Analysis of Crimean-Congo Hemorrhagic Fever Incidents with Dynamically Weighted Particle Filter

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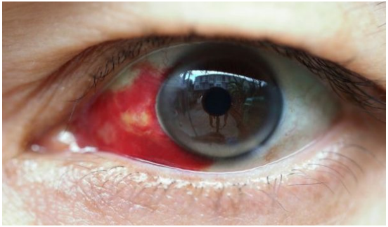
Outline

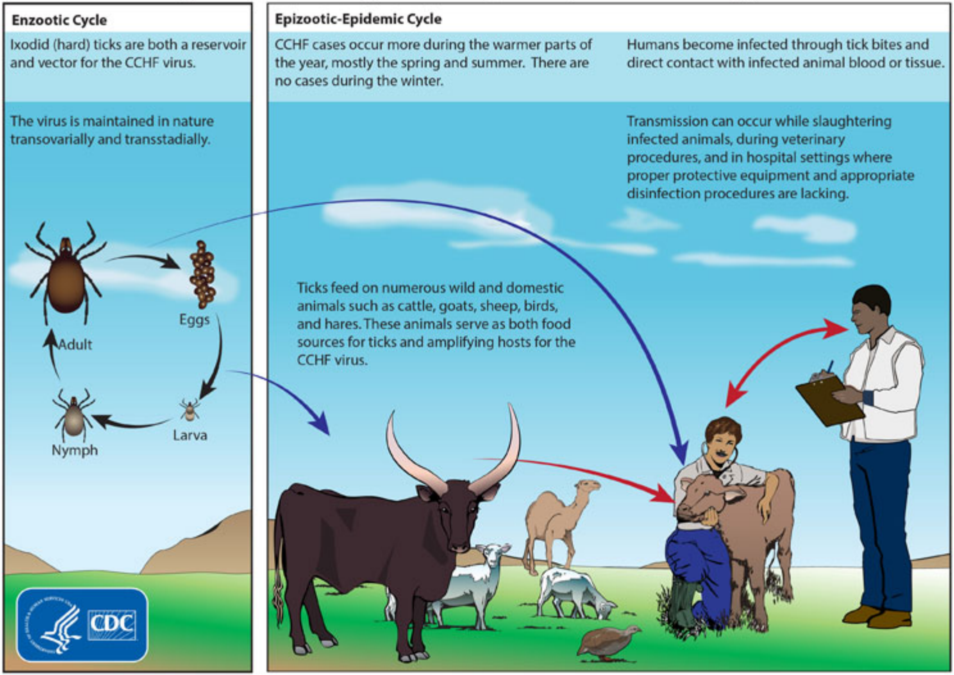
- Crimean-Congo Hemorrhagic Fever Incidents in Turkey
- A Dynamic Model for CCHF Incidents
- Estimation with Dynamically Weighted Particle Filter
- Concluding Remarks

Crimean-Congo Hemorrhagic Fever



- Caused by Crimean-Congo Hemorrhagic virus.
  $\Rightarrow$ 4-5 days of non-bleeding phase: headache, high fever, nausea, abdominal pain, muscle pain, diarrhea
  $\Rightarrow$ Bleeding in the eyes, the throat and in the stomach: hypotension, relative bradycardia, conjunctivitis, skin flushing and rash
  $\Rightarrow$ 10-20 days for convalescence
- No vaccine is available, and treatment is mostly supportive.

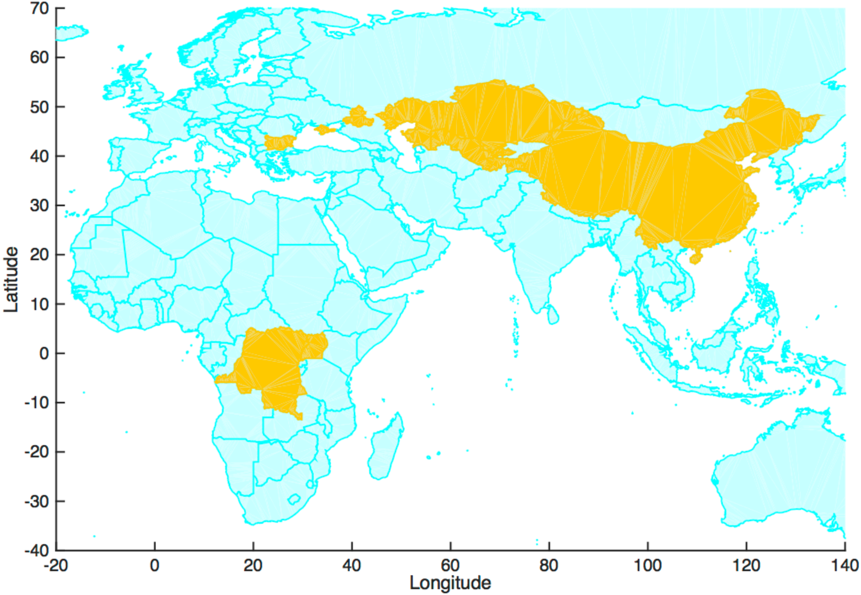
Crimean-Congo Hemorrhagic Fever (CCHF) Virus Ecology



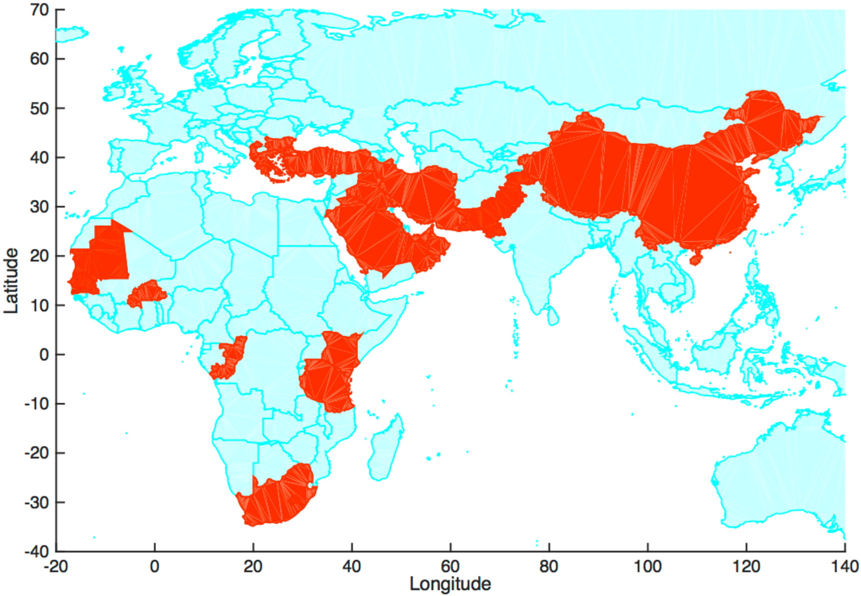
CCHF Characteristics

- CCHF is a fatal viral infection disease with the fatality rate of up to 30%.
- The virus is transmitted to human from ticks and livestock animals, as well as human.
- The great majority of incidents are from agriculture and/or husbandry:
  - Ergönül [1] reported that almost 90% of the cases around year 2007 outbreak in Turkey were farmers.
- CCHF has been found in parts of Africa, Asia, Eastern Europe and Middle East.
- The region of prevalent CCHF is growing as the following.
  - Before 1970: Soviet Union, Zaire, Uganda and China.
  - 1970-2000s: Africa, Eastern Europe, Middle East and China.

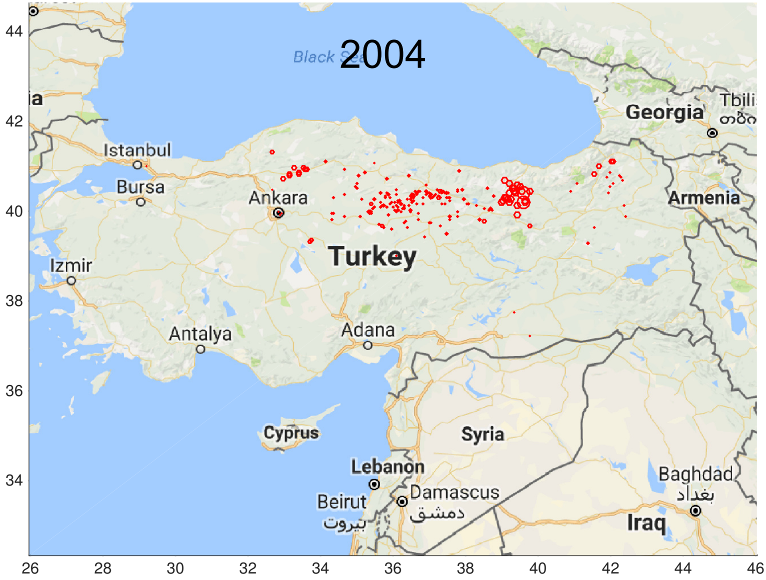
CCHF Reported before 1970



CCHF Reported between 1970-2000s



CCHF Incidents in Turkey



CCHF Incidents in Turkey

- From year 2004 to year 2012, there were 7040 CCHF incidents observed at 3514 locations in 73 cities.
- We consider CCHF incidents per 500,000 population of each city in Turkey:
  - Incidents are nonnegative integers.
  - Incidents are time dependent.
  - Incidents from neighboring locations are associated with each other.
- We analyze the incidents of 9 months from March to November in each year.
  - There is no incident during the winter season.
  - Incidents show a periodic pattern from March to November.
  - Incidents also show a nonlinear pattern over cities.

Dynamic Model for CCHF Incidents

- The CCHFs are sequentially observed at 79 months over 73 cities in Turkey.

- At a given time, incidents show a nonlinear relationship with location and take nonnegative integer values.

  $\Rightarrow$ Response Model: Poisson regression model and log-link function with the radial basis function (RBF) network.

- Some variables and parameters in the response model change by time.

  $\Rightarrow$ Transition Model: Linear or nonlinear models for state variables and time-varying parameters.

Response Model

- For a given time t, let Y_t denote the number of incidents at a location $\boldsymbol{X}_t = (\text{latitude}, \text{longitude})_t$ with $E(Y_t | \boldsymbol{X}_t) = \psi_t$.
- Then, we may model Y_t by the following generalized linear mixed model

$$
\begin{aligned}
Y_t &\sim \text{Poisson}(\psi_t) \\
L_t &= \log \psi_t + \delta_t = f_t(\boldsymbol{X}_t) + \delta_t,
\end{aligned}
$$

  where

  - $f_t(\boldsymbol{X}_t)$ is the RBF network,
  - $\delta_t \sim N(0, \sigma_t^2)$ is an independent random error,
  - L_t is a latent variable.

RBF network

- Let $\boldsymbol{X}_t^m$ denote the set of all terms of m-degree polynomial, e.g., for $\boldsymbol{X}_t = (X_{t1}, X_{t2})$, $\boldsymbol{X}_t^2 = (X_{t1}^2, X_{t1}X_{t2}, X_{t2}^2)$.
- Consider K_t knots $\boldsymbol{\mu}_t = (\boldsymbol{\mu}_{t,1}, \ldots, \boldsymbol{\mu}_{t,K_t})$ and let $z_{t,k}$ denote the distance from $\boldsymbol{X}_t$ to $\boldsymbol{\mu}_{t,k}$, e.g., $z_{t,k} = \| \boldsymbol{X}_t - \boldsymbol{\mu}_{t,k} \|$.
- Then, for a radial basis function ϕ on $z_{t,k}$ as in Holmes [2], RBF network is defined as a linear combination with smoothing penalty γ_t

$$f_t(\boldsymbol{X}_t) = \beta_{t,0} + \sum_{m=1}^{M} \boldsymbol{X}_t^m \boldsymbol{\beta}_{t,m} + \sum_{k=1}^{K_t} \phi(z_{t,k}) \beta_{t,M_b+k} = \boldsymbol{d}_t \boldsymbol{\beta}_t,$$

where

  - $\boldsymbol{d}_t = [1, X_{t1}, \ldots, X_{t2}^m, \ldots, \phi(z_{t,K_t})]$ is a design row vector,
  - $\boldsymbol{\beta}_t = [\beta_{t,0}, \beta_{t,1}, \ldots, \beta_{t,M_b}, \ldots, \beta_{t,M_b+K_t}]$ is a vector of regression coefficients, for $M_b = 1 + M(M+3)/2$.

Transition Model

- Consider normal random errors ϵ_{t_0}, ϵ_{t_1}, ϵ_{t_2} and ϵ_{t_3}.
- From time t to time $t+1$, all nuisance parameters in the response model $\boldsymbol{\lambda}_t = (\sigma_t^2, \gamma_t, K_t, \boldsymbol{\mu}_t)$ are transited by:

$$\log \sigma_{t+1}^2 = C_1 \log \sigma_t^2 + \epsilon_{t_1}, \quad \log \gamma_{t+1}^{-1} = C_2 \log \gamma_t^{-1} + \epsilon_{t_2},$$

$$\pi(K_{t+1}|K_t) = \begin{cases} 1/3, & \text{for } K_{t+1} = K_t \\ 1/3, & \text{for } K_{t+1} = K_t + 1 \\ 1/3, & \text{for } K_{t+1} = K_t - 1 \end{cases},$$

$$\boldsymbol{\mu}_{t+1} = \begin{cases} \boldsymbol{\mu}_t + \epsilon_{t_3}, & \text{for } K_{t+1} = K_t \\ (\boldsymbol{\mu}_t, \mu_{t+1,K_{t+1}}) + \epsilon_{t_3}, & \text{for } K_{t+1} = K_t + 1 \\ (\boldsymbol{\mu}_t)_{[-i]} + \epsilon_{t_3}, & \text{for } K_{t+1} = K_t - 1 \end{cases}.$$

- The latent variable is also transited by:

$$L_{t+1} = L_t + \epsilon_{t_0}.$$

Directed Acyclic Graph for Model

$\boldsymbol{\mu}_t$ knot locations

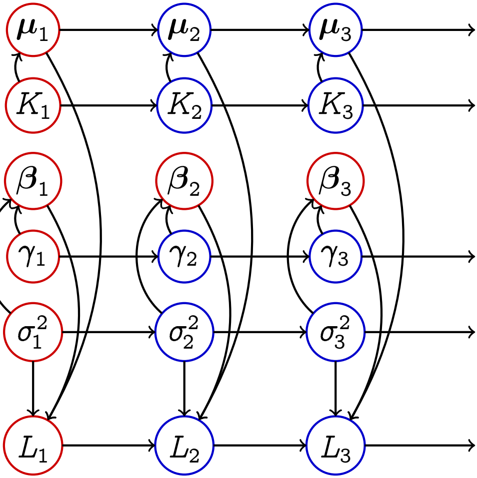
K_t no. of knots

$\boldsymbol{\beta}_t$ regression coeffs

γ_t smoothness

σ_t^2 error variance

L_t latent variable



Prior Distributions

- Only for time $t = 1$, assign prior distributions for nuisance parameters $\boldsymbol{\lambda}_1$:
  - Number of knots: $K_1 \sim$ Uniform,
  - Knot locations: $\boldsymbol{\mu}_1 | \boldsymbol{X}_1 \sim N(\boldsymbol{X}_1, \boldsymbol{I}_2)$,
  - Nuisance parameters: $\sigma_1^2 \sim IG(A_y, B_y)$, $\gamma_1 \sim G(A_\gamma, B_\gamma)$.
- For $t > 1$, the transition model projects $\boldsymbol{\lambda}_t$ based on $\boldsymbol{\lambda}_{t-1}$.
- For all times, we assign a conjugate Gaussian prior distribution for regression coefficients $\boldsymbol{\beta}_t$:

$$\begin{aligned}(\beta_{t,0}, \boldsymbol{\beta}_{t,1}^T, \ldots, \boldsymbol{\beta}_{t,M}^T) &\propto 1, \\ (\beta_{t,M_b+1}, \ldots, \beta_{t,M_b+K_t}) &\sim N\left(0, \frac{\sigma_t^2}{\gamma_t} \boldsymbol{I}_{K_t}^*\right).\end{aligned}$$

Full Conditional Distributions

- Regression coefficients:

$$\boldsymbol{\beta}_t | \cdot \sim N(\boldsymbol{V}_t^{-1}\boldsymbol{D}_t^T\boldsymbol{L}_t, \sigma_t^2\boldsymbol{V}_t^{-1}), \quad t \geq 1,$$

where $\boldsymbol{D}_t$ is a design matrix that accumulate row vectors $[\mathbf{1}, \boldsymbol{X}_t^m, \phi(z_{t,1}), \ldots, \phi(z_{t,K_t})]$ corresponding to all locations, $\boldsymbol{L}_t$ is a vector of latent variables and $\boldsymbol{V}_t = \boldsymbol{D}_t^T\boldsymbol{D}_t + \gamma_t\boldsymbol{I}^*$.

- Nuisance parameters:

$$\sigma_1^2 | \cdot \sim IG[A_y + (n+K_1)/2, \{B_y^{-1} + (\mathrm{RSS}_1 + \gamma_1\boldsymbol{\beta}_1^T\boldsymbol{I}^*\boldsymbol{\beta}_1)/2\}^{-1}],$$
$$\gamma_1 | \cdot \sim G[A_\gamma + K_1/2, \{B_\gamma^{-1} + (\boldsymbol{\beta}_1^T\boldsymbol{I}^*\boldsymbol{\beta}_1)/(2\sigma_1^2)\}^{-1}],$$

where $\mathrm{RSS}_1 = (\boldsymbol{L}_1 - \boldsymbol{D}_1\boldsymbol{\beta}_1)^T(\boldsymbol{L}_1 - \boldsymbol{D}_1\boldsymbol{\beta}_1)$.

- We use latent variables L_1 instead of Poisson responses Y_1:

$$L_1 | \cdot \propto p(Y_1 | L_1) N(\boldsymbol{d}_1\boldsymbol{\beta}_1, \sigma_1^2)$$

Projection by Transition Model

- For $t > 1$, we can generate $\boldsymbol{\lambda}_t = (\gamma_t, \sigma_t^2, K_t, \boldsymbol{\mu}_t)$ with a data assimilation method.

- Defining $\boldsymbol{\lambda}_0$ by the hyperparameters, transition model bring the target distribution for $\boldsymbol{\lambda}_t$:

$$p\{\boldsymbol{\lambda}_{0:t} \big| \boldsymbol{X}_{1:t}, \boldsymbol{L}_{1:t}\}$$

Data Assimilation Methods

- Kalman Filters:
  - Extended Kalman Filter (EKF)
  - Ensemble Kalman Filter (EnKF)
- Particle Filters:
  - Sequential Importance Sampling (SIS)
  - Rejection Control (RC)
  - Dynamically Weighted Importance Sampling (DWIS)

Kalman Filter

- Dynamic Model: Y_t is observed sequentially on state and time-varying parameters X_t.

$$\begin{aligned} Y_t &= B_t X_t + \epsilon_t, \quad \epsilon_t \sim N(0, \Sigma_e), \\ X_t &= A_t X_{t-1} + U_t, \quad U_t \sim N(0, \Sigma_u). \end{aligned}$$

- Forecast (predict): Project state variable,

$$\widetilde{X}_t = A_t \widehat{X}_{t-1} \sim N(m_t, S_t).$$

- Update (match): Correct the projection

$$\widehat{X}_t = \widetilde{X}_t + K_t(Y_t - B_t \widetilde{X}_t) \sim N(\mu_t, \Sigma_t),$$

where

$$\begin{aligned} K_t &= S_t B_t^T (B_t S_t B_t^T + \Sigma_e)^{-1}, \\ \mu_t &= E(\widetilde{X}_t | Y_t = B_t m_t) = m_t + K_t(Y_t - B_t m_t), \\ \Sigma_t &= \text{cov}(\widetilde{X}_t | \widetilde{Y}_t) = S_t - K_t B_t S_t. \end{aligned}$$

Particle Filter

- Dynamic Model: We relax the linearity and normality by considering nonlinear f_t and g_t and non-Gaussian ϵ_t and U_t

$$\begin{aligned} Y_t &= f_t(X_t) + \epsilon_t, \\ X_t &= g_t(X_{t-1}) + U_t. \end{aligned}$$

- Approximate the target (filtering) distribution $\pi(X_{1:t}|Y_{1:t})$ by a weighted set of particles, $\{\widetilde{X}_{1:t}^{(1)}, \dots, \widetilde{X}_{1:t}^{(N_t)}\}$, through importance sampling for a stream of particle i:

$$\begin{aligned} \text{Sample :} \quad & \widetilde{X}_{1:t}^{(i)} \text{ from } q(X_{1:t}|Y_{1:t}), \\ \text{Weight :} \quad & w_t^{(i)} = \frac{\pi(X_{1:t}|Y_{1:t})}{q(X_{1:t}|Y_{1:t})}, \end{aligned}$$

where q is a proposal distribution.

Sampling Methods for Particle Filters

- Sequential Importance Sampling:
  - Apply the decomposition to the target density π and proposal density q.

$$
\begin{aligned}
\pi(X_{1:t}|\,Y_{1:t}) &= \pi(X_t|X_{1:t-1},\,Y_{1:t})\,\pi(X_{1:t-1}|\,Y_{1:t}) \\
&= \pi(X_1|\,Y_1)\prod_{k=1}^{t}\pi(X_k|X_{1:k-1},\,Y_{1:k})
\end{aligned}
$$

  - Collects only one-step ahead samples with established previous samples.

- Rejection Control: If a stream of samples has less weight than the threshold, send it back to $t = 1$.

- Dynamically Weighted Importance Sampling: At each time investigate the streams and then prune and enrich them.

Dynamically Weighted Particle Filter

- For CCHF, we need to sample $\boldsymbol{\lambda}_{1:t}$ from the target distribution $p\left\{\boldsymbol{\lambda}_{0:t} \middle| \boldsymbol{X}_{1:t}, \boldsymbol{L}_{1:t}\right\}$.
- Assuming a Markovian structure:

$$p\{\boldsymbol{\lambda}_{0:t}|\boldsymbol{X}_{1:t},\boldsymbol{L}_{1:t}\} \propto p\{\boldsymbol{\lambda}_{0:t-1}|\boldsymbol{X}_{1:t-1},\boldsymbol{L}_{1:t-1}\}\, p(\boldsymbol{\lambda}_t|\boldsymbol{\lambda}_{t-1},\boldsymbol{X}_t,\boldsymbol{L}_t),$$

where $p(\boldsymbol{\lambda}_t|\boldsymbol{\lambda}_{t-1},\boldsymbol{X}_t,\boldsymbol{L}_t)$ is the marginalized over $\boldsymbol{\beta}_t$

$$\begin{aligned} p(\boldsymbol{\lambda}_t|\boldsymbol{\lambda}_{t-1},\boldsymbol{X}_t,\boldsymbol{L}_t) &\propto \int p(\boldsymbol{X}_t,\boldsymbol{L}_t|\boldsymbol{\lambda}_t,\boldsymbol{\beta}_t)\pi(\boldsymbol{\beta}_t|\boldsymbol{\lambda}_t)\pi(\boldsymbol{\lambda}_t|\boldsymbol{\lambda}_{t-1})\,d\boldsymbol{\beta}_t \\ &\propto \pi(\boldsymbol{\lambda}_t|\boldsymbol{\lambda}_{t-1})\,\exp\left\{-\mathrm{RSS}_t/2\sigma_t^2\right\}. \end{aligned}$$

- As an effective sampling scheme, as Liang [3] and Ryu [4] used, DWPF is a combination of SIS and DWIS algorithms.
- DWIS consists of dynamic weighting and population control scheme.

Dynamic Weighting

1. Update the weight upper and lower bounds:

$$(W_{lt}, W_{ut}) \leftarrow \begin{cases} (W_{lt}/a, W_{ut}/a), & \text{if } N_t < N_{low}, \\ (aW_{lt}, aW_{ut}), & \text{if } N_t > N_{up}, \\ (W_{lt}, W_{ut}), & \text{otherwise.} \end{cases}$$

for $i = 1, \ldots, N_t$ do

2. Draw $\boldsymbol{\lambda}_*^{(i)}$ from a proposal distribution $q(\boldsymbol{\lambda}|\boldsymbol{\lambda}_t^{(i)})$:

$$r_t^{(i)} = w_t^{(i)} \frac{p(\boldsymbol{\lambda}_*^{(i)}|\boldsymbol{\lambda}_{0:t-1}^{(i)})\, q(\boldsymbol{\lambda}_t^{(i)}|\boldsymbol{\lambda}_*^{(i)})}{p(\boldsymbol{\lambda}_t^{(i)}|\boldsymbol{\lambda}_{0:t-1}^{(i)})\, q(\boldsymbol{\lambda}_*^{(i)}|\boldsymbol{\lambda}_t^{(i)})}.$$

3. Update the weight with $\delta_t = \left[c_1 + c_2 W_{u,t}^{1+c_3}\right]^{-1}$:

$$\hat{\boldsymbol{\lambda}}_t^{(i)} = \boldsymbol{\lambda}_*^{(i)}, \quad \hat{w}_t^{(i)} = (1+\delta_t) r_t^{(i)}$$

endfor

Population Control Scheme

- Adaptive pruned-enriched population control:

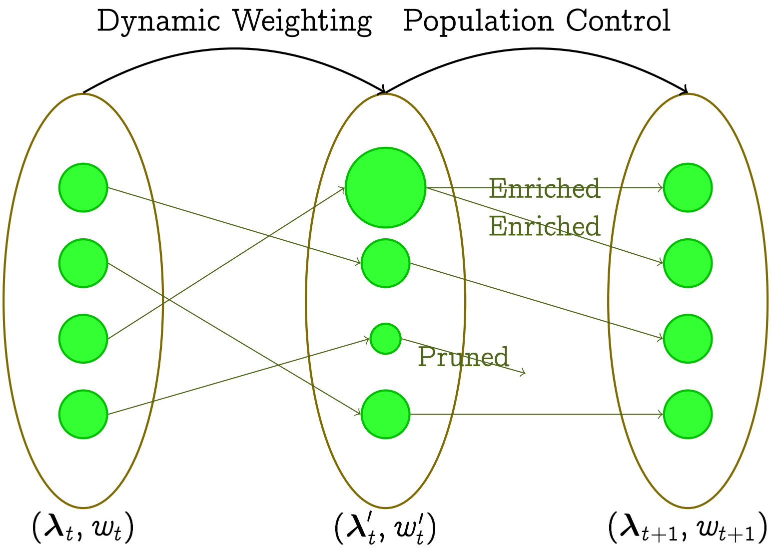
$$(\widehat{\boldsymbol{\lambda}}_t^{(i)}, \widehat{w}_t^{(i)}) \ \Rightarrow \ (\boldsymbol{\lambda}_t^{(i')}, w_t^{(i')})$$

- Algorithm: for $i = 1, \ldots, N_t$
  1. If $\widehat{w}_t^{(i)} < W_{lt}$, prune with probability $1 - \frac{\widehat{w}_t^{(i)}}{W_{lt}}$, or keep the particle w.p. $\frac{\widehat{w}_t^{(i)}}{W_{lt}}$ and set the weight to W_{lt}.
  2. If $\widehat{w}_t^{(i)} > W_{ut}$, enrich the particle with $h_t = \lfloor \frac{\widehat{w}_t^{(i)}}{W_{ut}} + 1 \rfloor$ replications of particles and the adjusted weight $\frac{\widehat{w}_t^{(i)}}{h_t}$.
- Assess: If $N_t' \notin (N_{\min}, N_{\max})$, adjust weight bounds:

$$(W_{lt}, W_{ut}) \leftarrow \begin{cases} (aW_{lt}, aW_{ut}), & \text{if } N_t' > N_{\max}, \\ (W_{lt}/a, W_{ut}/a), & \text{if } N_t' < N_{\min}, \end{cases}$$

and keep the process until $N_t' \in (N_{\min}, N_{\max})$.

Dynamically Weighted Importance Sampling



DWPF Procedure - Stage 1

1. Sample: Sample $\widehat{\boldsymbol{\lambda}}_1^{(i)}$ from $p(\boldsymbol{\lambda}_1|\boldsymbol{X}_1, \boldsymbol{L}_1)$, and set $\widehat{w}_1^{(i)} = 1$ for $i = 1, \ldots, N_0$. These form the initial population $(\widehat{\boldsymbol{\lambda}}_1, \widehat{\boldsymbol{w}}_1)$, and N_0 is called the initial population size.
2. DWIS: Generate $(\boldsymbol{\lambda}_1, \boldsymbol{w}_1)$ from $(\widehat{\boldsymbol{\lambda}}_1, \widehat{\boldsymbol{w}}_1)$ using DWIS, with $p(\boldsymbol{\lambda}_1|\boldsymbol{X}_1, \boldsymbol{L}_1)$ as the target distribution.

DWPF Procedure - Stage 2

1. Extrapolation: Generate $\widehat{\boldsymbol{\lambda}}_2^{(i)}$ from $\boldsymbol{\lambda}_1^{(i)}$, with the extrapolation operator $q(\boldsymbol{\lambda}_2 | \boldsymbol{\lambda}_1^{(i)}, \boldsymbol{X}_{1:2}, \boldsymbol{L}_{1:2})$, and set

$$\widehat{w}_2^{(i)} = w_1^{(i)} \frac{p(\boldsymbol{\lambda}_1^{(i)}, \widehat{\boldsymbol{\lambda}}_2^{(i)} | \boldsymbol{X}_{1:2}, \boldsymbol{L}_{1:2})}{p(\lambda_1^{(i)} | L_1) q(\widehat{\boldsymbol{\lambda}}_2^{(i)} | \boldsymbol{\lambda}_1^{(i)}, \boldsymbol{X}_{1:2}, \boldsymbol{L}_{1:2})}$$

   for each $i = 1, 2, \ldots, N_1$.

2. DWIS: Generate $(\boldsymbol{\lambda}_2, \boldsymbol{w}_2)$ from $(\widehat{\boldsymbol{\lambda}}_2, \widehat{\boldsymbol{w}}_2)$ using DWIS, with $p(\boldsymbol{\lambda}_{1:2} | \boldsymbol{X}_{1:2}, \boldsymbol{L}_{1:2})$ as the target distribution.

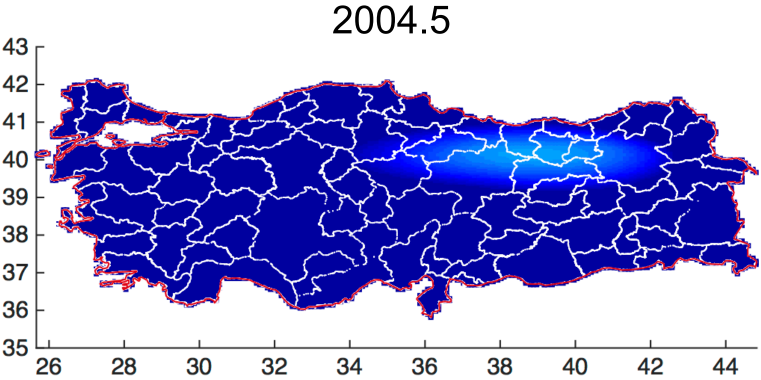
1. Extrapolation: Generate $\widehat{\boldsymbol{\lambda}}_t^{(i)}$ from $\boldsymbol{\lambda}_{t-1}^{(i)}$, with the extrapolation operator $q(\boldsymbol{\lambda}_t | \boldsymbol{\lambda}_{1:t-1}^{(i)}, \boldsymbol{X}_{1:t}, \boldsymbol{L}_{1:t})$ and set

$$\widehat{w}_t^{(i)} = w_{t-1}^{(i)} \frac{p(\boldsymbol{\lambda}_{1:t-1}^{(i)}, \widehat{\boldsymbol{\lambda}}_t^{(i)} | \boldsymbol{X}_{1:t}, \boldsymbol{L}_{1:t})}{p(\boldsymbol{\lambda}_{1:t-1}^{(i)} | \boldsymbol{X}_{1:t-1}, \boldsymbol{L}_{1:t-1}) q(\widehat{\boldsymbol{\lambda}}_t^{(i)} | \boldsymbol{\lambda}_{1:t-1}^{(i)}, \boldsymbol{X}_{1:t}, \boldsymbol{L}_{1:t})}$$

   for each $i = 1, 2, \ldots, N_{t-1}$.

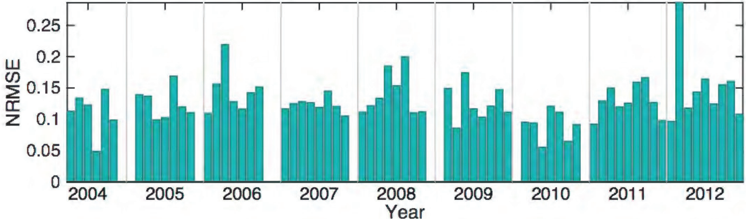
2. DWIS: Generate $(\boldsymbol{\lambda}_t, \boldsymbol{w}_t)$ from $(\widehat{\boldsymbol{\lambda}}_t, \widehat{\boldsymbol{w}}_t)$ using DWIS, with $p(\boldsymbol{\lambda}_{1:t} | \boldsymbol{X}_{1:t}, \boldsymbol{L}_{1:t})$ as the target distribution.

Predicted Prevalence of CCHF

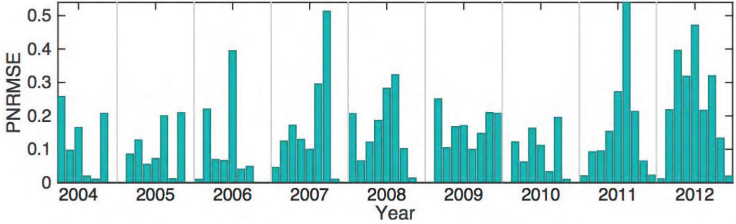


Prediction Performance

$$\text{NRMSE}_t = \sqrt{\frac{1}{73}\sum_{i=1}^{73}(Y_{it} - \widehat{\psi}_{it})^2}/\text{range}(Y_{1t}, \ldots, Y_{73t})$$



$$\text{PNRMSE}_t = \sqrt{\frac{1}{23}\sum_{i=1}^{23}(T_{it} - \widehat{\varphi}_{it})^2}/\text{range}(Y_{1t}, \ldots, Y_{73t})$$



Concluding Remarks

- The proposed model is suitable for modeling and mapping relative risk of CCHF incidents in cities of Turkey.
- It also delivers the results in a timely manner using an effective computation method DWPF.
- Estimated CCHF propagation reveals:
  - Birds and population of wild pigs in the region are suspected for a disease to move in the direction of north to south.
  - Mostly farmers, interact and share the same habitant and living space with those animals.
  - Due to economic reasons, farmers give up on precaution and do not pay attention health information.
- A timely control of the tick population and immunization of the livestock animals are also highly recommended.

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