# Removing the Gap between Annual and Sub-Annual Statistics based on Different Data Sources

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## Abstract

Benchmarking monthly or quarterly statistics to annual data is a common practice in many National Statistical Institutes. The benchmarking problem arises when time series data for the same target variable are measured at different frequencies in different data sources. One might expect that four quarterly values add up to one annual value, but because of differences in the data sources and processing methods, this is often not the case. As inconsistencies may confuse users of statistics, these are often not tolerated.

Several mathematical methods are available that remove inconsistencies between low- and high frequency data. These methods are adjustment methods that alter the high-frequency values at macro level. Traditionally, these methods are applied in the National Accounts, but these methods can also be applied to other application areas. Statistics Netherlands is currently in the process of implementing a benchmarking method for business statistics.

In this application monthly survey data have to be reconciled with quarterly VAT-register data. A well-known Denton method is planned to be used for this reconciliation process. Denton methods are very popularly applied, because of their relative simplicity. However, in a number of papers in the literature it is argued that another method has to be preferred: the Causey-Trager Growth rates preservation method (GRP). We will compare the Denton method and GRP, and examine relevant aspects of these methods for practical applications.

**Key Words:** Data integration, Benchmarking, Growth Rate Preservation

## 1. Introduction

Benchmarking monthly and quarterly series to annual data is a frequent occurring problem for many National Statistical Institutes. Benchmarking arises when data for the same target variable are measured at different frequencies.

One might expect that a temporal aggregation relationship between annual and sub annual time series is fulfilled, e.g. that four quarterly values add up to one annual value. But in practise, this is often not the case, for instance because quarterly data are available from an administrative data source and annual data from a survey. Benchmarking is the process to restore consistency.

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In this process high-frequency (e.g. monthly) values are adjusted to align with low-frequency (e.g. annual) ‘benchmarks’. Low-frequency values are fixed, as it is supposed that these data describe levels and long-term trends better than high-frequency sources. At the same time, short-term movements of high-frequency data are preserved as much as possible, as short-term statistics provide the only information on short-term change.

Benchmarking methods are often applied in the field of National Accounts. These methods can however also be applied to other application areas, like business statistics. Statistics Netherlands is currently in the process of implementing benchmarking methods for reconciling monthly and quarterly business statistics.

Several benchmarking methods are available in literature. Especially well-known are: Denton Proportionate First Differences (PFD) by Denton (1971), and Growth Rates Preservation (GRP) by Causey and Trager (1981; see also Trager, 1982, and Bozik and Otto, 1988). Denton methods are very popularly applied, because of their relative simplicity. However, in literature it is generally agreed that GRP is grounded on the strongest theoretical foundation (Bloem et al. 2001, p 100), as GRP explicitly preserves period-to-period percentage change of preliminary series.

The aim of this paper is to demonstrate that GRP suffers from a drawback that is, to the best of authors’ knowledge, not described in literature. A second aim is to present an alternative method for GRP. The current paper summarizes main findings of a forthcoming paper by Daalmans *et al.* (2016). However, contrary to the future paper, simulation results of current paper are not based on national accounts application, but on business statistics.

First, in Section 2, we will give a formal description of the Denton PFD and GRP benchmarking methods. Section 3 describes an unknown drawback of GRP. In Section 4 a new benchmarking method is proposed, that can be used as an alternative for GRP. Section 5 gives results of a business statistics application. Finally, Section 6 concludes this paper.

## 2. Temporal benchmarking methods

In this section we present a formal description of the Denton PFD and Growth Rates Preservation (GRP) benchmarking methods.
Suppose that initial high-frequency values are denoted $p_t$, $t$ = 1,…,$n$, where $n$ stands for the total number of observation of a time-series. According to GRP, benchmarked values $x_t$, $t$ = 1,…,$n$ are obtained as a solution to the following optimization problem:

$$\min_{x_t} f_F^{GRP}(\mathbf{x}) \text{ subject to } \mathbf{Ax} = \mathbf{b},$$

where

$$f_F^{GRP}(\mathbf{x}) = \sum_{t=2}^{n} \left( \frac{x_t}{x_{t-1}} - \frac{p_t}{p_{t-1}} \right)^2, \qquad (2.1)$$

The GRP criterion to be minimized, $f_F^{GRP}(\mathbf{x})$, explicitly relates to growth rates: it minimizes squared differences between growth rates of preliminary and benchmarked values. Subscript $F$ in this criterion stands for "Forward", later a "Backward" minimization function will be defined.

The linear system of equalities **Ax**=**b** contains temporal aggregation constraints, for instance that four quarterly values in a year have to sum up to the corresponding annual value. In this expression **x** is the target vector of high-frequency values, containing $x_t$, $t$ = 1,…,$n$, **b** is a vector of low-frequency values, and **A** is a temporal aggregation matrix converting high- into low-frequency values.

The Denton PFD benchmarked estimates are obtained as the solution to the following constrained quadratic minimization problem

$$\min_{x_t} f_F^{PFD}(\mathbf{x}) \text{ subject to } \mathbf{Ax} = \mathbf{b},$$

where

$$f_F^{PFD}(\mathbf{x}) = \sum_{t=2}^{n} \left( \frac{x_t}{p_t} - \frac{x_{t-1}}{p_{t-1}} \right)^2, \tag{2.2}$$

where the objective function is based on a popularly applied variant of the original Denton PFD method, as proposed by Cholette (1984). The Denton PFD criterion to be minimized, $f_F^{PFD}(\mathbf{x})$, is a sum of squared linear terms, which is easier to deal with than the nonlinear GRP objective function.

## 3. Time reversibility

Time reversibility means that it does not matter whether a method is applied forward or backward in time. The motivation of this principle is that in a benchmarking operation the direction of time does not have any naturally preferred direction.

In the context of benchmarking, time reversibility means that if we were to revert a time series, apply benchmarking, and revert the benchmarked series back again, we should get exactly the same results as benchmarking the original series. Put differently: from the benchmarked results it should not be possible to see whether benchmarking has been applied forward or backward in time.

Benchmarking a reverted time series, according to GRP and Denton PFD, respectively, comes down to minimizing the following objective functions

$$f_B^{GRP}(\mathbf{x}) = \sum_{t=2}^{n} \left( \frac{x_{t-1}}{x_t} - \frac{p_{t-1}}{p_t} \right)^2 \tag{3.1}$$

and

$$f_B^{PFD}(\mathbf{x}) = \sum_{t=2}^{n} \left( \frac{x_{t-1}}{p_{t-1}} - \frac{x_t}{p_t} \right)^2, \tag{3.2}$$

where subscript "B" stands for backwards. These objective functions are obtained from the forward objective functions by interchanging $t$ and $t$ – 1. Contrary to standard, forward benchmarking, minimization of (3.1) or (3.2) will be called 'backward benchmarking',

Denton PFD satisfies the time reversibility property, as it follows that $f_F^{PFD}(\mathbf{x}) = f_B^{PFD}(\mathbf{x})$, but GRP does not since $f_F^{GRP}(\mathbf{x}) \neq f_B^{GRP}(\mathbf{x})$.

In many practical applications “forward” benchmarking is applied, for example reconciliation of Dutch Supply and Use tables (Bikker *et al.*, 2013). However, after a revision, revised time series are constructed ‘back in time’, by using backward objective functions. It is highly undesirable that there are any differences in outcomes that can be purely explained from a difference in ‘time direction’, for instance because for non-symmetric benchmarking methods, like GRP, timing of the most important economic events, e.g. the peaks and troughs of a crisis, may be different, depending on the direction of time of a benchmarking method (see Daalmans *et al.* 2016).

## 4. Alternative benchmarking technique

In Daalmans *et al.* (2016) two alternative methods for GRP are proposed. In this section we describe one; a method called GRPS, a time-symmetric variant of growth-rate preservation. The GRPS objective function is given by

$$f_S^{GRP}(\mathbf{x}) = \sum_{t=2}^{n}\left(\frac{x_t}{x_{t-1}} - \frac{p_t}{p_{t-1}}\right)^2 + \sum_{t=2}^{n}\left(\frac{x_{t-1}}{x_t} - \frac{p_{t-1}}{p_t}\right)^2, \qquad (4.1)$$

where subscript *S* stands for ‘symmetric’.
GRPS simultaneously preserves forward and backward growth rates. It can easily be derived that GRPS satisfies time reversibility.

## 5. Empirical test

A simulation exercise is conducted to assess the impact of the time reversibility problem for a practical application.

This application deals with reconciliation of monthly and quarterly turnover for business statistics. Currently, Statistics Netherlands publishes monthly and quarterly turnover indices for industrial sectors. Statistics Netherlands is in the process of implementing a Denton method for reconciling monthly and quarterly ‘industrial’ data.

For services industries, a monthly publication does not exist. Unlike for industrial industries, a monthly survey is not available for services industries. Cost and response burden are the main barriers. There is however a monthly data source available, based on Value Added Tax (VAT) registration. But, because of selectivity problems, it was concluded that monthly VAT cannot be used as the only data source. Selectivity arises because monthly VAT reporting is compulsory for a small selective group of enterprises and voluntarily provided by a bigger (but declining) group of enterprises. Because of selectivity problems, relatively large discrepancies between monthly VAT and quarterly growth rates are observed.

In this section we consider an experimental application of benchmarking methods on Dutch Services industries data. On one hand one may doubt the usefulness of this, since benchmarking supposes that all initial discrepancies can be explained from random noise; a requirement that is obviously not satisfied. On the other hand, application of benchmarking methods to problems that require large adjustment most clearly provides insight into properties of different methods. Thus, the benchmarking process that is conducted here is suitable for educational purposes, not directly for compiling official statistics.

The used data set contains five years of monthly and quarterly turnover for 28 branches of Dutch Services Industries. Our aim is to compare the degree of forward, backward and simultaneous movement preservation between:

- Denton PFD: the standard proportionate first difference variant of Denton
- GRPF: standard forward variant of growth rates preservation
- GRPB: backward growth rate preservation
- GRPS: Simultaneous growth rate preservation, as introduced in Section 4.

Denton PFD and GPRS are time symmetric methods; GRPF and GPRB are not symmetric in time.
To compare the degree of growth rate preservation a relative criterion is used. This criterion compares an objective function value with its optimum value. It is defined as follows:

$$\text{Relative Difference (RD)} = 100\% * \frac{\text{Method} - \text{Opt}}{\text{Opt}}$$

where *Method* and *Opt* stand for the optimum objective function values for a specific benchmarking method and the best method, respectively. Please note, that as best methods for forward, backward and simultaneous movement preservation we respectively use GRPF, GPRB and GRPS.

**Table 1:** Forward movement preservation;

| *Method* | *Number of time series by category of relative distance (w.r.t. GRPF obj.)* | | | | |
|---|---|---|---|---|---|
| | *[0%, 5%)* | *[5%, 10%)* | *[10%, 20%)* | *[20%, 50%)* | *≥ 50%* |
| *Denton PFD* | 6 | 5 | 6 | 5 | 2 |
| *GRPF* | 24 | 0 | 0 | 0 | 0 |
| *GRPB* | 1 | 0 | 4 | 6 | 13 |
| *GRPS* | 4 | 5 | 6 | 6 | 3 |

**Table 2:** Backward movement preservation;

| *Method* | *Number of time series by category of relative distance (w.r.t. GRPB obj.)* | | | | |
|---|---|---|---|---|---|
| | *[0%, 5%)* | *[5%, 10%)* | *[10%, 20%)* | *[20%, 50%)* | *≥ 50%* |
| *Denton PFD* | 4 | 4 | 5 | 6 | 5 |
| *GRPF* | 1 | 1 | 3 | 8 | 11 |
| *GRPB* | 24 | 0 | 0 | 0 | 0 |
| *GRPS* | 6 | 5 | 5 | 8 | 0 |

**Table 3:** Simultaneous movement preservation;

| *Method* | *Number of time series by category of relative distance (w.r.t. GRPS obj.)* | | | | |
|---|---|---|---|---|---|
| | *[0%, 5%)* | *[5%, 10%)* | *[10%, 20%)* | *[20%, 50%)* | *≥ 50%* |
| *Denton PFD* | 18 | 3 | 0 | 0 | 3 |
| *GRPF* | 4 | 5 | 5 | 5 | 5 |
| *GRPB* | 5 | 6 | 5 | 5 | 3 |
| *GRPS* | 24 | 0 | 0 | 0 | 0 |

Tables 1-3 show that:

- GRPF poorly preserves backward movement; Denton PFD and GRPS better preserves backward movement than GRPF;
- Similarly, GRPB poorly preserves forward movement; Denton PFD and GRPS better preserves forward movement than GRPB;
- With regard to simultaneous movement preservation, Denton PFD closely approximates the optimal GRPS method. With respect to forward and backward movement preservation, the performance of GRPS and Denton PFD is almost the same.

It follows that the non-symmetric methods only perform well on the specific criterion that is intended to be optimized. (e.g. GRPF only works well for forward growth rate preservation; not for all other criteria). The time symmetric methods, Denton PFD and GPRS, perform reasonably well on all criteria. A comparison of the two time symmetric methods shows that the computational easier Denton PFD method is a very strong competitor for the optimal “GRPS-method.

The reader is referred to Daalmans *et al.* (2016) for a simulation on a larger data set, that can be considered more representative for official statistics.

## 6. Conclusions

When statistical output is compiled at different frequencies, e.g. monthly and quarterly, users of statistics may expect that a certain temporal aggregation relation is fulfilled, e.g. three monthly values that add up to one quarterly value. But, because of differences in the data sources and processing methods, consistency is not automatically accomplished. Benchmarking is an adjustment method to achieve consistency. Benchmarking is often applied in National Accounts, but the problem is also relevant for business statistics.

Two well-known benchmarking methods are Denton Proportionate First Differences (*PFD*) and Growth Rates Preservation (*GRP*). In the literature it is often mentioned that GRP has the strongest theoretical foundation. In this paper we argue however that GRP has an important drawback, namely that it does not satisfy the time reversibility property. According to this property it should not matter whether forward or backward growth rates are preserved. In other words: benchmarking an original time series, $t$=1,…,$n$, or to a ‘reverted’ time series, $t$=$n$,…,1 should lead to the same result. Since there is no preferred direction of time, any benchmarking method should satisfy the time reversibility property.

As an alternative of GRP, we propose a new method, called simultaneous growth rate preservation (GRPS), a method that preserves forward and backward growth rates at the same time. We have seen in a simulation study that GRPS' results are very well approximated by Denton PFD. Because of this, and because Denton PFD has the advantage of being simpler, Denton PFD can be advised as well for many applications.

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