (unobserved) true location ζ_t and movement (e.g., speed v_t and direction angle) that result in the observed locations.
- Optional vector $\boldsymbol{\theta}_t$ contains additional hypothesized parameters, here including the behavior type, denoted λ_t.

Sequentially update estimates of $\mathbf{x}_t$ using new observed $\mathbf{y}_t$:

$$\begin{aligned} \text{Prior on initial state:} \quad \mathbf{x}_0 &\sim p(\mathbf{x}_0) \\ \text{State/dynamic equation:} \quad \mathbf{x}_t &\sim \ell(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \boldsymbol{\theta}_t) \\ \text{Measurement equation:} \quad \mathbf{y}_t &\sim m(\mathbf{y}_t \mid \mathbf{x}_t, \boldsymbol{\theta}_t) \end{aligned}$$

1-D robot SSM

- Illustrate principles of 2-D shark movement, which includes direction, using a simpler example of a robot moving along the 1-D real line.
- At time t, its sensor measures position $z_t \in \mathbb{R}$ with error; the unknown true location at t is ζ_t.
- Time gaps Δ_t are a constant Δ_Υ, so $\mathbf{x}_t$ are modeled on the same clock times as observations $\mathbf{y}_t$. This is relaxed later for sharks.
- Velocity $v_t \in \mathbb{R}$ between true ζ_t and ζ_{t+1} is random $\sim \mathcal{N}(\boldsymbol{\alpha}, \sigma)$.
- We will later allow the robot to have either a slow and fast mode at each t, analogous to the shark foraging/transiting. Thus v_t will have a bimodal mixture distribution with two values of $\boldsymbol{\alpha}$ to estimate. For simplicity, assume now v_t is unimodal.
- States $\mathbf{x}_t = \begin{bmatrix} \zeta_t & v_t \end{bmatrix}$ are modeled sequentially by learning which values best predict the observed $\mathbf{y}_t = z_{t+1}$.

1-D robot SSM

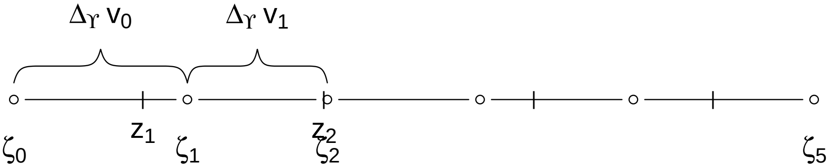
State equation:

Velocities v_t have single distribution with mean $\boldsymbol{\alpha}$

$$\mathbf{x}_t = \begin{bmatrix} \zeta_t \\ v_t \end{bmatrix} \sim \ell(\quad \mathbf{x}_t \,|\mathbf{x}_{t-1}, \boldsymbol{\theta} = \boldsymbol{\alpha} \quad) = \mathcal{N}_2 \left(\begin{bmatrix} \zeta_{t-1} + (\Delta_\Upsilon)(v_{t-1}) \\ \boldsymbol{\alpha} \end{bmatrix}, \mathbf{Q}_t \right)$$

Measurement equation:

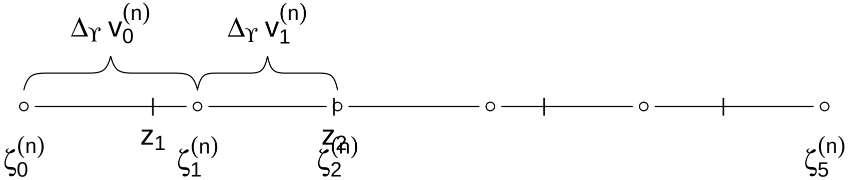
$$\mathbf{y}_t = \begin{bmatrix} z_{t+1} \end{bmatrix} \sim m(\quad \mathbf{y}_t \,|\mathbf{x}_t \quad) = \mathcal{N} \left(\begin{bmatrix} \zeta_t + (\Delta_\Upsilon)(v_t) \end{bmatrix}, \mathbf{R}_t \right)$$



Particle filters (PFs)

- Particle filters (PFs) generate multiple (N) simulations ('particles') of the process (e.g. movement) variables $\mathbf{x}_t = \begin{bmatrix} \zeta_t & v_t \end{bmatrix}$ to match the observations (locations) $\mathbf{y}_t = z_{t+1}$.
- The particle set $\{\mathbf{x}_t^{(n)}\}_{n=1}^N$ empirically estimate a posterior distribution of guesses of the value of the true unobserved $\mathbf{x}_t$.
- At each step t, each particle n calculates predictive density $d(\hat{\mathbf{y}}_t \mid \mathbf{x}_{t-1}^{(n)})$. Calculate a weight $w_t^{(n)}$ as $\propto d(\mathbf{y}_t \mid \mathbf{x}_{t-1}^{(n)})$, this density evaluated at the observed value $\mathbf{y}_t$.
- The higher $w_t^{(n)}$ is, the more likely particle n's modeled movement $\mathbf{x}_t^{(n)}$ is to resulted in the observed $\mathbf{y}_t$, and so is a better guess.
- Particle sets are resampled by the weights $\{w_t^{(n)}\}_{n=1}^N$, so particles with more likely estimates are favored for future prediction.

- We use the **conditional dynamic linear model** (CDLM; Carvalho 2010, et al.) PF formulation the 1-D robot. Here each particle n simulates its own values of $\zeta^{(n)}$ and $v_t^{(n)}$.
- We use an animation to illustrate the 1-D PF. These `shiny` programs will be available to-be-released package `animalEKF`.
- Click here for video of demonstration of package's `cdlm_robot()` function.

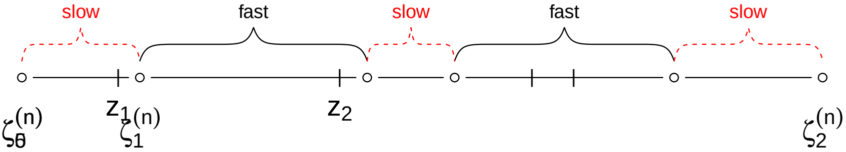


1-D robot with two 'behaviors'

- The 1-D robot can have one of two speed modes (denoted by λ_t) at each time point t: slow (0) and fast (1), analogous to shark foraging and transiting.
- Let $\mathrm{E}(v_t \mid \lambda_t = \text{slow}) = \boldsymbol{\alpha_0}$ and $\mathrm{E}(v_t \mid \lambda_t = \text{fast}) = \boldsymbol{\alpha_1}$, where $|\alpha_1| > |\alpha_0|$. By separately modeling movement for each behavior λ_t, we can infer which behavior (unobserved) occurred by seeing which movement type best predicts the observed location $\mathbf{y}_t$.
- For each particle n, $w_{t|\boldsymbol{k}}^{(n)} \propto d(\mathbf{y}_t \mid \mathbf{x}_{t-1}^{(n)}, \boldsymbol{\lambda_t = k})$ is the predictive likelihood of observation $\mathbf{y}_t$ if behavior $\boldsymbol{\lambda_t = k}$ occurred. The $\boldsymbol{k}$ for which $w_{t|\boldsymbol{k}}^{(n)}$ is highest is the most probable behavior.
- Particles are resampled by unconditional weights $w_t^{(n)} = \sum_k w_{t|\boldsymbol{k}}^{(n)}$. Behavior $\lambda_t^{(n)} = \boldsymbol{k}$ is drawn by the relative values of $w_{t|\boldsymbol{k}}^{(n)}$ after resampling.

Two-behavior (λ) CDLM visualization

- The transition probabilities $p_{i \to j}$ between slow/fast modes λ_t affect the conditional weights $w_{t|k}^{(n)}$.
- Click here for video of demonstration of package's `cdlm_robot_twostate()` function of modeling a dual-mode trajectory.



CDLM with interpolation

- In reality, unlike the robot example, telemetry observations $\mathbf{y}_t$ typically do not occur at constant length Δ_Υ time gaps.
- However, for proper parameter updates we want to model the movement $\mathbf{x}_t$ at these equal time intervals. Δ_Υ should be large enough to represent a 'distinct' movement but short enough to allow finer resolution of trajectories (e.g., 2 minutes).
- Denote state movement at these intervals by $\mathbf{x}_c$ at times Υ_c. For simplicity let $\Upsilon_0 = 0$ and $\Upsilon_c = \Upsilon_{c-1} + \Delta_\Upsilon$.
- For simplicity, sharks are modeled as moving in a straight line with constant speed v_c, depending on the behavior type λ_c, in each short time interval $(\Upsilon_c, \Upsilon_{c+1}]$.

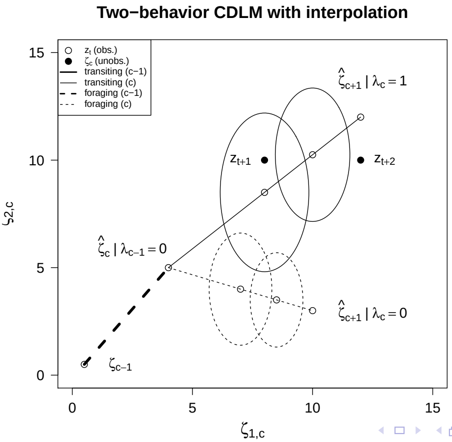
CDLM with interpolation

In any constant-length interval $(\Upsilon_c, \Upsilon_{c+1}]$, either there are or aren't observations $\mathbf{y}_t$ recorded.

- If there are no observations, simulate movement from $\mathbf{x}_c$ to $\mathbf{x}_{c+1}$ by a single random draw of behavior $\lambda_c \mid \lambda_{c-1}$, then drawing $\mathbf{x}_c \sim \ell(\cdot \mid \mathbf{x}_{c-1}, \boldsymbol{\lambda_c = k})$ from the SSM equation.
- If at least one observation is in the interval, predict straight-line movement $\mathbf{x}_c \mid (\mathbf{x}_{c-1}, \boldsymbol{\lambda_c = k})$ from Υ_c to Υ_{c+1} for each behavior $\boldsymbol{k}$.
- Let $\{\Delta_t^*\}$ be the irregular time gaps from the interval Υ_c to the first, and between each observation. For each $\mathbf{y}_t$, predict location $\hat{\mathbf{y}}_t \mid (\boldsymbol{\lambda_c = k}, \Delta_t^*)$ along the straight line at the time of observation t.
- Calculate behavior-conditional weights $w_{c|\boldsymbol{k}}^{(n)}$ by jointly how close observations $\{\mathbf{y}_t\}$ are to predictions $\{\hat{\mathbf{y}}_t\}$. The behavior k whose straight-line movement best jointly fits <u>all</u> observed locations (for which k $w_{c|\boldsymbol{k}}^{(n)}$ is highest) is the most likely.

Shark EKF with interpolation: observations $\mathbf{y}_t$

- Linear movement from ζ_c to ζ_{c+1} for each behavior $\lambda_c \in \{0, 1\}$, to best fit observations $\mathbf{y}_t = \mathbf{z}_{t+1}$ and $\mathbf{y}_{t+1} = \mathbf{z}_{t+2}$ in the interval.
- Ellipses along lines between $\hat{\zeta}_c$ and $\hat{\zeta}_{c+1}$ centered at $(\hat{\mathbf{z}}_{t+1}, \hat{\mathbf{z}}_{t+2}) \mid \lambda_c$.
- Error ellipse sizes depend on time gaps Δ_t^* between observations.



Conclusions and continuations

- Conclusion: are able to learn bimodal speed v_t distribution indicative of two behavior mixture for both observed and synthetic trajectories. Also incorporate behavioral interactions between sharks to jointly model 'schooling' tendencies.
- Use cross-validation to demonstrate that interpolation PF works better than simple non-statistical Euclidean connect-the-dots method for modeling omitted observations.
- Adapt a model-fit measure like Bayes Factor to PFs to demonstrate that parameters match a given shark's trajectory better than another's.

Selected references

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